Solving Linear Inequalities
Chapter Overview and Pacing

LESSON OBJECTIVES

**6-1 Solving Inequalities by Addition and Subtraction** *(pp. 318–323)*
- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

**6-2 Solving Inequalities by Multiplication and Division** *(pp. 324–331)*
*Preview:* Use algebra tiles to solve inequalities.
- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

**6-3 Solving Multi-Step Inequalities** *(pp. 332–337)*
- Solve linear inequalities involving more than one operation.
- Solve linear inequalities involving the Distributive Property.

**6-4 Solving Compound Inequalities** *(pp. 339–344)*
- Solve compound inequalities containing the word and and graph their solution sets.
- Solve compound inequalities containing the word or and graph their solution sets.

**6-5 Solving Open Sentences Involving Absolute Value** *(pp. 345–351)*
- Solve absolute value equations.
- Solve absolute value inequalities.

**6-6 Graphing Inequalities in Two Variables** *(pp. 352–358)*
- Graph inequalities on the coordinate plane.
- Solve real-world problems involving linear inequalities.
*Follow-Up:* Use a graphing calculator to investigate graphs of inequalities.

**Study Guide and Practice Test** *(pp. 359–363)*
**Standardized Test Practice** *(pp. 364–365)*

**Chapter Assessment**

**PACING (days)**

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<th>LESSON</th>
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</table>

**TOTAL** | **12** | **11** | **7** | **6**

An electronic version of this chapter is available on StudentWorks™. This backpack solution CD-ROM allows students instant access to the Student Edition, lesson worksheet pages, and web resources.
## Chapter Resource Manager

### Chapter 6 Resource Masters

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<td>379–392, 396–398</td>
</tr>
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</table>

*Key to Abbreviations:* GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual

**ELL** Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.
Solving Inequalities by Addition and Subtraction

To solve an equation, isolate the variable so that it has a coefficient of 1 on one side of the equal sign. An inequality is solved the same way. The Addition Property of Inequality is used in the same way as the Addition Property of Equality. It states that any number can be added to each side of an inequality and the result is a true inequality. The same is true for the Subtraction Property of Inequality: a number can be subtracted from each side of an inequality and the result is a true inequality.

There are infinitely many solutions to an inequality. The solutions to inequalities can be written in set builder notation, for example \( \{x \mid x > 3\} \). This is read as the set of all numbers \( x \) such that \( x \) is greater than 3.

The number found when solving an inequality is a boundary that is sometimes included in the solution and sometimes not. It is included in the solution if the inequality sign is \( \leq \) or \( \geq \), but it is not included if the symbol is \( < \) or \( > \). If the boundary number is included, a solid dot is placed at that point on the number line. If the number is not included, use an open circle. Then draw an arrow to the right if the rest of the solution set is greater than the boundary, or to the left if the rest of the solution set is less than the boundary.

Solving Inequalities by Multiplication and Division

Inequalities that include multiplication or division of the variable can also be solved. The same principles as found in the Multiplication and Division Properties of Equality are used, with one main difference. If an inequality is multiplied or divided by the same negative number on each side, the inequality symbol is reversed. The symbol must be reversed to result in a true inequality. The inequality sign is not reversed if each side is multiplied or divided by the same positive number. You multiply or divide by a negative number only if the coefficient of the variable is negative.

Solving Multi-Step Inequalities

Solve multi-step inequalities using the same process as for solving multi-step equations. Work backward using inverse operations to undo the operations. After each side is simplified using the Distributive Property and/or combining like terms, work in the opposite order of the order of operations. The Addition and Subtraction Properties of Inequality are applied first, followed by the Multiplication and Division Properties of Inequality.
If the solution is an untrue statement, such as \( 4 > 8 \), there is no solution. If the solution results in a statement that is always true, such as \( 5 > 3 \), then the solution is the set of all real numbers. A solution can always be checked by substituting it back into the inequality.

### Solving Compound Inequalities

In a compound inequality, one variable is related to two different amounts with two inequality signs. The signs may be the same or they may be different. If \( \text{and} \) is written between the inequalities, or the variable expression is between the two inequality signs, the graph is the intersection of the two inequalities. This is because the solution must be true for both inequalities. If \( \text{or} \) is written between the two inequalities, the graph is the union of the two inequalities. This is because the solution can be true for either inequality.

### Solving Open Sentences Involving Absolute Value

An absolute value open sentence can be an equation or an inequality. The value inside the absolute value symbols could be positive or negative. The absolute value represents the distance a number is from zero on a number line. Absolute value equations can be solved by graphing them or by writing them as a compound sentence and solving algebraically. To solve algebraically, write the expression inside the absolute value symbol, the \( < \) or \( > \), and the value to the right of the sign. Then write the expression, the opposite inequality sign, and the opposite value. Solve both inequalities and write the solution set as an intersection. To solve the second case you follow the same process, only write the solution set as a union using \( \text{or} \) before solving both inequalities.

### Graphing Inequalities in Two Variables

The solution set of an inequality, like that of an equation, is all ordered pairs that make the statement true. Similar to the solution set of an equation in two variables, the solution set of an inequality in two variables is graphed on a coordinate plane. However, the solution set of an inequality is not linear. It does have a linear boundary, but it covers a region called a half-plane.

First graph the inequality as if it contained an equal sign like an equation. This is the boundary line. If the inequality is \( < \) or \( > \), then the line is dashed. A solid line is graphed for \( \leq \) and \( \geq \). These relate to the circle and dot on a number line. Select a point in either half-plane and test it in the inequality. \((0, 0)\) is a good point to use if it is not on the boundary line. If the resulting statement is true, shade the half-plane that contains the point. If the statement is false, shade the other half plane.
### Chapter 6: Solving Linear Inequalities

#### Key to Abbreviations:
- TWE = Teacher Wraparound Edition
- CRM = Chapter Resource Masters

#### Type | Student Edition | Teacher Resources | Technology/Internet
--- | --- | --- | ---
**Ongoing** | Prerequisite Skills, pp. 317, 323, 331, 337, 344, 351
Practice Quiz 1, p. 331
Practice Quiz 2, p. 344 | 5-Minute Check Transparencies
*Prerequisite Skills Workbook*, pp. 79–80, 83–84
Quizzes, CRM pp. 393–394
Mid-Chapter Test, CRM p. 395
www.algebra1.com/self_check_quiz
www.algebra1.com/extra_examples

**Mixed Review** | pp. 323, 331, 337, 344, 351, 357 | Cumulative Review, CRM p. 396

**Error Analysis** | Find the Error, pp. 329, 348
Common Misconceptions, p. 326 | Find the Error, *TWE* pp. 329, 348
Unlocking Misconceptions, *TWE* pp. 321, 334
Tips for New Teachers, *TWE* pp. 323, 334

**Standardized Test Practice** | pp. 323, 328, 329, 331, 337, 343, 351, 357, 363, 364–365 | *TWE* pp. 364–365
Standardized Test Practice, *CRM* pp. 397–398 | Standardized Test Practice
CD-ROM
www.algebra1.com/standardized_test

**Open-Ended Assessment** | Writing in Math, pp. 323, 331, 337, 343, 351, 357
Open Ended, pp. 321, 328, 334, 341, 348, 355
Standardized Test, p. 365 | Modeling: *TWE* pp. 323, 351
Speaking: *TWE* pp. 337, 357
Writing: *TWE* pp. 331, 344
Open-Ended Assessment, CRM p. 391 | ExamView® Pro (see below)
MindJogger Videoquizzes
www.algebra1.com/vocabulary_review
www.algebra1.com/chapter_test

**Chapter Assessment** | Study Guide, pp. 359–362
Practice Test, p. 363 | Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 379–384
Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 385–390
Vocabulary Test/Review, CRM p. 392 | For more information on *Intervention and Assessment*, see pp. T8–T11.

### Yearly ProgressPro

#### Algebra Lesson | Yearly ProgressPro Skill Lesson(s)
--- | ---
6-1 | Solving Inequalities by Addition and Subtraction
Graphing Inequalities
6-2 | Solving Inequalities by Multiplication and Division
6-3 | Solving Multi-Step Inequalities
6-4 | Solving Compound Inequalities
Graphing Compound Inequalities
6-5 | Solving Open Sentences Involving Absolute Value
6-6 | Graphing Inequalities in Two Variables

For more information on Yearly ProgressPro, see p. 188.

### ExamView® Pro

Use the networkable ExamView® Pro to:
- Create multiple versions of tests.
- Create modified tests for Inclusion students.
- Edit existing questions and add your own questions.
- Use built-in state curriculum correlations to create tests aligned with state standards.
- Change English tests to Spanish and vice versa.
Reading and Writing in Mathematics

Glencoe Algebra 1 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

**Student Edition**
- Foldables Study Organizer, p. 317
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 321, 328, 334, 341, 348, 355)
- Reading Mathematics, p. 338
- Writing in Math questions in every lesson, pp. 323, 331, 337, 343, 351, 357
- Reading Study Tip, pp. 319, 339, 340
- WebQuest, p. 357

**Teacher Wraparound Edition**
- Foldables Study Organizer, pp. 317, 359
- Study Notebook suggestions, pp. 321, 324, 328, 334, 338, 341, 348, 355
- Modeling activities, pp. 323, 351
- Speaking activities, pp. 337, 357
- Writing activities, pp. 331, 344
- Differentiated Instruction, (Verbal/Linguistic), p. 320

**Additional Resources**
- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (Chapter 6 Resource Masters, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (Chapter 6 Resource Masters, pp. 347, 353, 359, 365, 371, 377)
- Vocabulary PuzzleMaker software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- Teaching Mathematics with Foldables provides suggestions for promoting cognition and language.
- Reading and Writing in the Mathematics Classroom
- WebQuest and Project Resources

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Taking good notes will help students become actively involved in the learning process. For each lesson, have students read and then write notes about the topic.

You may wish to show them the sample notes for Lesson 6-5 at the right to use as a guide. Afterward, allow class time for students to discuss their notes. Encourage students to talk about the procedures used in the lesson for solving the problems and how they addressed those procedures in their notes.

**Lesson 6-5**
Set up an absolute value inequality.

**Example:** $|x + 8| < 10$

**Case 1:** The value inside the absolute value symbols is less than 10.
$x + 8 < 10$

**Case 2:** The value inside the absolute value symbols is greater than $-10$.
$x - 8 > -10$
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Inequalities are used to represent various real-world situations in which a quantity must fall within a range of possible values. For example, figure skaters and gymnasts frequently want to know what they need to score to win a competition. That score can be represented by an inequality. You will learn how a competitor can determine what score is needed to win in Lesson 6-1.

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 6 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 6 test.
Prerequisite Skills  To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lessons 6-1 and 6-3  Solve Equations
Solve each equation.  (For review, see Lessons 3-2, 3-4, and 3-5.)  \(6. -19\)
1. \(t + 31 = 84\)  53  2. \(b - 17 = 23\)  40  3. \(18 = 27 + f\)  -9
4. \(m = \frac{2}{3} = \frac{1}{6}\)  5. \(3v - 45 = 4v\)  -45  6. \(5m + 7 = 4m - 12\)  4
7. \(3y + 4 = 16\)  4  8. \(7a + 5 - 3a = 4\)  1
9. \(\frac{1}{2}k - 4 = 7\)  22  10. \(4.3b + 1.8 = 8.25\)  1.5
11. \(6s - 12 = 2(s + 2)\)  12. \(n - 3 = \frac{n + 1}{2}\)  7

For Lesson 6-5  Evaluate Absolute Values
Find each value.  (For review, see Lesson 2-1.)
13. \(|-8|\)  8  14. \(|20|\)  20  15. \(|-30|\)  30  16. \(|-1.5|\)  1.5
17. \(|14 - 7|\)  7  18. \(|1 - 16|\)  15  19. \(|2 - 3|\)  1  20. \(|7 - 10|\)  3

For Lesson 6-6  Graph Equations with Two Variables
Graph each equation.  (For review, see Lesson 4-5.)  21-28. See pp. 365A-365D.
21. \(2x + 2y = 6\)  22. \(x - 3y = -3\)  23. \(y = 2x - 3\)  24. \(y = -4\)
25. \(x = -\frac{1}{2}y\)  26. \(3x - 6 = 2y\)  27. \(15 = 3(x + y)\)  28. \(2 - x = 2y\)

Solving Linear Inequalities  Make this Foldable to help you organize your notes. Begin with two sheets of notebook paper.

Step 1  Fold and Cut
Fold one sheet in half along the width. Cut along the fold from each edge to the margin.

Step 2  Fold a New Paper and Cut
Fold in half along the width. Cut along the fold between the margins.

Step 3  Fold
Insert the first sheet through the second sheet and align the folds.

Step 4  Label
Label each page with a lesson number and title.

Reading and Writing  As you read and study the chapter, fill the journal with notes, diagrams, and examples of linear inequalities.

Organization of Data and Journal Writing  After students make their Foldable journals, have them label each page to correspond to a lesson in the chapter. Students can use their Foldables to take notes, record concepts, and define terms. They can also use them to record the direction and progress of learning, to describe positive and negative experiences during learning, to write about personal associations and experiences, and to list examples of ways in which new knowledge has or will be used in their daily life.

For more information about Foldables, see *Teaching Mathematics with Foldables*.
If any number is added to each side of a true inequality, the resulting inequality is also true.

**Vocabulary**
- set-builder notation

**How are inequalities used to describe school sports?**

Ask students:
- Is the number of schools that offer volleyball greater than or less than the number of schools that offer track and field? less than
- Suppose 1200 schools added track and field, and 1200 added volleyball. Would there be more schools offering track and field, or volleyball? track and field
- Sports The kinds of sports offered in schools often reflects the sports that are popular in the region. Which sports are offered at your school? Which have the most participants?

**USA TODAY Snapshots®**

Girls gear up for high school sports

High school girls are playing sports in record numbers, almost 2.7 million in the 1999-2000 school year. Most popular girls sports by number of schools offering each program:

<table>
<thead>
<tr>
<th>Sport</th>
<th>Number of Schools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basketball</td>
<td>14,587</td>
</tr>
<tr>
<td>Volleyball</td>
<td>16,526</td>
</tr>
<tr>
<td>Track and Field</td>
<td>13,426</td>
</tr>
<tr>
<td>Softball</td>
<td>13,009</td>
</tr>
<tr>
<td>Cross Country</td>
<td>11,227</td>
</tr>
</tbody>
</table>

Source: National Federation of State High School Associations

By Ellen J. Horrow and Alejandro Gonzalez, USA TODAY

Solve linear inequalities by using addition.

**Example 1** Solve by Adding

Solve \( t - 45 \leq 13 \). Then check your solution.

\[
\begin{align*}
\text{Add } 45 \text{ to each side.} \\
\text{This means all numbers less than or equal to 58.}
\end{align*}
\]

**CHECK** Substitute 58, a number less than 58, and a number greater than 58.

<table>
<thead>
<tr>
<th>Let ( t = 58 ).</th>
<th>Let ( t = 50 ).</th>
<th>Let ( t = 60 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>58 - 45 ( \leq ) 13</td>
<td>50 - 45 ( \leq ) 13</td>
<td>60 - 45 ( \leq ) 13</td>
</tr>
<tr>
<td>13 ( \leq ) 13</td>
<td>5 ( \leq ) 13</td>
<td>15 ( \leq ) 13</td>
</tr>
</tbody>
</table>

The solution is the set of all numbers less than or equal to 58.

**Key Concept**

**Addition Property of Inequalities**

- **Words**
  - If any number is added to each side of a true inequality, the resulting inequality is also true.
- **Symbols**
  - For all numbers \( a, b, \) and \( c \), the following are true.
  1. If \( a > b \), then \( a + c > b + c \).
  2. If \( a < b \), then \( a + c < b + c \).
- **Example**
  - \( 2 + 6 < 7 + 6 \)
  - \( 8 < 13 \)

This property is also true when \( > \) and \( < \) are replaced with \( \geq \) and \( \leq \).
The solution of the inequality in Example 1 was expressed as a set. A more concise way of writing a solution set is to use *set-builder notation*. The solution in set-builder notation is \{t | t ≤ 58\}.

The solution to Example 1 can also be represented on a number line.

1. The heavy arrow pointing to the left shows that the inequality includes all numbers less than 58.
2. The dot at 58 shows that 58 is included in the inequality.

**Example 2**  
Graph the Solution

Solve \(7 < x - 4\). Then graph it on a number line.

1. \(7 < x - 4\)  
   *Original inequality*
2. \(7 + 4 < x - 4 + 4\)  
   *Add 4 to each side.*
3. \(11 < x\)  
   *Simplify.*

Since \(11 < x\) is the same as \(x > 11\), the solution set is \(\{x | x > 11\}\).

1. The circle at 11 shows that 11 is not included in the inequality.
2. The heavy arrow pointing to the right shows that the inequality includes all numbers greater than 11.

**Solve Inequalities by Subtraction**  
Subtraction can also be used to solve inequalities.

**Key Concept**  
*Subtraction Property of Inequalities*

- **Words**  
  If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

- **Symbols**  
  For all numbers \(a\), \(b\), and \(c\), the following are true.
  1. If \(a > b\), then \(a - c > b - c\).
  2. If \(a < b\), then \(a - c < b - c\).

- **Example**  
  \(17 > 8\)  
  \(17 - 5 > 8 - 5\)
  \(12 > 3\)

This property is also true when \(>\) and \(<\) are replaced with \(\geq\) and \(\leq\).

**Example 3**  
Solve by Subtracting

Solve \(19 + r \geq 16\). Then graph the solution.

1. \(19 + r \geq 16\)  
   *Original inequality*
2. \(19 + r - 19 \geq 16 - 19\)  
   *Subtract 19 from each side.*
3. \(r \geq -3\)  
   *Simplify.*

The solution set is \(\{r | r \geq -3\}\).

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**Online Lesson Plans**

USA TODAY Education’s Online site offers resources and interactive features connected to each day’s newspaper. *Experience TODAY*, USA TODAY’s daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.
Terms with variables can also be subtracted from each side to solve inequalities.

**Example 4 Variables on Both Sides**

Solve $5p + 7 > 6p$. Then graph the solution.

1. Original inequality: $5p + 7 > 6p$
2. Subtract $5p$ from each side: $7 > p$
3. Simplify: $7 > p$

Since $7 > p$ is the same as $p < 7$, the solution set is $\{p \mid p < 7\}$.

Verbal problems containing phrases like greater than or less than can often be solved by using inequalities. The following chart shows some other phrases that indicate inequalities.

<table>
<thead>
<tr>
<th>Inequalities</th>
<th>&lt;</th>
<th>&gt;</th>
<th>≤</th>
<th>≥</th>
</tr>
</thead>
<tbody>
<tr>
<td>• less than</td>
<td>• greater than</td>
<td>• at most</td>
<td>• at least</td>
<td></td>
</tr>
<tr>
<td>• fewer than</td>
<td>• more than</td>
<td>• no more than</td>
<td>• no less than</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• less than or equal to</td>
<td>• greater than or equal to</td>
<td></td>
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</tbody>
</table>

**Example 5 Write and Solve an Inequality**

Write an inequality for the sentence below. Then solve the inequality.

*Four times a number is no more than three times that number plus eight.*

1. Four times a number: $4n$
2. Three times that number: $3n$
3. Plus eight: $+ 8$

- Original inequality: $4n \leq 3n + 8$
- Subtract $3n$ from each side: $n \leq 8$
- Simplify: $n \leq 8$

The solution set is $\{n \mid n \leq 8\}$.

**Example 6 Write an Inequality to Solve a Problem**

**OLYMPICS** Yulia Raskina scored a total of 39.548 points in the four events of rhythmic gymnastics. Yulia Barsukova scored 9.883 in the rope competition, 9.900 in the hoop competition, and 9.916 in the ball competition. How many points did Barsukova need to score in the ribbon competition to surpass Raskina and win the gold medal?

- Words: Barsukova’s total must be greater than Raskina’s total.
- Variable: Let $r$ = Barsukova’s score in the ribbon competition.

**Differentiated Instruction**

**Verbal/Linguistic** If students are having difficulty choosing the correct symbol for the problem’s wording, have them use the chart above to write the common inequality phrases on index cards and the appropriate inequality symbol on the back of each card. As students solve verbal problems such as Examples 5 and 6, they can pick the card that has the same wording as the problem. The back of the card will reveal the appropriate inequality symbol to use.
Solve the inequality.

\[
\]
\[
29.699 + r > 39.548
\]
\[
\]
\[
r > 9.849
\]

Barsukova needed to score more than 9.849 points to win the gold medal.

Check for Understanding

Concept Check
1. Sample answers: \( y + 1 < -2, y - 1 < -4, y + 3 < 0 \)

Guided Practice

1. OPEN ENDED List three inequalities that are equivalent to \( y < -3 \).
2. Compare and contrast the graphs of \( a < 4 \) and \( a \leq 4 \). See margin.
3. Explain what \( |b| \geq -5 \) means. The set of all numbers \( b \) such that \( b \) is greater than or equal to \(-5\).
4. Which graph represents the solution of \( m + 3 > 7? \)
   - b. Picture of graph b.
   - c. Picture of graph c.
   - d. Picture of graph d.

Solve each inequality. Then check your solution, and graph it on a number line.
5. \( \frac{a + 4}{2} < \frac{-2}{2} \)  \( \{ a \} < -2 \)
6. \( 2b + 4 \leq \frac{5}{2} \)  \( \{ b \} \geq \frac{-5}{2} \)
7. \( t - 7 \leq 5 \)  \( \{ t \} \leq 12 \)
8. \( y - 2.5 \geq 3.1 \)  \( \{ y \} \geq 5.6 \)
9. \( 5.2r + 6.7 \geq 6.2r \)  \( \{ r \} \geq 6.7 \)
10. \( 7p \leq 6p - 2 \)  \( \{ p \} \leq -2 \)

Define a variable, write an inequality, and solve each problem. Then check your solution. 11–12. Sample answer: Let \( n = \) the number.
11. A number decreased by 8 is at most 14. \( n - 8 \leq 14; \{ n \} \leq 22 \)
12. A number plus 7 is greater than 2. \( n + 7 > 2; \{ n \} > -5 \)

Application
13. HEALTH Chapa’s doctor recommended that she limit her fat intake to no more than 60 grams per day. This morning, she ate two breakfast bars with 3 grams of fat each. For lunch she ate pizza with 21 grams of fat. If she follows her doctor’s advice, how many grams of fat can she have during the rest of the day? no more than 33 g

* indicates increased difficulty

Practice and Apply

Match each inequality with its corresponding graph.
14. \( x - 3 \geq -2 \)  d. Picture of graph d.
15. \( x + 7 \leq 6 \)  f. Picture of graph f.
16. \( 4x > 3x - 1 \)  a. Picture of graph a.
17. \( 8 + x < 9 \)  c. Picture of graph c.
18. \( 5 \leq x + 6 \)  e. Picture of graph e.
19. \( x - 1 > 0 \)  b. Picture of graph b.

Unlocking Misconceptions

Rewriting Inequalities An equation such as \( x = 5 \) can be rewritten as \( 5 = x \) because of the Symmetric Property of Equality. Because of this property, students may incorrectly assume that they can rewrite an inequality such as \( 3 > y \) as \( y > 3 \). Remind students that the inequality sign always points to the smaller value. In \( 3 > y \), it points to \( y \), so to write the expression with \( y \) on the left, use \( y < 3 \).
28. (y) y > -8
31. (w) w ≥ 1
32. (y) y < 1
33. (a) a ≤ -5
34. (h) h < 0.36
35. (x) x ≥ 0.36
36. (a) a > -1
37. (I) p ≤ 1

42. 30 ≤ n + (-8); (n) n ≥ 38
43. 2n > n + 14; (n) n > 14
44. n + (-7) ≤ 18; (n) n ≤ 25

46. BIOLOGY Adult Nile crocodiles weigh up to 2200 pounds. If a young Nile crocodile weighs 157 pounds, how many pounds might it be expected to gain in its lifetime? no more than 2043 lb

47. ASTRONOMY There are at least 200 billion stars in the Milky Way. If 1100 of these stars can be seen in a rural area without the aid of a telescope, how many stars in the galaxy cannot be seen in this way? at least 199,999,989,900 stars

48. BIOLOGY There are 3500 species of bees and more than 600,000 species of insects. How many species of insects are not bees? more than 596,500 species

49. BANKING City Bank requires a minimum balance of $1500 to maintain free checking services. If Mr. Hayashi knows he must write checks for $1300 and $947, how much money should he have in his account before writing the checks? at least $3747

50. GEOMETRY The length of the base of the triangle at the right is less than the height of the triangle. What are the possible values of x? more than 8 in.

51. SHOPPING Terrell has $65 to spend at the mall. He bought a T-shirt for $18 and a belt for $14. If Terrell still wants to buy a pair of jeans, how much can he spend on the jeans? no more than $33

52. SOCCER The Centerville High School soccer team plays 18 games in the season. The team has a goal of winning at least 60% of its games. After the first three weeks of the season, the team has won 4 games. How many more games must the team win to meet their goal? at least 7 more games
53. CRITICAL THINKING Determine whether each statement is always, sometimes, or never true.
   a. If \(a < b\) and \(c < d\), then \(a + c < b + d\). always
   b. If \(a < b\) and \(c < d\), then \(a + c \geq b + d\). never
   c. If \(a < b\) and \(c < d\), then \(a - c = b - d\). sometimes

**HEALTH** For Exercises 54 and 55, use the following information.
Hector’s doctor told him that his cholesterol level should be below 200. Hector’s cholesterol is 225.
54. Let \(p\) represent the number of points Hector should lower his cholesterol. Write an inequality with \(225 - p\) on one side. \(225 - p < 200\)
55. Solve the inequality. \((p|p > 25)\)

56. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See pp. 365A–365D.
   How are inequalities used to describe school sports?
   Include the following in your answer:
   • an inequality describing the number of schools needed to add girls’ track and field so that the number is greater than the number of schools currently participating in girls’ basketball.

57. Which inequality is not equivalent to \(x \leq 12\)? C
   \(A\) \(x - 7 \leq 5\) \(B\) \(x + 4 \leq 16\) \(C\) \(x - 1 \leq 13\) \(D\) \(12 \geq x\)

58. Which statement is modeled by \(n + 6 \geq 5?\) A
   \(A\) The sum of a number and six is at least five.
   \(B\) The sum of a number and six is at most five.
   \(C\) The sum of a number and six is greater than five.
   \(D\) The sum of a number and six is no greater than five.

**Maintain Your Skills**

**Mixed Review**

59. Would a scatter plot for the relationship of a person’s height to the person’s grade on the last math test show a positive, negative, or no correlation? (Lesson 5-7)

Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of each equation. (Lesson 5-6)
60. \((1, -3); y = 3x - 2\) \(61. (0, 4); x + y = -3\) \(62. (-1, 2); 2x - y = 1\)
   \(y = 3x - 6\) \(y = -x + 4\) \(y = 2x + 4\)

Find the next two terms in each sequence. (Lesson 4-8)
63. \(7, 13, 19, 25, \ldots\) \(64. 243, 81, 27, 9, \ldots\) \(65. 3, 6, 12, 24, \ldots\)
   \(31, 37\) \(64. 243, 81, 27, 9, \ldots\) \(65. 3, 6, 12, 24, \ldots\)

Solve each equation if the domain is \([-1, 3, 5]\). (Lesson 4-4)
66. \(y = -2x\) \(67. y = 7 - x\) \(68. 2x - y = 6\)

69. \(6x = 42\) \(70. \frac{1}{9} = 14\) \(71. \frac{2}{3}y = 14\) \(72. 3m = 435\)

73. \(\frac{4}{7}x = 28\) \(74. 5.3x = 11.13\) \(75. \frac{a}{35} = 7\) \(76. 8p = 35\)

**Open-Ended Assessment**

**Modeling** Draw a large number line on an overhead transparency. Using a washer or a coin, and a ribbon or a strip of paper, graph several different inequalities on the overhead. Use the coin to indicate a dot on the number line and the washer to indicate a circle. The ribbon is the ray representing points greater or less than the starting value. Have students identify the inequalities you graph, identifying the significance of the circle or dot.

**Tips for New Teachers**

Students should always check their solutions, but they often hurry to finish their assignments and omit this step. Remind students that checking solutions is especially important with inequalities because the direction of the inequality sign often gets changed when writing solutions in set-builder notation.

**Getting Ready for Lesson 6-2**

**PREREQUISITE SKILL** In Lesson 6-2, students learn how to solve inequalities using multiplication and division. The process is almost identical to the process for solving equations using multiplication and division. Use Exercises 69–76 to determine your students’ familiarity with solving these types of equations.
### Algebra Activity

#### A Preview of Lesson 6-2

**Getting Started**

**Objective** Use algebra tiles to solve inequalities.

**Materials**
- algebra tiles
- equation mat
- self-adhesive notes

**Teach**

1. You may wish to do the example as a demonstration.
2. Make sure the inequality sign on the self-adhesive note is pointed in the correct direction to match the inequality.
3. Once they have isolated the $x$ tiles, remind students to separate the 1 tiles into equal groups to correspond to the number of $x$ tiles.
4. If the $x$ tiles end up on the right side of the inequality, students may rotate the mat 180 degrees to read the inequality with the variable on the left side.

**Assess**

Have students work in small groups for Exercises 1–7. Observe to determine if they are able to verbalize the activities in Exercises 1–4. Students should conclude after Exercises 6–7 that when they multiply or divide both sides of an inequality by a negative number, the direction of the inequality sign changes.

### Solve $-2x \geq 6$.

**Step 1** Model the inequality.

Use a self-adhesive note to cover the equals sign on the equation mat. Then write a $\geq$ symbol on the note. Model the inequality.

**Step 2** Remove zero pairs.

Since you do not want to solve for a negative $x$ tile, eliminate the negative $x$ tiles by adding 2 positive $x$ tiles to each side. Remove the zero pairs.

**Step 3** Remove zero pairs.

Add 6 negative 1 tiles to each side to isolate the $x$ tiles. Remove the zero pairs.

**Step 4** Group the tiles.

Separate the tiles into 2 groups.

Model and Analyze

Use algebra tiles to solve each inequality.

1. $-4x < 12 \quad \{x \mid x > -3\}$
2. $-2x > 8 \quad \{x \mid x < -4\}$
3. $-3x \geq -6 \quad \{x \mid x \leq 2\}$
4. $-5x \leq -5 \quad \{x \mid x \geq 1\}$

5. In Exercises 1–4, is the coefficient of $x$ in each inequality positive or negative? **negative**

6. Compare the inequality symbols and locations of the variable in Exercises 1–4 with those in their solutions. What do you find? 6–7. See pp. 365A–365D.

7. Model the solution for $2x \geq 6$. What do you find? How is this different from solving $-2x \geq 6$?
**Solving Inequalities by Multiplication and Division**

**What You’ll Learn**
- Solve linear inequalities by using multiplication.
- Solve linear inequalities by using division.

**Why are inequalities important in landscaping?**
Isabel Franco is a landscape architect. To beautify a garden, she plans to build a decorative wall of either bricks or blocks. Each brick is 3 inches high, and each block is 12 inches high. Notice that $3 < 12$.

A wall 4 bricks high would be lower than a wall 4 blocks high.

\[ 3 \times 4 \quad ? \quad 12 \times 4 \]
\[ 12 < 48 \]

**SOLVE INEQUALITIES BY MULTIPLICATION**
If each side of an inequality is multiplied by a positive number, the inequality remains true.

\[
\begin{align*}
8 \quad &\quad ? \quad 5 \\
8(2) \quad &\quad ? \quad 5(2) \\
16 \quad &\quad > \quad 10 \\
5(4) \quad &\quad ? \quad 9(4) \\
20 \quad &\quad < \quad 36 \\
\end{align*}
\]

This is not true when multiplying by negative numbers.

\[
\begin{align*}
5 \quad &\quad ? \quad 3 \\
5(-2) \quad &\quad ? \quad 3(-2) \\
-10 \quad &\quad < \quad -6 \\
-6(-5) \quad &\quad ? \quad 8(-5) \\
30 \quad &\quad > \quad -40 \\
\end{align*}
\]

If each side of an inequality is multiplied by a negative number, the direction of the inequality symbol changes. These examples illustrate the **Multiplication Property of Inequalities**.

**Key Concept**

**Multiplying by a Positive Number**

- **Words** If each side of a true inequality is multiplied by the same positive number, the resulting inequality is also true.
- **Symbols** If $a$ and $b$ are any numbers and $c$ is a positive number, the following are true:
  - If $a > b$, then $ac > bc$, and if $a < b$, then $ac < bc$.

---

**Resource Manager**

**Workbook and Reproducible Masters**
- **Chapter 6 Resource Masters**
  - Study Guide and Intervention, pp. 349–350
  - Skills Practice, p. 351
  - Practice, p. 352
  - Reading to Learn Mathematics, p. 353
  - Enrichment, p. 354
  - Assessment, p. 393

- **Parent and Student Study Guide Workbook**, p. 47
- **School-to-Career Masters**, p. 12

**Transparencies**
- 5-Minute Check Transparency 6-2
- Answer Key Transparencies

**Technology**
- Interactive Chalkboard
SOLVE INEQUALITIES BY MULTIPLICATION

**Teaching Tip** Remind students that \( \frac{b}{7} \) is the same as \( \frac{1}{7}b \). To isolate \( b \), multiply by the reciprocal of \( \frac{1}{7} \), which is 7.

1. Solve \( \frac{8}{3} < 12 \). Then check your solution. \( \{g \mid g < 36\} \)
2. Solve \( -\frac{3}{4}d \geq 6 \). \( \{d \mid d \leq -8\} \)
3. Write an inequality for the sentence below. Then solve the inequality. Four-fifths of a number is at most twenty. \( \frac{4}{5}r \leq 20 \). \( \{r \mid r \leq 25\} \)

### Key Concept

**Multiplying by a Negative Number**

- **Words** If each side of a true inequality is multiplied by the same negative number, the direction of the inequality symbol must be reversed so that the resulting inequality is also true.
- **Symbols** If \( a \leq b \) are any numbers and \( c \) is a negative number, the following are true:
  - if \( a > b \), then \( ac < bc \), and if \( a < b \), then \( ac > bc \).

This property also holds for inequalities involving \( \geq \) and \( \leq \).

You can use this property to solve inequalities.

#### Example 1

**Multiply by a Positive Number**

Solve \( \frac{b}{7} \geq 25 \). Then check your solution.

\[
\begin{align*}
\frac{b}{7} & \geq 25 & \text{Original inequality} \\
(7) \frac{b}{7} & \geq (7)25 & \text{Multiply each side by 7. Since we multiplied by a positive number, the inequality symbol stays the same.} \\
b & \geq 175 & \text{Let } b = 175. \quad \text{Let } b = 140. \quad \text{Let } b = 210. \\
\frac{175}{7} & \geq 25 & \frac{140}{7} & \geq 25 & \frac{210}{7} & \geq 25 \\
25 \geq 25 & \checkmark & 20 \geq 25 & \checkmark & 30 \geq 25 & \checkmark \\
\end{align*}
\]

The solution set is \( \{b \mid b \geq 175\} \).

#### Example 2

**Multiply by a Negative Number**

Solve \( -\frac{2}{5}p < -14 \).

\[
\begin{align*}
-\frac{2}{5}p & < -14 & \text{Original inequality} \\
\left( -\frac{5}{2} \right) \left( -\frac{2}{5}p \right) & > \left( -\frac{5}{2} \right) (-14) & \text{Multiply each side by } -\frac{5}{2} \text{ and change } < \text{ to } >. \\
p & > 35 & \text{The solution set is } \{p \mid p > 35\}. \\
\end{align*}
\]

#### Example 3

**Write and Solve an Inequality**

Write an inequality for the sentence below. Then solve the inequality.

One fourth of a number is less than \( -7 \).

\[
\begin{align*}
\frac{1}{4} \times n & < -7 & \text{Original inequality} \\
\frac{1}{4}n & < -7 \\
(4)\frac{1}{4}n & < (4)(-7) & \text{Multiply each side by 4 and do not change the inequality’s direction.} \\
n & < -28 & \text{The solution set is } \{n \mid n < -28\}. \\
\end{align*}
\]
Solve Inequalities by Division

Dividing each side of an inequality by the same number is similar to multiplying each side of an equality by the same number. Consider the inequality $6 < 15$.

Divide each side by 3. Divide each side by $-3$.

\[
\begin{align*}
6 &< 15 \\
6 \div 3 &< 15 \div 3 \\
2 &< 5 \\
6 \div (-3) &< 15 \div (-3) \\
-2 &> -5
\end{align*}
\]

Since each side is divided by a positive number, the direction of the inequality symbol remains the same.

Since each side is divided by a negative number, the direction of the inequality symbol is reversed.

These examples illustrate the **Division Property of Inequalities**.

**Key Concept**

**Dividing by a Positive Number**

- **Words**: If each side of a true inequality is divided by the same positive number, the resulting inequality is also true.
- **Symbols**: If $a$ and $b$ are any numbers and $c$ is a positive number, the following are true.
  
  $\text{if } a > b \text{, then } \frac{a}{c} > \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

**Dividing by a Negative Number**

- **Words**: If each side of a true inequality is divided by the same negative number, the direction of the inequality symbol must be reversed so that the resulting inequality is also true.
- **Symbols**: If $a$ and $b$ are any numbers and $c$ is a negative number, the following are true.
  
  $\text{if } a > b \text{, then } \frac{a}{c} < \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

This property also holds for inequalities involving $\leq$ and $\geq$.

**Example 4** Divide by a Positive Number

Solve $14h > 91$.

$14h > 91$ Original inequality

$\frac{14h}{14} > \frac{91}{14}$ Divide each side by 14 and do not change the direction of the inequality sign.

$h > 6.5$

**CHECK**

<table>
<thead>
<tr>
<th>$h = 6.5$</th>
<th>$h = 7$</th>
<th>$h = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$14h &gt; 91$</td>
<td>$14h &gt; 91$</td>
<td>$14h &gt; 91$</td>
</tr>
<tr>
<td>$14(6.5) &gt; 91$</td>
<td>$14(7) &gt; 91$</td>
<td>$14(6) &gt; 91$</td>
</tr>
<tr>
<td>$91 &gt; 91$</td>
<td>$98 &gt; 91$</td>
<td>$84 &gt; 91$</td>
</tr>
</tbody>
</table>

The solution set is $\{h | h > 6.5\}$.

Since dividing is the same as multiplying by the reciprocal, there are two methods to solve an inequality that involve multiplication.

**Differentiated Instruction**

**Kinesthetic** Have students write an inequality involving a negative coefficient of the variable on their paper, using a self-adhesive note for the inequality symbol, such as $-12x > 24$. Tell them that they are going to change all the signs in the inequality, so everything is its opposite. The expression becomes $-12x < -24$. Students now can divide without having to worry about the inequality sign.
5 Solve \(-8q < 136\) using two methods. \(q \mid q > -17\)

Teaching Tip Another way to check the solution is to rework the problem using a different method.

6 Which inequality does not have the solution \(\{x \mid x > 6\}\)? B
   A. \(-2x < -12\)
   B. \(-6x > -72\)
   C. \(\frac{5}{6}x > 5\)
   D. \(-\frac{1}{8}x < -\frac{3}{4}\)

Example 5 Divide by a Negative Number

Solve \(-5t \geq 275\) using two methods.

Method 1 Divide.
\[-5t \geq 275\]
\[t \leq -55\] Simplify.

Method 2 Multiply by the multiplicative inverse.
\[-5t \geq 275\]
\[\left(\frac{1}{-5}\right)(-5t) \leq \left(\frac{1}{-5}\right)275\] Multiply each side by \(-\frac{1}{5}\) and change \(\geq\) to \(\leq\).
\[t \leq -55\] Simplify.

The solution set is \(\{t \mid t \leq -55\}\).

You can use the Multiplication Property and the Division Property for Inequalities to solve standardized test questions.

Example 6 The Word “not”

Multiple-Choice Test Item

Which inequality does not have the solution \(\{y \mid y \leq -5\}\)?

A. \(-7y \geq 35\)
B. \(2y \leq -10\)
C. \(\frac{7}{5}y \geq -7\)
D. \(-\frac{y}{4} \geq \frac{5}{4}\)

Read the Test Item
You want to find the inequality that does not have the solution set \(\{y \mid y \leq -5\}\).

Solve the Test Item
Consider each possible choice.

A. \(-7y \geq 35\)
\[-\frac{7}{7}y \leq \frac{35}{7}\]
\[y \leq 5\] The answer is C.

Check for Understanding

1. Explain why you can use either the Multiplication Property of Inequalities or the Division Property of Inequalities to solve \(-7r \leq 28\). See margin.

2. OPEN ENDED Write a problem that can be represented by the inequality \(\frac{3}{4}x > 9\).

Answer

1. You could solve the inequality by multiplying each side by \(-\frac{1}{7}\) or by dividing each side by \(-7\). In either case, you must reverse the direction of the inequality symbol.

Standardized Test Practice

Example 6 Some questions can be answered without solving each equation or inequality given. Have students examine each inequality in Example 6 to determine what inequality sign should be included in the solution set without working it out. For A and D, the sign becomes \(\leq\). For B, it stays \(\leq\). C is the remaining choice.
3. **Guided Practice**

Ilonia and Zachary are solving $-9b \leq 18$.

![Inequality Solution]

Who is correct? Explain your reasoning.

4. Which statement is represented by $7n \geq 14$? **a**
   a. Seven times a number is at least 14.
   b. Seven times a number is at most 14.
   c. Seven times a number is less than 14.
   d. Seven times a number is greater than 14.

5. Which inequality represents five times a number is less than 25? **c**
   a. $5n > 25$
   b. $5n \leq 25$
   c. $5n < 25$
   d. $5n \geq 25$

Solve each inequality. Then check your solution.

6. $-15x > 75$
7. $\frac{1}{9} < -12$
8. $-\frac{2}{3}b \leq -9$
9. $25f \geq 9$

Define a variable, write an inequality, and solve each problem. Then check your solution. **10–11. Sample answer:** Let $n =$ the number.

10. The opposite of four times a number is more than 12. $-4n > 12$; $n | n < -3$
11. Half of a number is at least 26. $\frac{1}{2}n \geq 26$; $(n | n \geq 52)$

12. Which inequality does not have the solution set $\{x | x > 4\}$? **B**
   
   $\begin{align*}
   &\text{A: } -5x < -20 \\
   &\text{B: } 6x < 24 \\
   &\text{C: } \frac{1}{3}x > \frac{4}{5} \\
   &\text{D: } -\frac{3}{4}x < -3
   \end{align*}$

**Practice and Apply**

Match each inequality with its corresponding statement.

13. $\frac{1}{5}n > 10$ **d**
   a. Five times a number is less than or equal to ten.
   b. One fifth of a number is no less than ten.
   c. Five times a number is greater than ten.
   d. Five times a number is greater than or equal to ten.

14. $5n \leq 10$ **a**
15. $5n > 10$ **e**
16. $-5n < 10$ **f**
17. $\frac{1}{5}n \geq 10$ **b**
18. $5n < 10$ **c**

Solve each inequality. Then check your solution. **19–34. See margin.**

19. $6x \leq 144$
20. $7t > 84$
21. $-14d \geq 84$
22. $-16z \leq -64$
23. $\frac{m}{5} \geq 7$
24. $\frac{b}{10} \leq 5$
25. $-\frac{r}{7} < -7$
26. $-\frac{a}{11} > 9$
27. $\frac{5}{8}y \geq -15$
28. $\frac{2}{3}u < 6$
29. $-\frac{3}{4}g \leq -33$
30. $-\frac{2}{5}p > 10$
31. $-2.5w < 6.8$
32. $-0.8s > 6.4$
33. $\frac{15c}{-7} > \frac{3}{14}$
34. $\frac{4m}{5} < -\frac{3}{15}$

**Answers**

19. $|g| \leq 24$
20. $|t| > 12$
21. $|d| \leq -6$
22. $|z| \geq 4$
23. $|m| \geq 35$
24. $|b| \leq 50$
25. $|r| > 49$
26. $|a| < -99$
27. $|y| \geq -24$
28. $|v| < 9$
29. $|q| \geq 44$
30. $|p| < -25$
31. $|w| > -2.72$
32. $|s| < -8$
33. $c \geq -\frac{1}{10}$
34. $m \geq -\frac{1}{4}$
Solve Inequalities by Multiplication

1. Multiplication Property of Inequalities
   - If the same positive number is multiplied, the resulting inequality is the same.
   - If the same negative number is multiplied, the direction of the inequality must be reversed for the resulting inequality to be true.

Example

\[
\begin{align*}
\text{Given:} & \quad \frac{3}{4}x > 12 \\
\text{Solution:} & \quad x > 16
\end{align*}
\]

Solve \( \frac{3}{4}x < 15 \).

Example

\[
\begin{align*}
\text{Given:} & \quad 2n < 12 \\
\text{Solution:} & \quad n < 6
\end{align*}
\]

Solve \( 4n < 15 \).

Example

\[
\begin{align*}
\text{Given:} & \quad 5n < 30 \\
\text{Solution:} & \quad n < 6
\end{align*}
\]

Solve \( 6n < 36 \).

Example

\[
\begin{align*}
\text{Given:} & \quad \frac{1}{6}x < 2 \\
\text{Solution:} & \quad x < 12
\end{align*}
\]

Solve \( \frac{1}{6}x > 12 \).

Example

\[
\begin{align*}
\text{Given:} & \quad n - 30; \quad \{n \in N \} \\
\text{Solution:} & \quad n < 30
\end{align*}
\]

Solve \( n - \frac{3}{4} \).

Example

\[
\begin{align*}
\text{Given:} & \quad \{n \in N \} \\
\text{Solution:} & \quad \{n \in N \}
\end{align*}
\]

Solve \( \{n \in N \} \).

Example

\[
\begin{align*}
\text{Given:} & \quad \{n \in N \} \\
\text{Solution:} & \quad \{n \in N \}
\end{align*}
\]

Solve \( \{n \in N \} \).

Example

\[
\begin{align*}
\text{Given:} & \quad n - 30; \quad \{n \in N \} \\
\text{Solution:} & \quad n < 30
\end{align*}
\]

Solve \( n - \frac{3}{4} \).

Example

\[
\begin{align*}
\text{Given:} & \quad \{n \in N \} \\
\text{Solution:} & \quad \{n \in N \}
\end{align*}
\]

Solve \( \{n \in N \} \).

Example

\[
\begin{align*}
\text{Given:} & \quad n - 30; \quad \{n \in N \} \\
\text{Solution:} & \quad n < 30
\end{align*}
\]

Solve \( n - \frac{3}{4} \).

Example

\[
\begin{align*}
\text{Given:} & \quad \{n \in N \} \\
\text{Solution:} & \quad \{n \in N \}
\end{align*}
\]

Solve \( \{n \in N \} \).

Example

\[
\begin{align*}
\text{Given:} & \quad n - 30; \quad \{n \in N \} \\
\text{Solution:} & \quad n < 30
\end{align*}
\]

Solve \( n - \frac{3}{4} \).

Example

\[
\begin{align*}
\text{Given:} & \quad \{n \in N \} \\
\text{Solution:} & \quad \{n \in N \}
\end{align*}
\]

Solve \( \{n \in N \} \).

Example

\[
\begin{align*}
\text{Given:} & \quad n - 30; \quad \{n \in N \} \\
\text{Solution:} & \quad n < 30
\end{align*}
\]

Solve \( n - \frac{3}{4} \).

Example

\[
\begin{align*}
\text{Given:} & \quad \{n \in N \} \\
\text{Solution:} & \quad \{n \in N \}
\end{align*}
\]

Solve \( \{n \in N \} \).
54. **CIVICS** For a candidate to run for a county office, he or she must submit a petition with at least 6000 signatures of registered voters. Usually only 85% of the signatures are valid. How many signatures should a candidate seek on a petition? At least 7059 signatures.

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See margin.

**Why are inequalities important in landscaping?**

Include the following in your answer:
- an inequality representing a brick wall that can be no higher than 4 feet, and
- an explanation of how to solve the inequality.

56. The solution set for which inequality is not represented by the following graph?

![Graph](image)

(A) \( \frac{-x}{5} \leq 1 \)  
(B) \( \frac{x}{5} \leq -1 \)  
(C) \( -9x \leq 45 \)  
(D) \( 2.5x \leq -12.5 \)

57. Solve \( \frac{-7}{8} t < \frac{14}{15} \). C

(A) \( t > \frac{16}{5} \)  
(B) \( |t| < \frac{16}{15} \)  
(C) \( |t| > \frac{16}{15} \)  
(D) \( |t| < \frac{16}{15} \)

**Maintain Your Skills**

**Mixed Review** Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)* 58–60. See margin for graphs.

58. \( s - 7 < 12 \)  
59. \( g + 3 \leq -4 \)  
60. \( 7 > n + 2 \)  

**61.** Draw a scatter plot that shows a positive correlation. *(Lesson 5-7)*

**Writing** Have students write a one-paragraph summary of what they think is the most important thing to remember about solving inequalities by multiplication or division. Students will likely suggest that the most important thing to remember is to change the direction of the inequality symbol when multiplying or dividing by negative numbers. Ask student volunteers to read their paragraphs to the class.

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. *(Lesson 3-6)*

62. \( x + 3 = 2x \)  
63. \( y = 2 \)  
64. \( y = \frac{1}{3}x + 9 \)

65. Solve each proportion. *(Lesson 3-6)*

66. \( h(2) = 8 \)  
67. \( h(w) = 3w + 2 \)  
68. \( h(r - 6) = 3r - 16 \)

**Practice Quiz 1** The quiz provides students with a brief review of the concepts and skills in Lessons 6-1 and 6-2. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

**Quiz (Lessons 6-1 and 6-2)** is available on p. 393 of the *Chapter 6 Resource Masters.*

**Answers**

55. Inequalities can be used to compare the heights of walls. Answers should include the following.
- If \( x \) represents the number of bricks and the wall must be no higher than 4 ft or 48 in., then \( 3x \leq 48 \).
- To solve this inequality, divide each side by 3 and do not change the direction of the inequality. The wall must be 16 bricks high or fewer.

56. See pp. 365A–365D for graphs.

Solve each inequality. Then check your solution, and graph it on a number line. *(Lesson 6-1)*

1. \( h - 16 > -13 \)  
2. \( r + 3 \leq -1 \)  
3. \( 4 \geq p + 9 \)  
4. \( -3 < a - 5 \)  
5. \( 7g \leq 6g - 1 \)

Solve each inequality. Then check your solution. *(Lesson 6-2)*

6. \( 15z \geq 105 \)  
7. \( \frac{9}{5} < 7 \)  
8. \( -\frac{3}{7} y > 15 \)  
9. \( -156 < 12r \)  
10. \( -\frac{2}{5} w \leq -\frac{1}{2} \)

58. \( 15 16 17 18 19 20 21 22 23 \)

59. \( -8 -7 -6 -5 -4 -3 -2 -1 0 \)

60. \( 0 1 2 3 4 5 6 7 8 \)
1 Focus

5-Minute Check Transparency 6-3 Use as a quiz or a review of Lesson 6-2.

Mathematical Background notes are available for this lesson on p. 316C.

Building on Prior Knowledge

Solving multi-step inequalities is no different from solving multi-step equations except when multiplying or dividing by a negative value. Students should understand that they don’t have to learn a whole new process, but just a special rule when using negatives.

How are linear inequalities used in science?

Ask students:

• What would the inequality \( F < -31 \) represent? The temperatures at which chlorine is not a gas.

• What expression was substituted for \( F \) to represent the temperature of the boiling point of chlorine in degrees Celsius? \( \frac{9}{5}C + 32 \)

What You’ll Learn

• Solve linear inequalities involving more than one operation.
• Solve linear inequalities involving the Distributive Property.

How are linear inequalities used in science?

The boiling point of a substance is the temperature at which the element changes from a liquid to a gas. The boiling point of chlorine is \(-31^\circ F\). That means chlorine will be a gas for all temperatures greater than \(-31^\circ F\). If \( F \) represents temperature in degrees Fahrenheit, the inequality \( F > -31 \) represents the temperatures for which chlorine is a gas.

If \( C \) represents degrees Celsius, then \( F = \frac{9}{5}C + 32 \). You can solve \( \frac{9}{5}C + 32 > -31 \) to find the temperatures in degrees Celsius for which chlorine is a gas.

SOLVE MULTI-STEP INEQUALITIES The inequality \( \frac{9}{5}C + 32 > -31 \) involves more than one operation. It can be solved by undoing the operations in the same way you would solve an equation with more than one operation.

Example 1 Solve a Real-World Problem

SCIENCE Find the temperatures in degrees Celsius for which chlorine is a gas.

\[
\frac{9}{5}C + 32 > -31 \quad \text{Original inequality}
\]

\[
\frac{9}{5}C + 32 - 32 > -31 - 32 \quad \text{Subtract 32 from each side.}
\]

\[
\frac{9}{5}C > -63 \quad \text{Simplify.}
\]

\[
\left(\frac{5}{9}\right)\left(\frac{9}{5}C\right) > \left(\frac{5}{9}\right)(-63) \quad \text{Multiply each side by} \ \frac{5}{9}.
\]

\[
C > -35 \quad \text{Simplify.}
\]

Chlorine will be a gas for all temperatures greater than \(-35^\circ C\).

When working with inequalities, do not forget to reverse the inequality sign whenever you multiply or divide each side by a negative number.

Example 2 Inequality Involving a Negative Coefficient

Solve \(-7b + 19 < -16\). Then check your solution.

\[
-7b + 19 < -16 \quad \text{Original inequality}
\]

\[
-7b + 19 - 19 < -16 - 19 \quad \text{Subtract 19 from each side.}
\]

\[
-7b < -35 \quad \text{Simplify.}
\]

\[
\frac{-7b}{-7} > \frac{-35}{-7} \quad \text{Divide each side by} \ -7 \text{ and change} < \text{ to} > .
\]

\[
b > 5 \quad \text{Simplify.}
\]
CHECK To check this solution, substitute 5, a number less than 5, and a number greater than 5.

Let \( b = 5 \).
- \(-7b + 19 < -16\)
- \(-7(5) + 19 < -16\)
- \(-35 + 19 < -16\)
- \(-16 < -16\)

Let \( b = 4 \).
- \(-7b + 19 < -16\)
- \(-7(4) + 19 < -16\)
- \(-28 + 19 < -16\)
- \(-9 < -16\)

Let \( b = 6 \).
- \(-7b + 19 < -16\)
- \(-7(6) + 19 < -16\)
- \(-42 + 19 < -16\)
- \(-23 < -16\)

The solution set is \( \{b \mid b > 5\} \).

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.

Three times a number minus eighteen is at least five times the number plus twenty-one.

<table>
<thead>
<tr>
<th>Three times a number</th>
<th>minus</th>
<th>eighteen</th>
<th>is at least</th>
<th>five times the number</th>
<th>plus</th>
<th>twenty one</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3n)</td>
<td>(-)</td>
<td>(18)</td>
<td>(\geq)</td>
<td>(5n)</td>
<td>(+)</td>
<td>(21)</td>
</tr>
</tbody>
</table>

\[3n - 18 \geq 5n + 21\]

Add 18 to each side.
\[-2n \geq 39\]

Divide each side by \(-2\) and change to \(\leq\).
\[n \leq -19.5\]

A graphing calculator can be used to solve inequalities.

Teaching Tip Examples 1 and 2 were solved in two steps. Example 3 requires three steps. Other problems will require even more steps. Remind students that when they solve multi-step inequalities, they should always undo operations in the reverse of the order of operations. This means undoing addition or subtraction first to isolate the variable, then multiplying or dividing to make the coefficient 1.

3. \(x < 2;\ y = 1\) for those values of \(x\) for which the inequality is true; \(y = 0\) for those values of \(x\) for which the inequality is not true.

www.algebra1.com/extra_examples/sol

Example 3 requires three steps. Two were solved in two steps. Other problems will require even more steps. Remind students that when they solve multi-step inequalities, they should always undo operations in the reverse of the order of operations. This means undoing addition or subtraction first to isolate the variable, then multiplying or dividing to make the coefficient 1.

3 Write an inequality for the sentence below. Then solve the inequality. Four times a number plus twelve is less than a number minus three.

\(4n + 12 < n - 3; \ (n | n < -5)\)
SOLVE INEQUALITIES INVOLVING THE DISTRIBUTIVE PROPERTY

In-Class Examples

4 Solve \(-8 - (c + 3) \leq 6c + 3(2 - c). \{c | c \geq -\frac{1}{4}\}

5 Solve \(-7(s + 4) + 11s \geq 8s - 2(2s + 1). \emptyset

Concept Check

Distributive Property Ask students to identify the first step they must do when solving inequalities that have grouping symbols. Use the Distributive Property to remove the grouping symbols.

Practice/Apply

Guided Practice

3. Justify each indicated step.

\[
\begin{align*}
3(a - 7) + 9 &\leq 21 \\
3a - 21 + 9 &\leq 21 \\
3a &\leq 21 \\
3a - 12 - 12 &\leq 21 - 12 \\
3a - 24 &\leq 9 \\
3a &\leq 33 \\
3 &\leq 33 \\
a &\leq 11
\end{align*}
\]

Check for Understanding

1. Compare and contrast the method used to solve \(-5h + 6 = -7\) and the method used to solve \(-5h + 6 \leq -7\). See margin.

2. OPEN ENDED Write a multi-step inequality with the solution graphed below. Sample answer: \(2x + 4 > 2\)

3. Justify each indicated step.

a. \(?\) Distributive Property

b. \(?\) Add 12 to each side.

c. \(?\) Divide each side by 3.

DAILY INTERVENTION

Unlocking Misconceptions

Students may incorrectly assume that the solution of all inequalities in which the variable has been eliminated is the empty set \(\emptyset\). Remind students that they must simplify the inequality to see whether it is a true statement. If the inequality is true, the solution set is all real numbers. Only when the inequality is untrue is the solution set the empty set.
Solve each inequality. Then check your solution.

4. \(-4y - 23 < 19\)
5. \(\frac{2}{3}r + 9 \geq 3\)
6. \(7b + 11 > 9b - 13\)
7. \(-5(q + 4) > 3(q - 4)\)
8. \(3 + 5t \leq 3(t - 1) - 4(2 - t)\)

9. Define a variable, write an inequality, and solve the problem below. Then check your solution.

\[7 \text{ minus two times a number is less than three times the number plus thirty-two.}\]

Sample answer: Let \(n\) be the number; \(7 - 2n < 3n + 32; |n| n > -5\).

10. SALES A salesperson is paid \$22,000 a year plus 5% of the amount of sales made. What is the amount of sales needed to have an annual income greater than \$35,000? more than \$260,000

**Application**

**Practice and Apply**

**Justify each indicated step.**

11. \[\frac{2}{5}w + 7 \leq -9\]
12. \[m > \frac{15 - 2m}{3}\]

**Answers**

1. To solve both the equation and the inequality, you first subtract 6 from each side and then divide each side by -5. In the equation, the equal sign does not change. In the inequality, the inequality symbol is reversed because you divided by a negative number.

12a. Multiply each side by \(-3\) and change > to <.
12b. Add \(2m\) to each side.
12c. Multiply each side by \(-1\) and change < to >.

13. \(4(t - 7) \leq 2(t + 9)\)

**Differentiated Instruction**

**Interpersonal** Some students benefit from working with a partner so that they can talk through the process being used. Group students in pairs to solve inequalities. Once both students agree on the solution, have them test several values to help verify that their solution is correct.
MUSIC PRACTICE

A number is less than one fourth the sum of three times the number and four. Solution.

10. Solve each inequality. Then check your solution.

How are linear inequalities used in science?

To solve linear inequalities involving more than one variable, follow these steps:

1. Use the Distributive Property to remove any grouping symbols.
2. Combine like terms.
3. Add or subtract the same variable terms or constants on both sides.
4. Multiply or divide to undo operations.
5. Reverse the direction of the inequality symbol if both sides were multiplied or divided by a negative number.
6. Be sure the variable is by itself on one side of the final inequality.

LABOR

For Exercises 48–50, use the following information.

A union worker made $500 per week. His union sought a one-year contract and went on strike. Once the new contract was approved, it provided for a 4% raise.

48. Assume that the worker was not paid during the strike. Given his raise in salary, how many weeks could he strike and still make at least as much for the next 52 weeks as he would have made without a strike? no more than 2 weeks

49. How would your answer to Exercise 48 change if the worker had been making $600 per week? no change

50. How would your answer to Exercise 48 change if the worker’s union provided him with $150 per week during the strike? up to 2.8 weeks

51. NUMBER THEORY

Find all sets of two consecutive positive odd integers whose sum is no greater than 18. 7, 9; 5, 7; 3, 5; 1, 3

52. NUMBER THEORY

Find all sets of three consecutive positive even integers whose sum is less than 40. 10, 12, 14; 8, 10, 12; 6, 8, 10; 4, 6, 8; 2, 4, 6

Enrichment, p. 360

Carlos Montezuma

During his lifetime, Carlos Montezuma (1867–1912) was one of the most influential Native Americans in the United States. He was not only a spiritual leader for his tribe, the Apache, but also a leader in the struggle for the rights of Native American peoples. The Yuma people, who differ from the Apache in language and culture, were the primary enemies of the Apache. Montezuma was born in the state of Arizona. He was a Yuma-Apache person. In 1856, the Apache were forced to move to a reservation in Wyoming. Montezuma and his people lived in a small village near the river. In 1881, Montezuma was killed by a group of Apache who had been led by Geronimo. Montezuma’s death helped to end the Apache wars. The Yuma people were later allowed to return to their homeland. Montezuma’s life is remembered for his dedication to his people and his efforts to preserve their culture. Montezuma taught his people how to make pottery and how to live in harmony with nature. He also taught them the Apache religion, which includes the use of sacred plants and the performance of dances. Montezuma was a leader in the Apache nation and was respected by all of his people. He died in 1912, but his legacy lives on through his books and through the Apache people who continue to honor his memory. Montezuma’s life is considered a model of leadership and a symbol of strength and courage. His death is remembered as a loss for the Apache people and for all of the Native American peoples. Montezuma’s legacy is still celebrated today, and his books continue to inspire his people and people around the world.
53. **Writing in Math**  Answer the question that was posed at the beginning of the lesson.  **See margin.**

How are linear inequalities used in science?
Include the following in your answer:
- an inequality for the temperatures in degrees Celsius for which bromine is a gas, and
- a description of a situation in which a scientist might use an inequality.

54. What is the first step in solving \( \frac{y - 5}{9} \geq 13 \)?  **D**  
   **A**  Add 5 to each side.  
   **B**  Subtract 5 from each side.  
   **C**  Divide each side by 9.  
   **D**  Multiply each side by 9.

55. Solve \( |t| + 2 < 8t - (6t - 10) \).  **C**  
   **A**  \( |t| < -6 \)  
   **B**  \( |t| > -6 \)  
   **C**  \( |t| < 4 \)  
   **D**  \( |t| > 4 \)

56. \( 3x + 7 > 4x + 9 \)  
57.  
58. \( 2(x - 3) < 3(2x + 2) \)

\( x < -2 \)  
\( x \leq 8 \)  
\( x > -3 \)

**Maintain Your Skills**

59. **Business**  The charge per mile for a compact rental car at Great Deal Rentals is $0.12. Mrs. Ludlow must rent a car for a business trip. She has a budget of $50 for mileage charges. How many miles can she travel without going over her budget?  **(Lesson 6-2)** up to 416 mi

Solve each inequality. Then check your solution, and graph it on a number line.  **(Lesson 6-1)** 60–62. See margin for graphs.

60. \( d + 13 \geq 22 \)  
61. \( t - 5 < 3 \)  
62. \( 4 > y + 7 \)

\( |t| < 8 \)
\( |t| < -3 \)

**Graphing Calculator**

Use a graphing calculator to solve each inequality.

63.  
64.  
65.  
66.  
67.  
68.  

\( |t| < -6 \)
\( |t| > -6 \)
\( |t| < 4 \)
\( |t| > 4 \)

**Mixed Review**

Write the point-slope form of an equation for a line that passes through each point with the given slope.  **(Lesson 5-5)**

63. \( (1, -3), m = 2 \)  
64. \( (-2, -1), m = -\frac{2}{5} \)  
65. \( (3, 6), m = 0 \)

\( y + 3 = 2(x - 1) \)
\( y + 1 = -\frac{2}{5}(x + 2) \)
\( y - 6 = 0 \)

Determine the slope of the line that passes through each pair of points.  **(Lesson 5-1)**

66. \( (3, -1), (4, -6) \)  
67. \( (-2, -4), (1, 3) \)  
68. \( (0, 3), (-2, -5) \)

Determine whether each equation is a linear equation. If an equation is linear, rewrite it in the form \( Ax + By = C \).  **(Lesson 4-5)**

69. \( 4x = 7 + 2y \)  
70. \( 2x^2 - y = 7 \)  
71. \( x = 12 \) yes; \( x + 0y = 12 \) yes; \( 4x - 2y = 7 \)

Solve each equation. Then check your solution.  **(Lesson 3-5)**

72. \( 2(x - 2) = 3x - (4x - 5) \)  
73. \( 5t - 7 = t + 3 \)

3 2.5

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**  Graph each set of numbers on a number line.  **(To review graphing integers on a number line, see Lesson 2-1)** 74–82. See pp. 365A–365D.

74. \( \{-2, 3, 5\} \)
75. \( \{-1, 0, 3, 4\} \)
76. \( \{-5, -4, -1, 1\} \)
77. \{integers less than 5\}
78. \{integers greater than -2\}
79. \{integers between 1 and 6\}
80. \{integers between -4 and 2\}
81. \{integers greater than or equal to -4\}
82. \{integers less than 6 but greater than -1\}

**Answers**

53. Inequalities can be used to describe the temperatures for which an element is a gas or a solid. Answers should include the following.
- The inequality for temperatures in degrees Celsius for which bromine is a gas is \( \frac{9}{5}C + 32 > 138 \).
- Sample answer: Scientists may use inequalities to describe the temperatures for which an element is a solid.
Discuss the meaning of the adjective compound. Ask students the meaning of the word and to give an example of something that is compound.

Review the polygons listed on this page. In order to determine whether compound statements are true, students must be familiar with the number of sides each polygon has.

**Sentence Structure** Ask students to recall the definition of a compound sentence from their language arts studies. Students should recall that a compound sentence has two independent clauses that are joined by a coordinating conjunction, punctuation, or both.

Explain that the compound statements in this activity are compound sentences in which the two independent clauses are joined by the coordinating conjunctions and or or.

**Getting Started**

**Teach**

**Teach**

**Assess**

<table>
<thead>
<tr>
<th>Study Notebook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ask students to summarize what they have learned about compound statements.</td>
</tr>
</tbody>
</table>

**ELL** English Language Learners may benefit from writing key concepts from this activity in their Study Notebooks in their native language and then in English.

**Reading Mathematics**

**Compound Statements**

Two simple statements connected by the words and or or form a compound statement. Before you can determine whether a compound statement is true or false, you must understand what the words and and or mean. Consider the statement below.

A triangle has three sides, and a hexagon has five sides.

For a compound statement connected by the word and to be true, both simple statements must be true. In this case, it is true that a triangle has three sides. However, it is false that a hexagon has five sides; it has six. Thus, the compound statement is false.

A compound statement connected by the word or may be exclusive or inclusive. For example, the statement “With your dinner, you may have soup or salad,” is exclusive. In everyday language, or means one or the other, but not both. However, in mathematics, or is inclusive. It means one or the other or both. Consider the statement below.

A triangle has three sides, or a hexagon has five sides.

For a compound statement connected by the word or to be true, at least one of the simple statements must be true. Since it is true that a triangle has three sides, the compound statement is true.

**Reading to Learn**

Determine whether each compound statement is true or false. Explain your answer. 1–12. See margin for explanations.

1. A hexagon has six sides, or an octagon has seven sides. **true**
2. An octagon has eight sides, and a pentagon has six sides. **false**
3. A pentagon has five sides, and a hexagon has six sides. **true**
4. A triangle has four sides, or an octagon does not have seven sides. **true**
5. A pentagon has three sides, or an octagon has ten sides. **false**
6. A square has four sides, or a hexagon has six sides. **true**
7. 5 < 4 or 8 < 6 **false**
8. \( -1 > 0 \) and \( 1 < 5 \) **false**
9. 4 > 0 and \( -4 < 0 \) **true**
10. 0 = 0 or \( -2 > -3 \) **true**
11. 5 ≠ 5 or \( -1 > -4 \) **true**
12. 0 > 3 and 2 > -2 **false**

**Answers**

1. true or false
2. true and false
3. true and true
4. false or true
5. false or false
6. true or true
7. false or false
8. false and true
9. true and true
10. true or true
11. false or true
12. false and true
Solving Compound Inequalities

What You’ll Learn

• Solve compound inequalities containing the word and and graph their solution sets.
• Solve compound inequalities containing the word or and graph their solution sets.

How are compound inequalities used in tax tables?

Richard Kelley is completing his income tax return. He uses the table to determine the amount he owes in federal income tax.

2002 Tax Tables

<table>
<thead>
<tr>
<th>If taxable income is</th>
<th>Single</th>
<th>Married filing jointly</th>
<th>Married filing separately</th>
<th>Head of a household</th>
</tr>
</thead>
<tbody>
<tr>
<td>At least $41,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$41,050</td>
<td>7423</td>
<td>5554</td>
<td>7975</td>
<td>6083</td>
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<td>6191</td>
</tr>
<tr>
<td>$41,500</td>
<td>7558</td>
<td>5629</td>
<td>8096</td>
<td>6204</td>
</tr>
<tr>
<td>$41,550</td>
<td>7571</td>
<td>5636</td>
<td>8110</td>
<td>6218</td>
</tr>
<tr>
<td>$41,600</td>
<td></td>
<td></td>
<td></td>
<td>6231</td>
</tr>
</tbody>
</table>

Source: IRS

Let c represent the amount of Mr. Kelley’s income. His income is at least $41,350 and it is less than $41,400. This can be written as $41,350 ≤ c < $41,400. When considered together, these two inequalities form a compound inequality. This compound inequality can be written without using and in two ways:

$41,350 ≤ c < $41,400 or $41,400 > c ≥ $41,350

Inequalities containing AND A compound inequality containing and is true only if both inequalities are true. Thus, the graph of a compound inequality containing and is the intersection of the graphs of the two inequalities. In other words, the solution must be a solution of both inequalities.

The intersection can be found by graphing each inequality and then determining where the graphs overlap.

Example 1 Graph an Intersection

Graph the solution set of $x < 3$ and $x ≥ -2$.

The solution set is $\{x | -2 ≤ x < 3\}$. Note that the graph of $x ≥ -2$ includes the point -2. The graph of $x < 3$ does not include 3.
In-Class Examples

**Teaching Tip** The symbol for intersection is \( \cap \). The solution set in Example 1 could be written as \( \{x \mid x < 3\} \cap \{x \mid x \geq -2\} \).

1. Graph the solution set of \( y \geq 5 \) and \( y < 12 \). The solution set is \( y \in [5, 12) \).

**Teaching Tip** Students may benefit from rewriting the compound inequality as two separate inequalities before they attempt to solve it.

2. Solve \( 7 < z + 2 \leq 11 \). Then graph the solution set. \( \{z \mid 5 < z \leq 9\} \).

**Career Choices**

**AVIATION** An airplane is experiencing heavy turbulence while flying at 30,000 feet. The control tower tells the pilot that he should increase his altitude to at least 33,000 feet or decrease his altitude to no more than 26,000 feet to avoid the turbulence. Write and graph a compound inequality that describes the altitude at which the airplane should fly.

**Words** The pilot has been told to fly at an altitude of at least 33,000 feet or no more than 26,000 feet.

**Variables** Let \( a \) be the plane’s altitude.

**Inequality**

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Plane’s altitude is at least 33,000 feet or</th>
<th>The altitude is no more than 26,000 feet.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \geq 33,000 ) or ( a \leq 26,000 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, graph the solution set.

**Differentiated Instruction**

**Visual/Spatial** Prepare a large number line with two dashed horizontal lines above it as a transparency or laminated sheet of paper. Give the students a compound inequality written as two statements. Have students use a dry-erase marker to plot the solution for each inequality on one of the dashed lines. If the inequality involves “and,” have them wipe away any parts that are not on both dashed lines. This leaves a clear picture of what part of the number line should be used for the solution.
**Example 4 Solve and Graph a Union**

Solve \(-3h + 4 < 19\) or \(7h - 3 > 18\). Then graph the solution set.

\[-3h + 4 < 19\] \hspace{1cm} \[7h - 3 > 18\]

\[-3h < 15\] \hspace{1cm} \[7h > 21\]

\[-h < -5\] \hspace{1cm} \[h > 3\]

The solution set is the union of the two graphs.

Graph \(h > -5\).

Graph \(h > 3\).

Find the union.

Notice that the graph of \(h > -5\) contains every point in the graph of \(h > 3\). So, the union is the graph of \(h > -5\). The solution set is \(\{h \mid h > -5\}\).

---

**Check for Understanding**

**Concept Check**

1. Describe the difference between a compound inequality containing and and a compound inequality containing or. See margin.

2. Write \(7\) is less than \(t\), which is less than \(12\) as a compound inequality. \(7 < t < 12\)

3. OPEN ENDED Give an example of a compound inequality containing and that has no solution. Sample answer: \(x < -2\) and \(x > 3\)

**Guided Practice**

**GUIDED PRACTICE KEY**

<table>
<thead>
<tr>
<th>Exercises</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>4–7</td>
<td>1</td>
</tr>
<tr>
<td>8–12</td>
<td>2, 4</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
</tr>
</tbody>
</table>

8–11. See margin for graphs.

8. \(\{w\mid 3 < w < 8\}\)

9. \(\{n\mid n \leq 2\) or \(n \geq 8\}\)

**Application**

13. **PHYSICAL SCIENCE** According to Hooke’s Law, the force \(F\) in pounds required to stretch a certain spring \(x\) inches beyond its natural length is given by \(F = 4.5x\). If forces between 20 and 30 pounds, inclusive, are applied to the spring, what will be the range of the increased lengths of the stretched spring? **About 4.44 \(\leq x \leq 6.67**
Graph the solution set of each compound inequality. 14–19. See margin.

14. \( x > 5 \) and \( x \leq 9 \)

15. \( s < -7 \) and \( s \leq 0 \)

16. \( r < 6 \) or \( r > 6 \)

17. \( m \geq -4 \) or \( m > 6 \)

18. \( 7 < d \leq 11 \)

19. \(-1 \leq g < 3 \)

Write a compound inequality for each graph.

20. \[
\begin{cases}
-5 & < x < 3 \\
0 & < x < 9
\end{cases}
\]

21. \[
\begin{cases}
-10 & < x < 3 \\
0 & < x < 9
\end{cases}
\]

22. \[
\begin{cases}
10 & < x < 13 \\
12 & < x < 15
\end{cases}
\]

23. \[
\begin{cases}
-10 & < x < 3 \\
0 & < x < 9
\end{cases}
\]

24. \[
\begin{cases}
-9 & < x < -4 \\
0 & < x < 9
\end{cases}
\]

25. \[
\begin{cases}
10 & < x < 19 \\
0 & < x < 9
\end{cases}
\]

26. **WEATHER** The Fujita Scale (F-scale) is the official classification system for tornado damage. One factor used to classify a tornado is wind speed. Use the information in the table to write an inequality for the range of wind speeds of an F3 tornado. \( 158 \leq w \leq 206 \)

27. **BIOLOGY** Each type of fish thrives in a specific range of temperatures. The optimum temperatures for sharks range from 18°C to 22°C, inclusive. Write an inequality to represent temperatures where sharks will not thrive. \( t \geq 18 \) or \( t \leq 22 \)

Solve each compound inequality. Then graph the solution set.

28. \( k + 2 > 12 \) and \( k + 2 \leq 18 \)

29. \( f + 8 \leq 3 \) and \( f + 9 \geq -4 \)

30. \( d - 4 > 3 \) or \( d - 4 \leq 1 \)

31. \( h - 10 < -21 \) or \( h + 3 < 2 \)

32. \( 3 < 2x - 3 < 15 \)

33. \( 4 < 2y - 2 < 10 \)

34. \( 3f - 7 = 5 \) and \( 2t + 6 = 12 \)

35. \( 8 > 5 - 3q \) and \( 5 - 3q > -13 \)

36. \( -1 < x < 3 \) or \( -1 < x < -4 \)

37. \( 3u + 11 \leq 13 \) or \( -3n \geq -12 \)

38. \( 2p - 2 \leq 4p - 8 \leq 3p - 3 \)

39. \( 3x + 12 \leq 6 \) or \( 3x - 18 \geq 7 \)

40. \( 4c - 2 < 10 \) or \( -3c < -12 \)

41. \( 0.3b > -6 \) or \( 3b + 16 < -8 + b \)

42. Eight less than a number is no more than 14 and no less than 5.

43. The sum of 3 times a number and 4 is between –8 and 10.

44. The product of –5 and a number is greater than 35 or less than 10.

45. One half a number is greater than 0 and less than or equal to 1.

46. **HEALTH** About 20% of the time you sleep is spent in rapid eye movement (REM) sleep, which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep? between 1.4 and 1.6 hours inclusive.

47. **SHOPPING** A store is offering a $30 mail-in rebate on all color printers. Luisana is looking at different color printers that range in price from $175 to $260. How much can she expect to spend after the mail-in rebate? between $145 and $230 inclusive.

**Extra Practice**
See page 834.
48. **FUND-RAISING** Rashid is selling chocolates for his school’s fund-raiser. He can earn prizes depending on how much he sells. So far, he has sold $70 worth of chocolates. How much more does he need to sell to earn a prize in category D? between $51 and $110 inclusive.

<table>
<thead>
<tr>
<th>Sales ($)</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–25</td>
<td>A</td>
</tr>
<tr>
<td>26–60</td>
<td>B</td>
</tr>
<tr>
<td>61–120</td>
<td>C</td>
</tr>
<tr>
<td>121–180</td>
<td>D</td>
</tr>
<tr>
<td>180+</td>
<td>E</td>
</tr>
</tbody>
</table>

49. **CRITICAL THINKING** Write a compound inequality that represents the values of \( x \) which make the following expressions false.
   a. \( x < 5 \) or \( x > 8 \) \( x \geq 5 \) and \( x \leq 8 \)
   b. \( x \leq 6 \) and \( x \geq 1 \) \( x > 6 \) or \( x < 1 \)

50. Write a compound inequality for the hearing range of humans and one for the hearing range of dogs. \( 20 \leq h \leq 20,000 \); \( 15 \leq d < 50,000 \)

51. What is the union of the two solution sets? the intersection?
   \( h \leq 20 \leq h \leq 20,000 \); \( h \leq 50,000 \)

52. Write an inequality or inequalities for the range of sounds that dogs can hear, but humans cannot. \( 15 \leq h < 20 \) or \( 20,000 < h \leq 50,000 \)

53. **RESEARCH** Use the Internet or other resource to find the altitudes in miles of the layers of Earth’s atmosphere, troposphere, stratosphere, mesosphere, thermosphere, and exosphere. Write inequalities for the range of altitudes for each layer. Sample answer: troposphere: \( a \leq 10 \); stratosphere: \( 10 < a \leq 30 \); mesosphere: \( 30 < a \leq 50 \); thermosphere: \( 50 < a \leq 400 \); exosphere: \( a > 400 \)

54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 365A–365D.

**How are compound inequalities used in tax tables?**
Include the following in your answer:
- A description of the intervals used in the tax table shown at the beginning of the lesson.
- A compound inequality describing the income of a head of a household paying $7024 in taxes.

55. Ten pounds of fresh tomatoes make between 10 and 15 cups of cooked tomatoes. How many cups does one pound of tomatoes make? A \( \frac{5}{2} \) cups B \( \frac{5}{2} \) cups C \( \frac{5}{6} \) cups D \( \frac{3}{2} \) cups E \( \frac{3}{2} \) cups

56. Solve \(-7 < x < 2 \). A \( -5 < x < 6 \) B \( -9 < x < 2 \) C \( -5 < x < 2 \)

57. **SOLVE COMPOUND INEQUALITIES** In Lesson 6-3, you learned how to use a graphing calculator to find the values of \( x \) that make a given inequality true. You can also use this method to test compound inequalities. The words and or or can be found in the LOGIC submenu of the TEST menu of a TI-83 Plus. Use this method to solve each of the following compound inequalities using your graphing calculator.
   \( a. |x| < -6 \) or \( x > 1 \) \( b. |x| = -2 \leq x \leq 8 \)
   \( a. x + 4 < -2 \) or \( x > 3 \)
   \( b. x \leq -3 \) or \( x > 6 \)

**Enrichment, p. 366**

**Some Properties of Inequalities**
The two interpretations of the order of the inequality are sometimes called the first and second members of the inequality.

If the inequality symbol in a compound inequality points in the same direction, the inequality has the property of the same direction.

If the inequality symbol in a compound inequality points in the opposite direction, the inequality has the property of the opposite direction.

The problem on this page will help you explore some properties of inequalities.

Three of the four statements below are true for all numbers \( a \) and \( b \). \( a, b, c, \) and \( d \). Write each statement in algebraic form. If the statement is true for all numbers, prove it. If it is not true, give an example to show that it fails.

1. If \( a \) is an inequality, and \( c \) is an inequality, then \( a \) and \( c \) can be combined by subtracting the numbers and reversing the symbols.

2. If \( a > b \), then \( a - c > b - c \).

3. If \( a < b \), then \( a + c < b + c \).

4. If \( a > b \), then \( a - c < b - c \).

5. If \( a > b \), then \( a + c > b + c \).

6. If \( a < b \), then \( a + c < b + c \).

7. If \( a > b \), then \( a - c < b - c \).

8. If \( a < b \), then \( a - c > b - c \).

9. If \( a > b \), then \( a + c > b + c \).

10. If \( a < b \), then \( a - c > b - c \).
Open-Ended Assessment

Writing Have students write a paragraph comparing and contrasting compound inequalities containing and with inequalities containing or. The paragraph should contain examples of the different types of inequalities and their graphs.

Getting Ready for Lesson 6-5

PREREQUISITE SKILL Students will learn how to solve open sentences involving absolute value in Lesson 6-5. A good grasp of what absolute value means will help them understand how this concept applies to expressions in open sentences. Use Exercises 73–80 to determine your students’ familiarity with finding absolute values.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 6-3 and 6-4. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Answers

7. 

8. 

9. 

10. 

Maintain Your Skills

Mixed Review

58. FUND-RAISING A university is running a drive to raise money. A corporation has promised to match 40% of whatever the university can raise from other sources. How much must the school raise from other sources to have a total of at least $800,000 after the corporation’s donation? (Lesson 6-3) at least $371,426.57

Solve each inequality. Then check your solution. (Lesson 6-2) 

59. $18d > 90$ 60. $-7b < 91$ 61. $\frac{t}{13} < 13$ 62. $-\frac{3}{8}b > 9$

Solve. Assume that $y$ varies directly as $x$. (Lesson 5-2) 

63. If $y = -8$ when $x = -3$, find $x$ when $y = 6$. 2.25

64. If $y = 2.5$ when $x = 0.5$, find $y$ when $x = 20$. 100

Express the relation shown in each mapping as a set of ordered pairs. Then state the domain, range, and inverse. (Lesson 4-3) 

65. 

66. 

67. 

Find the odds of each outcome if a die is rolled. (Lesson 2-6) 

68. A number greater than 2:1 69. not a 3:1

Find each product. (Lesson 2-3) 

70. $-\frac{5}{6}(\frac{-8}{5})\frac{1}{3}$ 71. $-100(4.7)$ 72. $-\frac{7}{12}(\frac{3}{7})\frac{3}{8}$

PREREQUISITE SKILL Find each value. (To review absolute value, see Lesson 2-1.) 

73. $|7|$ 74. $|10|$ 75. $|-1|$ 76. $|-3.5|$ 77. $|12 - 6|$ 78. $|5 - 9|$ 79. $|20 - 21|$ 80. $|3 - 18|$ 81. $|15|

Practice Quiz 2 Lessons 6-3 and 6-4 Solve each inequality. Then check your solution. (Lesson 6-3) 

1. $5 - 4b > -23 \quad (b \| b < 7)$

3. $3(t + 6) < 9 \quad (t \| t < -3)$

5. $2m + 5 \leq 4m - 1 \quad (m \| m \geq 3)$

6. $a < \frac{2a - 15}{3} \quad (a \| a < -15)$

Solve each compound inequality. Then graph the solution set. (Lesson 6-4) 

7. $x - 2 < 7$ and $x + 2 > 5 \quad (x \| 3 < x < 9)$

8. $2b + 5 \leq -1 \quad (b \| b \leq -3 \quad b \geq 0)$

9. $4m - 5 > 7 \quad (m \| m > 3 \quad m < -1)$

10. $a - 4 < 1 \quad (a \| a < 5)$

11. $a + 2 > 1 \quad (a \| 1 < a < 5)$

344 Chapter 6 Solving Linear Inequalities
**What You’ll Learn**

- Solve absolute value equations.
- Solve absolute value inequalities.

**How is absolute value used in election polls?**

Voters in Hamilton will vote on a new tax levy in the next election. A poll conducted before the election found that 47% of the voters surveyed were for the tax levy, 45% were against the tax levy, and 8% were undecided. The poll has a 3-point margin of error.

The margin of error means that the result may be 3 percentage points higher or lower. So, the number of people in favor of the tax levy may be as high as 50% or as low as 44%. This can be written as an inequality using absolute value.

\[ |x - 47| \leq 3 \]

The difference between the actual number and 47 is within 3 points.

**ABSOLUTE VALUE EQUATIONS**

There are three types of open sentences that can involve absolute value.

\[ |x| = n \quad |x| < n \quad |x| > n \]

Consider the case of \[ |x| = n \]. \[ |x| = 5 \] means the distance between 0 and \( x \) is 5 units.

If \[ |x| = 5 \], then \( x = -5 \) or \( x = 5 \). The solution set is \( \{-5, 5\} \).

When solving equations that involve absolute value, there are two cases to consider.

**Case 1** The value inside the absolute value symbols is positive.

**Case 2** The value inside the absolute value symbols is negative.

Equations involving absolute value can be solved by graphing them on a number line or by writing them as a compound sentence and solving it.
Example 1 Solve an Absolute Value Equation

Solve \(|a - 4| = 3\).

Method 1 Graphing

\(|a - 4| = 3\) means the distance between \(a\) and 4 is 3 units. To find \(a\) on the number line, start at 4 and move 3 units in either direction.

The solution set is \([1, 7]\).

Method 2 Compound Sentence

Write \(|a - 4| = 3\) as \(a - 4 = 3\) or \(a - 4 = -3\).

Case 1 Case 2

\[
\begin{align*}
    a - 4 &= 3 \\
    a - 4 + 4 &= 3 + 4 \\
    a &= 7
\end{align*}
\]

Add 4 to each side.

\[
\begin{align*}
    a - 4 &= -3 \\
    a - 4 + 4 &= -3 + 4 \\
    a &= 1
\end{align*}
\]

Simplify.

The solution set is \([1, 7]\).

Example 2 Write an Absolute Value Equation

Write an equation involving absolute value for the graph.

Find the point that is the same distance from 3 as the distance from 9. The midpoint between 3 and 9 is 6.

So, an equation is \(|x - 6| = 3\).

CHECK Substitute 3 and 9 into \(|x - 6| = 3\).

\[
\begin{align*}
    |x - 6| &= 3 \\
    |3 - 6| &= 3 \\
    |9 - 6| &= 3 \\
    |-3| &= 3 \quad |3| = 3 \\
    3 &= 3 \quad 3 = 3
\end{align*}
\]

ABSOLUTE VALUE INEQUALITIES Consider the inequality \(|x| < n\).

\(|x| < 5\) means that the distance from 0 to \(x\) is less than 5 units.

Therefore, \(x > -5\) and \(x < 5\). The solution set is \([-5 < x < 5]\).

DAILY INTERVENTION

Differentiated Instruction

Logical Some students may respond better to rewriting absolute value equations by applying the two situations (positive and negative) to the expression within the absolute value symbols. For example, \(|x| = 4\) can be written as \(x = 4\) or \(-x = 4\), which yields \(x = -4\). Example 1 can be written as \(a - 4 = 3\) or \(-(a - 4) = 3\). Then students can solve each equation.
The Algebra Activity explores an inequality of the form \(|x| < n\).

### Algebra Activity

#### Absolute Value

**Collect the Data**
- Work in pairs. One person is the timekeeper.
- Start timing. The other person tells the timekeeper to stop timing after he or she thinks that one minute has elapsed.
- Write down the time in seconds.
- Switch places. Make a table that includes the results of the entire class.

**Analyze the Data**
1. See students' work.
2. Determine the error by subtracting 60 seconds from each student's time.
3. What does a negative error represent? a positive error?
4. What is the absolute value of the error? Since absolute value cannot be negative, the absolute error is positive. If the absolute error is 6 seconds, write two possibilities for a student's estimated time of one minute. 54 s or 66 s
5. Graph the responses and highlight all values such that \(|60 - x| < 6\). How many guesses were within 6 seconds? See students' work.

When solving inequalities of the form \(|x| < n\), find the intersection of these two cases.

**Example 3** Solve an Absolute Value Inequality (<)

Solve \(|t + 5| < 9\). Then graph the solution set.

Write \(|t + 5| < 9\) as \(t + 5 < 9\) and \(t + 5 > -9\).

**Case 1** The value inside the absolute value symbols is less than the positive value of \(n\).

|t + 5| < 9

\(t + 5 < 9\)  Subtract 5 from each side.
\(t < 4\)  Simplify.

**Case 2** The value inside the absolute value symbols is greater than the negative value of \(n\).

\(t + 5 > -9\)

\(t + 5 - 5 > -9 - 5\)  Subtract 5 from each side.
\(t > -14\)  Simplify.

The solution set is \(\{t \mid -14 < t < 4\}\).

Consider the inequality \(|x| > n\). \(|x| > 5\) means that the distance from 0 to \(x\) is greater than 5 units.

Therefore, \(x < -5\) or \(x > 5\). The solution set is \(\{x \mid x < -5\ \text{or} \ x > 5\}\).

### Building on Prior Knowledge

Before presenting Example 3, remind students that the two cases presented represent a compound inequality involving and, which they learned to solve in Lesson 6-4.

#### Teaching Tip

Students may be confused about why \(|t + 5| < 9\) is rewritten as \(t + 5 < 9\) and \(t + 5 > -9\). An alternative method is to rewrite the inequality as \(t + 5 < 9\) and \(-(t + 5) < 9\). Then multiply each side of the second inequality by \(-1\) to yield \(t + 5 > -9\). This method makes the switch of the direction of the inequality more obvious, as students must make the switch when they divide each side by \(-1\).

### In-Class Example

**Example** Solve \(|s - 3| \leq 12\). Then graph the solution set. \(\{-9 \leq s \leq 15\}\)

-5 -4 -3 -2 -1 0 1 2 3 4 5 6

-15 -10 -5 0 5 10 15

---

**Study Tip**

Less Than

When an absolute value is on the left and the inequality symbol is < or \(\leq\), the compound sentence uses and.

**Absolute Value Inequalities**

| Lesson 6-5 Solving Open Sentences Involving Absolute Value | 347 |

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**Algebra Activity**

**Materials:** clock or watch that displays seconds
- Explain to students that the purpose of this activity is not to see which student can guess closest to the length of a minute, but to collect data for the rest of the activity.
- If students do not understand why the error cannot be negative, refer them to the example involving election poll results at the beginning of the lesson.
Solve an Absolute Value Inequality (>)

When solving inequalities of the form \( |x| > n \), find the union of these two cases.

**Case 1**  The value inside the absolute value symbols is greater than the positive value of \( n \).

**Case 2**  The value inside the absolute value symbols is less than the negative value of \( n \).

Example 4  Solve an Absolute Value Inequality (>)

Solve \( |2x + 8| \geq 6 \). Then graph the solution set.

Write \( |2x + 8| \geq 6 \) as \( 2x + 8 \geq 6 \) or \( 2x + 8 \leq -6 \).

**Case 1**

\[
2x + 8 \geq 6
\]
\[
2x \geq -2
\]
\[
x \geq -1
\]

**Case 2**

\[
2x + 8 \leq -6
\]
\[
2x \leq -14
\]
\[
x \leq -7
\]

The solution set is \( x \geq -1 \) or \( x \leq -7 \).

In general, there are three rules to remember when solving equations and inequalities involving absolute value.

**Concept Summary**

| \(|x| = n\) | \(x = -n\) or \(x = n\).
| \(|x| < n\) | \(x < -n\) and \(x > -n\).
| \(|x| > n\) | \(x > n\) or \(x < -n\).

These properties are also true when \(>\) or \(<\) is replaced with \(\geq\) or \(\leq\).

**Check for Understanding**

1. **Concept Check**
   - Compare and contrast the solution of \( |x - 2| > 6 \) and the solution of \( |x - 2| < 6 \). 1-2. See margin.

2. **Open Ended**
   Write an absolute value inequality and graph its solution set.

3. **Find the Error**
   Leslie and Holly are solving \( |x + 3| = 2 \).

**Leslie**

\[
x + 3 = 2 \quad \text{or} \quad x + 3 = -2
\]
\[
x + 3 - 3 = x - 2 \quad \text{or} \quad x + 3 - 3 = -2 - 3
\]
\[
x = -1 \quad \text{or} \quad x = -5
\]

**Holly**

\[
x + 3 = 2 \quad \text{or} \quad x + 3 = 2
\]
\[
x + 3 - 3 = x \quad \text{or} \quad x + 3 - 3 = 2 + 3
\]
\[
x = -1 \quad \text{or} \quad x = 5
\]

Who is correct? Explain your reasoning. Leslie; see margin for explanation.

**Answers**

1. The solution of \( |x - 2| > 6 \) includes all values that are less than \(-4\) or greater than \(8\). The solution of \( |x - 2| < 6 \) includes all values that are greater than \(-4\) and less than \(8\).

2. Sample answer: \( |x| > 2 \)

3. You need to consider the case when the value inside the absolute value symbols is positive and the case when the value inside the absolute value symbols is negative. So \( x + 3 = 2 \) or \( x + 3 = -2 \).
4. Which graph represents the solution of $|k| \leq 3$?  
   a. ![Graph A]
   b. ![Graph B]
   c. ![Graph C]
   d. ![Graph D]

5. Which graph represents the solution of $|x - 4| > 2$?  
   a. ![Graph A]
   b. ![Graph B]
   c. ![Graph C]
   d. ![Graph D]

6. Express the statement in terms of an inequality involving absolute value. Do not solve. $|g - 832| \leq 46$
   A jar contains 832 gumballs. Amanda’s guess was within 46 pieces.

7. Solve each open sentence. Then graph the solution set.
   7. $|r + 3| = 10$ (Answer: $-13, 7$)
   8. $|c - 2| < 6$ (Answer: $-4 < c < 8$)
   9. $|10 - w| > 5$ \(w < -5\) or \(w > 25\)
   10. $|2g + 5| \geq 7$ (Answer: \(g \geq -6\) or \(g \geq 1\))

   For each graph, write an open sentence involving absolute value.
   11. $|x - 1| = 3$
   12. $|x - 8| > 4$

13. MANUFACTURING: A manufacturer produces bolts which must have a diameter within 0.001 centimeter of 1.5 centimeters. What are the acceptable measurements for the diameter of the bolts?

   Match each open sentence with the graph of its solution set.
   14. $|x + 5| \leq 3$  
      a. ![Graph A]
      b. ![Graph B]
      c. ![Graph C]
      d. ![Graph D]

   15. $|x - 4| > 4$  
      a. ![Graph A]
      b. ![Graph B]
      c. ![Graph C]
      d. ![Graph D]

   16. $|2x - 8| = 6$  
      a. ![Graph A]
      b. ![Graph B]
      c. ![Graph C]
      d. ![Graph D]

   17. $|x + 3| \geq -1$  
      a. ![Graph A]
      b. ![Graph B]
      c. ![Graph C]
      d. ![Graph D]

   18. $|x| < 2$  
      a. ![Graph A]
      b. ![Graph B]
      c. ![Graph C]
      d. ![Graph D]

   19. $|8 - x| = 2$  
      a. ![Graph A]
      b. ![Graph B]
      c. ![Graph C]
      d. ![Graph D]

   Express each statement using an inequality involving absolute value. Do not solve.
   20. The pH of a buffered eye solution must be within 0.002 of a pH of 7.3.
   21. The temperature inside a refrigerator should be within 1.5 degrees of 38°F.
   22. Ramona’s bowling score was within 6 points of her average score of 98.
   23. The cruise control of a car set at 55 miles per hour should keep the speed within 3 miles per hour of 55. $|s - 55| \leq 3$
For each graph, write an open sentence involving absolute value.

40. \(|x| = 5\)
41. \(|x - 3| = 5\)
42. \(|x| \leq 3\)
43. \(|x + 3| < 4\)
44. \(|x - 1| > 2\)
45. \(|x + 10| \geq 2\)

For each graph, write an open sentence involving absolute value.

13. \(|x| = 4\)
14. \(|x - 1| = 2\)
15. \(|x + 4| = 2\)

46. Write an absolute value inequality for the length of a full-term pregnancy.
47. Solve the inequality for the length of a full-term pregnancy: \(|d| \leq 294\)

48. FIRE SAFETY The pressure of a typical fire extinguisher should be within 25 pounds per square inch (psi) of 195 psi. Write the range of pressures for safe extinguishers: \(|p - 170| < 220\)

49. HEATING A thermostat with a 2-degree differential will keep the temperature within 2 degrees Fahrenheit of the temperature set point. Suppose your home has a thermostat with a 3-degree differential. If you set the thermostat at 68°F, what is the range of temperatures in the house? \(65 \leq t \leq 71\)

50. ENERGY Use the margin of error indicated in the graph at the right to find the range of the percent of people who say protecting the environment should have priority over developing energy supplies.

51. TIRE PRESSURE Tire pressure is measured in pounds per square inch (psi). Tires should be kept within 2 psi of the manufacturer’s recommended tire pressure. If the recommended inflation pressure for a tire is 30 psi, what is the range of acceptable pressures?

52. CRITICAL THINKING State whether each open sentence is always, sometimes, or never true.

a. \(|x + 3| < -5\) never
b. \(|x - 6| > -1\) always
c. \(|x + 2| = 0\) sometimes

Health

For Exercises 46 and 47, use the following information.
The average length of a human pregnancy is 280 days. However, a healthy, full-term pregnancy can be 14 days longer or shorter.

46. \(|d - 280| \leq 14\)

50. 49–55% 51. \(|p - 28| \leq 32\)
53. PHYSICAL SCIENCE  Li-Cheng must add 3.0 milliliters of sodium chloride to a solution. The sodium chloride must be within 0.5 milliliter of the required amount. How much sodium chloride can she add and obtain the correct results?

54. ENTERTAINMENT  Luis Gomez is a contestant on a television game show. He must guess within $1500 of the actual price of a car without going over to win the car. The actual price of the car is $18,000. What is the range of guesses in which Luis can win the vehicle? \( p \mid 16,500 \leq p \leq 18,000 \)

55. CRITICAL THINKING  The symbol \( \leq \) means plus or minus.
   a. If \( x = 3 \pm 1.2 \), what are the values of \( x \)? 1.8, 4.2
   b. Write \( x = 3 \pm 1.2 \) as an expression involving absolute value.  \( |x - 3| = 1.2 \)

56. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson. See margin.

How is absolute value used in election polls?
Include the following in your answer:
• an explanation of how to solve the inequality describing the percent of people who are against the tax levy, and
• a prediction of whether you think the tax levy will pass and why.

57. Choose the replacement set that makes \( |x + 5| = 2 \) true. B
   \[ A \quad \{-3, 3\} \quad B \quad \{-3, -7\} \quad C \quad \{2, -2\} \quad D \quad \{-3, -7\} \]

58. What can you conclude about \( x \) if \( -6 < |x| < 6 \)? C
   \[ A \quad -x \geq 0 \quad B \quad x \leq 0 \quad C \quad -6 < x < 6 \quad D \quad -x > 6 \]

**Maintain Your Skills**

**Mixed Review**

59. FITNESS  To achieve the maximum benefits from aerobic activity, your heart rate should be in your target zone. Your target zone is the range between 60% and 80% of your maximum heart rate. If Rafael’s maximum heart rate is 190 beats per minute, what is his target zone? (Lesson 6-4)

   Solve each inequality. Then check your solution. (Lesson 6-3)
   \[ 2m + 7 > 17 \quad \text{or} \quad 2 \leq 3 - 3x \leq 7 \quad w \leq 15 \]

   Find the slope and \( y \)-intercept of each equation. (Lesson 5-4)
   \[ 2x + y = 4 \quad 2y - 3x = 4 \quad 3x \quad 2 \quad 2 \]

   Solve each equation or formula for the variable specified. (Lesson 3-8)
   \[ l = \text{pr} \quad r = \frac{l}{t} \quad e - 2y = 3z \quad a + 5 = 7x \]

   Find each sum or difference. (Lesson 2-2)
   \[ -13 + 8 \quad -5 \quad -13.2 - 6.1 \quad -19.3 \quad -4.7 - (-8.9) \]

   Name the property illustrated by each statement. (Lesson 1-6)
   \[ 10x + 10y = 10(x + y) \quad \text{Distributive Property} \quad (2 + 3)a + 7 = 5a + 7 \quad \text{Substitution Property} \]

Getting Ready for the Next Lesson

**PREREQUISITE SKILL**  Graph each equation.

(To review graphing linear equations, see Lesson 4-5) 74–79. See pp. 365A–365D.

74. \( y = 3x + 4 \) 75. \( y = -2 \) 76. \( x + y = 3 \)
77. \( y - 2x = -1 \) 78. \( 2y - x = -6 \) 79. \( 2(x + y) = 10 \)

**Online Lesson Plans**

USA TODAY Education’s Online site offers resources and interactive features connected to each day’s newspaper. Experience TODAY, USA TODAY’s daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

**Lesson 6-5 Solving Open Sentences Involving Absolute Value 351**
**Graphing Inequalities in Two Variables**

**What You’ll Learn**
- Graph inequalities on the coordinate plane.
- Solve real-world problems involving linear inequalities.

**Vocabulary**
- half-plane
- boundary

**How are inequalities used in budgets?**

Hannah budgets $30 a month for lunch. On most days, she brings her lunch. She can also buy lunch at the cafeteria or at a fast-food restaurant. She spends an average of $3 for lunch at the cafeteria and an average of $4 for lunch at a restaurant. How many times a month can Hannah buy her lunch and remain within her budget?

Let \( x \) represent the number of days she buys lunch at the cafeteria, and let \( y \) represent the number of days she buys lunch at a restaurant. Then the following inequality can be used to represent the situation:

\[
3x + 4y \leq 30
\]

There are many solutions of this inequality.

**GRAPH LINEAR INEQUALITIES** The solution set of an inequality in two variables is the set of all ordered pairs that satisfy the inequality. Like a linear equation in two variables, the solution set is graphed on a coordinate plane.

**Example 1 Ordered Pairs that Satisfy an Inequality**

From the set \((1, 6), (3, 0), (2, 2), (4, 3)\), which ordered pairs are part of the solution set for \(3x + 2y < 12\)?

Use a table to substitute the \( x \) and \( y \) values of each ordered pair into the inequality.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( 3x + 2y &lt; 12 ) True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>3(1) + 2(6) ( &lt; 12 ) false</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3(3) + 2(0) ( &lt; 12 ) true</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3(2) + 2(2) ( &lt; 12 ) true</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3(4) + 2(3) ( &lt; 12 ) false</td>
</tr>
</tbody>
</table>

The ordered pairs \((3, 0), (2, 2)\) are part of the solution set of \(3x + 2y < 12\). In the graph, notice the location of the two ordered pairs that are solutions for \(3x + 2y < 12\) in relation to the line.

**Resource Manager**

- **Workbook and Reproducible Masters**
  - **Chapter 6 Resource Masters**
    - Study Guide and Intervention, pp. 373–374
    - Skills Practice, p. 375
    - Practice, p. 376
    - Reading to Learn Mathematics, p. 377
    - Enrichment, p. 378
    - Assessment, p. 394
  - **Graphing Calculator and Spreadsheet Masters**, p. 34

- **Parent and Student Study Guide Workbook**, p. 51

- **Transparencies**
  - 5-Minute Check Transparency 6-6
  - Answer Key Transparencies

- **Technology**
  - Interactive Chalkboard
The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a half-plane. An equation defines the boundary or edge for each half-plane.

In-Class Examples

From the set \{(3, 3), (0, 2), (2, 4), (1, 0)\}, which ordered pairs are part of the solution set for \(4x + 2y > 8\)? \(\{(3, 3), (2, 4)\}\)

**Teaching Tip** Students may need a quick refresher on slope-intercept form before they graph inequalities. Remind students that slope-intercept form is \(y = mx + b\). Graph \(2y - 4x > 6\).

**Study Tip**

Dashed Line
- Like a circle on a number line, a dashed line on a coordinate plane indicates that the boundary is not part of the solution set.

Solid Line
- Like a dot on a number line, a solid line on a coordinate plane indicates that the boundary is included.

Consider the graph of \(y > 4\). First determine the boundary by graphing \(y = 4\), the equation you obtain by replacing the inequality sign with an equals sign. Since the inequality involves \(y\)-values greater than 4, but not equal to 4, the line should be dashed. The boundary divides the coordinate plane into two half-planes.

To determine which half-plane contains the solution, choose a point from each half-plane and test it in the inequality.

Try (3, 0). Try (5, 6).

\[
\begin{align*}
y > 4 & \quad y = 0 & \quad y > 4 & \quad y = 6 \\
0 > 4 & \quad \text{false} & \quad 6 > 4 & \quad \text{true}
\end{align*}
\]

The half-plane that contains (5, 6) contains the solution. Shade that half-plane.

**Example 2 Graph an Inequality**

Graph \(y - 2x \leq -4\).

**Step 1** Solve for \(y\) in terms of \(x\).

\[
y - 2x \leq -4 \quad \text{Original inequality}
\]

\[
y - 2x + 2x \leq -4 + 2x \quad \text{Add } 2x \text{ to each side.}
\]

\[
y \leq 2x - 4 \quad \text{Simplify.}
\]

**Step 2** Graph \(y = 2x - 4\). Since \(y \leq 2x - 4\) means \(y < 2x - 4\) or \(y = 2x - 4\), the boundary is included in the solution set. The boundary should be drawn as a solid line.

(continued on the next page)
Solve Real-World Problems

In-Class Example

3. Journalism Lee Cooper writes and edits short articles for a local newspaper. It generally takes her an hour to write an article and about a half-hour to edit an article. If Lee works up to 8 hours a day, how many articles can she write and edit in one day?

Answers (p. 355)

1. The graph of \( y = x + 2 \) is a line. The graph of \( y < x + 2 \) does not include the boundary \( y = x + 2 \), and it includes all ordered pairs in the half-plane that contains the origin.

2. Sample answer: \( x > y \)

3. If the test point results in a true statement, shade the half-plane that contains the point. If the test point results in a false statement, shade the other half-plane.

Example 3 Write and Solve an Inequality

Advertising Rosa Padilla sells radio advertising in 30-second and 60-second time slots. During every hour, there are up to 15 minutes available for commercials. How many commercial slots can she sell for one hour of broadcasting?

Step 1 Let \( x \) equal the number of 30-second commercials. Let \( y \) equal the number of 60-second or 1-minute commercials. Write an open sentence representing this situation.

\[
\frac{1}{2} \text{ min times the number of 30-s commercials} + \frac{1}{2} \times \text{ number of 1-min commercials} \leq 15 \text{ min}
\]

Step 2 Solve for \( y \) in terms of \( x \).

\[
\frac{1}{2}x + y \leq 15 \quad \text{Original inequality}
\]

\[
\frac{1}{2}x + y - \frac{1}{2}x \leq 15 - \frac{1}{2}x \quad \text{Subtract } \frac{1}{2}x \text{ from each side.}
\]

\[
y \leq 15 - \frac{1}{2}x \quad \text{Simplify.}
\]

Step 3 Since the open sentence includes the equation, graph \( y = 15 - \frac{1}{2}x \) as a solid line. Test a point in one of the half-planes, for example \((0, 0)\). Shade the half-plane containing \((0, 0)\) since \(0 \leq 15 - \frac{1}{2}(0)\) is true.
Step 4  Examine the solution.
   • Rosa cannot sell a negative number of commercials. Therefore, the domain and range contain only nonnegative numbers.
   • She also cannot sell half of a commercial. Thus, only points in the shaded half-plane whose x- and y-coordinates are whole numbers are possible solutions.

One solution is (12, 8). This represents twelve 30-second commercials and eight 60-second commercials in a one hour period.

Check for Understanding

Concept Check
1–3. See margin.

1. Compare and contrast the graph of \( y = x + 2 \) and the graph of \( y < x + 2 \).
2. OPEN ENDED Write an inequality in two variables and graph it.
3. Explain why it is usually only necessary to test one point when graphing an inequality.

Guided Practice

Determine which ordered pairs are part of the solution set for each inequality.

4. \( y \leq x + 1 \), \((-1, 0)\), \((3, 2)\), \((2, 5)\), \((-2, 1)\) \((-1, 0)\), \((3, 2)\)
5. \( y > 2x \), \((2, 6)\), \((0, -1)\), \((3, 5)\), \((-1, -2)\) \((2, 6)\)
6. Which graph represents \( y - 2x \geq 2? \)

Graph each inequality. 7–10. See margin.

7. \( y \geq 4 \)
8. \( y \leq 2x - 3 \)
9. \( 4 - 2x < -2 \)
10. \( 1 - y > x \)

Application

11. ENTERTAINMENT Coach Riley wants to take her softball team out for pizza and soft drinks after the last game of the season. She doesn’t want to spend more than $60. Write an inequality that represents this situation and graph the solution set.
   \( 12x + 3y \leq 60; \) See margin for graph.

About the Exercises...

Organization by Objective
   • Graph Linear Inequalities: 12–37
   • Solve Real-World Problems: 38–44

Odd/Even Assignments
Exercises 12–37 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Average: 13–37 odd, 38–41, 45–63
Advanced: 12–36 even, 40–63
Solving Linear Inequalities

11. Does Satchi have enough money to buy 2 videos and 3 CDs?

12. Graph each inequality.

Pre-Activity

How are inequalities used in budgets?

Reading the Lesson

1. Complete the chart to show which type of line is needed for each symbol.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Graph of Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;</td>
<td>Dashed line</td>
</tr>
<tr>
<td>≤</td>
<td>Dashed line</td>
</tr>
<tr>
<td>≥</td>
<td>Solid line</td>
</tr>
<tr>
<td>&gt;</td>
<td>Solid line</td>
</tr>
</tbody>
</table>

Helping You Remember

3. The inequality symbol is < or >, make the boundary line dashed.

4. The symbol is ≤ or ≥, make the boundary line solid.

Reading to Learn Mathematics, p. 377

Graph each inequality.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>x &lt; -5</td>
<td>Dashed line</td>
</tr>
<tr>
<td>x ≦ 3</td>
<td>Solid line</td>
</tr>
<tr>
<td>x &gt; -3</td>
<td>Solid line</td>
</tr>
<tr>
<td>x ≧ -3</td>
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</tbody>
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Graph each inequality.

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</tr>
<tr>
<td>y ≧ -3</td>
<td>Solid line</td>
</tr>
</tbody>
</table>

Skills Practice, p. 375 and Practice, p. 376 (shown)

Determine which ordered pairs are part of the solution set for each inequality:

1. x + y ≥ 3, (4, 1), (1, 5)
2. x - y ≤ 2, (0, 1), (2, 0)
3. x + y > 5, (3, 1), (1, 3)

Match each inequality with its graph.

4. a. b. c. d.

Reading to Learn, Enrichment

Reading the Lesson

1. Complete the chart to show which type of line is needed for each symbol.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Graph of Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;</td>
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<td>≥</td>
<td>Solid line</td>
</tr>
<tr>
<td>&gt;</td>
<td>Solid line</td>
</tr>
</tbody>
</table>

Helping You Remember

3. The inequality symbol is < or >, make the boundary line dashed.

4. The symbol is ≤ or ≥, make the boundary line solid.

The origin cannot be used as a test point when it is on the boundary.

Enrichment, p. 378

Using Equations: Ideal Weight

You can find your ideal weight as follows:

A woman should weigh 100 pounds for the first 5 feet of height and 1 additional pound for each inch over 5 feet (5 feet = 60 inches). A woman should weigh 105 pounds for the first 5 feet of height and 1 additional pound for each inch over 5 feet (5 feet = 60 inches). A man should weigh 100 pounds for the first 5 feet of height and 1 additional pound for each inch over 5 feet (5 feet = 60 inches).

To determine your body's ideal weight, using tape and inches, measure around the waist of your lower back. If the tape and inches just touch, you have normal bone structure. If the tape or inches are too high, you have large-boned structure. The U.S. Postal Service limits the size of packages to those in which the length of the longest side plus the distance around the thickest part is less than or equal to 108 inches.

38. Write an inequality that represents this situation. \( l + d \leq 108 \)

39. Are there any restrictions on the domain or range? The solution set is limited to pairs of positive numbers.

Online Research Data Update What are the current postage rates and regulations? Visit www.algebra1.com/data_update to learn more.

SHIPPING For Exercises 40 and 41, use the following information.

A delivery truck is transporting televisions and microwaves to an appliance store. The weight limit for the truck is 4,000 pounds. The televisions weigh 77 pounds, and the microwaves weigh 55 pounds.

40. Write an inequality for this situation. \( 77 + 55m \leq 4000 \)

41. Will the truck be able to deliver 35 televisions and 25 microwaves at once? No, the weight will be greater than 4,000 pounds.
FALL DANCE  For Exercises 42–44, use the following information. Tickets for the fall dance are $5 per person or $8 for couples. In order to cover expenses, at least $1200 worth of tickets must be sold.

42. Write an inequality that represents this situation. \(5s + 8c \geq 1200\)
43. Graph the inequality. See margin.
44. If 100 single tickets and 125 couple tickets are sold, will the committee cover its expenses? Yes

45. CRITICAL THINKING  Graph the intersection of the graphs of \(y \leq x - 1\) and \(y \geq -x\). See margin.

46. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson. See margin.

How are inequalities used in budgets?
Include the following in your answer:
• an explanation of the restrictions placed on the domain and range of the inequality used to describe the number of times Hannah can buy her lunch, and
• three possible solutions of the inequality.

47. Which ordered pair is not a solution of \(y - 2x < -5\)? D
   \[A\] (2, -2) \hspace{1cm} \[B\] (-1, -8) \hspace{1cm} \[C\] (4, 1) \hspace{1cm} \[D\] (5, 6)

48. Which inequality is represented by the graph at the right? B
   \[A\] \(2x + y < 1\) \hspace{1cm} \[B\] \(2x + y > 1\) \hspace{1cm} \[C\] \(2x + y \geq 1\) \hspace{1cm} \[D\] \(2x + y \leq 1\)

Maintain Your Skills

Mixed Review
Solve each open sentence. Then graph the solution set. (Lesson 6-5)
49. \(|3 + 2x| = 11\) \(-7, 4\) 50. \(|x + 8| < 6\) \((x| -14 < x < -2\) 51. \(|2y + 5| \geq 3\) \((y| y \leq -4\text{ or } y \geq 1\)

Solve each compound inequality. Then graph the solution. (Lesson 6-4)
52. \(y + 6 > -1\) and \(y - 2 < 4\) 53. \(m + 4 < 2\text{ or } m - 2 > 1\) \((m| m < -2\text{ or } m > 3\)

State whether each percent of change is a percent of increase or decrease. Then find the percent of change. Round to the nearest whole percent. (Lesson 3-7)
54. original: 200 55. original: 100 56. original: 53
   new: 172 decrease; 14% new: 142 increase; 42% new: 75 increase; 42%

Solve each equation. (Lesson 3-4)
57. \(\frac{d - 2}{3} = 7\) 23 58. \(3n + 6 = -15\) -7 59. \(35 + 20h = 100\) 3.25

Simplify. (Lesson 2-4)
60. \(-\frac{64}{4} - 16\) 61. \(-\frac{27c}{9} - 3c\) 62. \(\frac{12a - 14b}{-2}\) 63. \(\frac{18y - 9}{3} - 6y - 3\) -6a + 7b

Open-Ended Assessment
Speaking  Have students explain why, when they use linear inequalities to solve real-world problems, the solution is often not the entire half-plane. The explanations should include an example.

Assessment Options
Quiz (Lesson 6-6) is available on p. 394 of the Chapter 6 Resource Masters.

Answers
46. The amount of money spent in each category must be less than or equal to the budgeted amount. How much you spend on individual items can vary. Answers should include the following.
   • The domain and range must be positive integers.
   • Sample answers: Hannah could buy 5 cafeteria lunches and 3 restaurant lunches, 2 cafeteria lunches and 5 restaurant lunches, or 8 cafeteria lunches and 1 restaurant lunch.

49. 50. 51. 52. 53.
You can use a TI-83 Plus graphing calculator to investigate the graphs of inequalities. Since graphing calculators only shade between two functions, enter a lower boundary as well as an upper boundary for each inequality.

**Graph two different inequalities on your graphing calculator.**

**Step 1** Graph \( y \leq 3x + 1. \)
- Clear all functions from the \( y= \) list.
  
  **KEYSTROKES:** \( y= \) CLEAR
- Graph \( y \leq 3x + 1 \) in the standard window.
  
  **KEYSTROKES:** 2nd DRAW 10, 3 X,T,0,n + 1 ENTER

The lower boundary is \( Y_{\text{min}} \) or \(-10\). The upper boundary is \( y = 3x + 1 \). All ordered pairs for which \( y \) is less than or equal to \( 3x + 1 \) lie below or on the line and are solutions.

**Step 2** Graph \( y - 3x \geq 1. \)
- Clear the drawing that is currently displayed.
  
  **KEYSTROKES:** 2nd DRAW 1
- Rewrite \( y - 3x \geq 1 \) as \( y \geq 3x + 1 \) and graph it.
  
  **KEYSTROKES:** 2nd DRAW 7 3 X,T,0,n + 1 , 10 ) ENTER

This time, the lower boundary is \( y = 3x + 1 \). The upper boundary is \( Y_{\text{max}} \) or \( 10 \). All ordered pairs for which \( y \) is greater than or equal to \( 3x + 1 \) lie above or on the line and are solutions.

**Exercises**

**2b. Sample answer:** \( \{(0, 4), (-1, 7), (2, 6), (4.2, -1.5)\} \)

1. Compare and contrast the two graphs shown above. See margin.

2. Graph the inequality \( y \geq -2x + 4 \) in the standard viewing window.
   a. What functions do you enter as the lower and upper boundaries? \( y = -2x + 4; Y_{\text{max}} \) or 10
   b. Using your graph, name four solutions of the inequality.

3. Suppose student movie tickets cost $4 and adult movie tickets cost $8. You would like to buy at least 10 tickets, but spend no more than $80.
   a. Let \( x \) = number of student tickets and \( y \) = number of adult tickets. Write two inequalities, one representing the total number of tickets and the other representing the total cost of the tickets. \( x + y \geq 10; 4x + 8y \leq 80 \)
   b. Which inequalities would you use as the lower and upper boundaries? \( y \geq -x + 10; y \leq -0.5x + 10 \)
   c. Graph the inequalities. Use the viewing window \([0, 20]\) scl: 1 by \([0, 20]\) scl: 1. See margin.
   d. Name four possible combinations of student and adult tickets. Sample answer: \( \{(8, 5), (10, 4), (14, 2), (20, 0)\} \)
Vocabulary and Concept Check

Addition Property of Inequalities (p. 318)  
boundary (p. 353)  
compound inequality (p. 339)  
Division Property of Inequalities (p. 327)  
half-plane (p. 353)  
intersection (p. 339)  
Multiplication Property of Inequalities (p. 325)  
set-builder notation (p. 319)  
Subtraction Property of Inequalities (p. 319)  
union (p. 340)

Choose the letter of the term that best matches each statement, algebraic expression, or algebraic sentence.

1. \(|w| \geq -14| \)  
a. Addition Property of Inequalities  
b. Division Property of Inequalities  
c. half-plane  
d. intersection  
e. Multiplication Property of Inequalities  
f. set-builder notation  
g. Subtraction Property of Inequalities  
h. union

Lesson-by-Lesson Review

6-1 See pages 318–323.

Solving Inequalities by Addition and Subtraction

Concept Summary

- If any number is added to each side of a true inequality, the resulting inequality is also true.
- If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

Examples

Solve each inequality.

1. \(f + 9 \leq -23\)

Original inequality

\(f + 9 \leq -23\)

Subtract.

\(f \leq -32\)

Simplify.

The solution set is \(\{f | f \leq -32\}\).

2. \(v - 19 > -16\)

Original inequality

\(v - 19 > -16\)

Add.

\(v > 3\)

Simplify.

The solution set is \(\{v | v > 3\}\).

Exercises

Solve each inequality. Then check your solution, and graph it on a number line. See Examples 1–5 on pages 318–320. 9–16. See pp. 365A–365D.

9. \(c + 51 > 32\)

10. \(r + 7 > -5\)

11. \(w - 14 \leq 23\)

12. \(a - 6 > -10\)

13. \(-0.11 \geq n - (-0.04)\)

14. \(2.3 < g - (-2.1)\)

15. \(7h \leq 6h - 1\)

16. \(5b > 4b + 5\)

17. Define a variable, write an inequality, and solve the problem. Then check your solution. Twenty-one is no less than the sum of a number and negative two.

Sample answer: Let \(n = \) the number; \(21 \geq n + (-2); \{n | n \leq 23\}\).

www.algebra1.com/vocabulary_review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 6 includes a page reference where each term was introduced.

Assessment A vocabulary test/review for Chapter 6 is available on p. 392 of the Chapter 6 Resource Masters.

Lesson-by-Lesson Review

For each lesson,
- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students’ mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Foldables Study Organizer

For more information about Foldables, see Teaching Mathematics with Foldables.

Have students look through the chapter to make sure they have included examples in their Foldable journal for each type of inequality they learned to solve.

Encourage students to refer to their Foldable journal while completing the Study Guide and Review and to use them in preparing for the Chapter Test.
6-2 Solving Inequalities by Multiplication and Division

Concept Summary
- If each side of a true inequality is multiplied or divided by the same positive number, the resulting inequality is also true.
- If each side of a true inequality is multiplied or divided by the same negative number, the direction of the inequality must be reversed.

Examples
Solve each inequality.

\[ \begin{align*}
1. & \quad -14g \geq 126 & 2. & \quad \frac{3}{4}d < 15 \\
& \quad g \leq \frac{126}{-14} & & \quad d > \frac{15}{\frac{3}{4}} \\
& \quad g \leq -9 & & \quad d < 20 \\
\end{align*} \]

Exercise
Solve each inequality. Then check your solution.
See Examples 1–5 on pages 326–328.

\[ \begin{align*}
18. & \quad v > 4 & 19. & \quad r \leq 6 \\
20. & \quad z \leq 5 & 21. & \quad m < -11 \\
22. & \quad b \geq -36 & 23. & \quad d < 65 \\
24. & \quad w > -33 & 25. & \quad p \leq -25 \\
\end{align*} \]

6-3 Solving Multi-Step Inequalities

Concept Summary
- Multi-step inequalities can be solved by undoing the operations.
- Remember to reverse the inequality sign when multiplying or dividing each side by a negative number.
- When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example
Solve \(4(n - 1) < 7n + 8\).

\[ \begin{align*}
4(n - 1) & < 7n + 8 & \text{Original inequality} \\
4n & - 4 < 7n + 8 & \text{Distributive Property} \\
4n - 7n & < 4 + 8 & \text{Subtract } 7n \text{ from each side.} \\
-3n & < 12 & \text{Simplify.} \\
-\frac{3n}{-3} & > \frac{12}{-3} & \text{Divide each side by } -3 \text{ and change } < \text{ to } >. \\
\quad n & > -4 & \text{Simplify.} \\
\end{align*} \]

The solution set is \(\{n \mid n > -4\}\).
Solve each inequality. Then check your solution.

27. \(-4k + 7 > 15\)
28. \(5 - 6n > -19\)
29. \(-5x + 3 < 3x + 19\)
30. \(15b - 12 > 7b + 60\)
31. \(-5(q + 12) < 3q - 4\)
32. \(7(q + 8) < 5(q + 2) + 4g\)
33. \(\frac{2(x + 2)}{3} \geq 4\)
34. \(\frac{1 - 7n}{5} > 10\)
35. Define a variable, write an inequality, and solve the problem. Then check your solution. Two thirds of a number decreased by 27 is at least 9.

Graph the solution set of each compound inequality.

1. \(x \geq -1\) and \(x > 3\)
2. \(x \leq 8\) or \(x < 2\)

The solution set is \([x \mid x > 3]\).

The solution set is \([x \mid x \leq 8]\).

Solve each compound inequality. Then graph the solution set.

36. \(-1 < p + 3 < 5\)
37. \(-3 < 2k - 1 < 5\)
38. \(3w + 8 < 2\) or \(w\) is a real number.
39. \(a - 3 \leq 8\) or \(a + 5 \geq 21\)
40. \(m + 8 < 4\) and \(3 - m < 5\)
41. \(10 - 2y > 12\) and \(7y < 4y + 9\)

Solving Open Sentences Involving Absolute Value

If \(|x| = n\), then \(x = -n\) or \(x = n\).

If \(|x| < n\), then \(x > -n\) and \(x < n\).

If \(|x| > n\), then \(x < -n\) or \(x > n\).
Study Guide and Review

Answers

42. -12 -8 -4 0 4 8 12 16 20 24 28
43. -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1
44. -16 -14 -12 -10 -8 -6 -4 -2 0 2 4
45. -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0
46. -15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5
47. -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1
48. -8 -7 -6 -5 -4 -3 -2 -1 0 1 2
49. -6 -5 -4 -3 -2 -1 0 1 2 3 4
50. y - 2x = -3
51. x + 2y = 4
52. y = 5x + 1
53. 2x - 3y = 6

Example

Solve |x + 6| = 15.

|x + 6| = 15

x + 6 = 15 or x + 6 = -15

x = 9 or x = -21

The solution set is {-21, 9}.

Exercises

Solve each open sentence. Then graph the solution set.

42. |w - 8| = 12 43. |q + 5| = 2 44. |h + 5| > 7 45. |w + 8| ≥ 1
46. |r + 10| < 3 47. |t + 4| ≤ 3 48. |2x + 5| < 4 49. |3d + 4| < 8
50. |r - 13| ≤ r ≤ 7 |t - 7| ≤ t ≤ -1 |x - 4| ≤ x ≤ -1 |d - 4| ≤ d ≤ 1

Graphing Inequalities in Two Variables

Concept Summary

- To graph an inequality in two variables:
  - Step 1 Determine the boundary and draw a dashed or solid line.
  - Step 2 Select a test point. Test that point.
  - Step 3 Shade the half-plane that contains the solution.

Example

Graph y ≥ x - 2.

Since the boundary is included in the solution, draw a solid line.
Test the point (0, 0).
y ≥ x - 2 Original inequality
0 ≥ 0 - 2 x = 0, y = 0
0 ≥ -2 true
The half plane that contains (0, 0) should be shaded.

Exercises

Determine which ordered pairs are part of the solution set for each inequality. See Example 1 on page 352.

50. 3x + 2y < 9, {(1, 3), (3, 2), (-2, 7), (-4, 11)} {(2, 7)}
51. 5 - y ≥ 4x, {(-2, -5), (1/2, 7), (-1, 6), (-3, 20)} {(-2, -5), (-1, 6)}
52. 1/2y ≤ x - 6, {(-4, 15), (5, 1), (3, 8), (-2, 25)} {(-4, 15), (5, 1)}
53. -2x < 8 - y, {(5, 10), (3, 6), (-4, 0), (-3, 6)} {(5, 10), (3, 6)}

Graph each inequality. See Example 2 on pages 353 and 354.
54. y - 2x < -3 55. x + 2y ≥ 4 56. y ≤ 5x + 1 57. 2x - 3y > 6
### Vocabulary and Concepts

1. Write the set of all numbers \( t \) such that \( t \) is greater than or equal to 17 in set-builder notation. \( \{ t \mid t \geq 17 \} \)

2. Show how to solve \( 6(a + 5) < 2a + 8 \). Justify your work. See pp. 365A–365D.

3. **OPEN ENDED** Give an example of a compound inequality that is an intersection and an example of a compound inequality that is a union. Sample answers:
   \( 2 < x < 8; \ x < 2 \ or \ x > 8 \)

4. Compare and contrast the graphs of \( |x| < 3 \) and \( |x| \geq 3 \). Both graphs have dots at 3 and -3. The graph of \( |x| \leq 3 \) is darkened between the two dots. The graph of \( |x| \geq 3 \) is darkened to the right of the dot at 3 and to the left of the dot at -3.

### Skills and Applications

**Solve each inequality. Then check your solution.**

5. \(-23 \leq g - 6 \) \( g \leq -17 \)

6. \( 9p < 8p - 18 \) \( p < -18 \)

7. \( d - 5 < 2d - 14 \) \( d > 9 \)

8. \( \frac{7}{2}w \geq -21 \) \( w \geq -24 \)

9. \( -22b \leq 99 \) \( b \geq -4.5 \)

10. \( 4m - 11 \geq 8m + 7(m) \leq -4.5 \)

11. \( -3(k - 2) > 12 \) \( k < 12 \)

12. \( \frac{f - 5}{3} > -3 \) \( f > -4 \)

13. \( 0.3(y - 4) \leq 0.8(2y + 2) \) \( y \leq 20 \)

14. **REAL ESTATE** A homeowner is selling her house. She must pay 7% of the selling price to her real estate agent after the house is sold. To the nearest dollar, what must the selling price of her house to have at least $110,000 after the agent is paid? \( \text{at least } \$118,280 \)

15. Solve \( 6 + |r| = 3 \)

16. Solve \( |d| > -2 \). \( d \) \( d \) is a real number.

**Solve each compound inequality. Then graph the solution set. 17–22. See pp. 365A–365D for graphs.**

17. \( r + 3 > 2 \) and \( 4r < 12 \) \( r \geq -1 \) \( r < 3 \)

18. \( 3n^2 + 2 \geq 17 \) or \( 3n + 2 \leq -1 \) \( n \geq -3 \) \( n \leq -1 \)

19. \( 9 + 2p > 3 \) and \( -13 > 8p + 3 \) \( p > -3 \) \( p < -2 \)

20. \( 2a - 5 < 7 \) \( a < 6 \)

21. \( |7 - 3s| \geq 2 \) \( s \leq \frac{2}{3} \) or \( s > 3 \)

22. \( |7 - 5z| < 3 \) \( z > 0.8 \) or \( z > 2 \)

Define a variable, write an inequality, and solve each problem. Then check your solution. 23–25. Sample answer: Let \( n \) be the number.

23. One fourth of a number is no less than -3. \( \frac{1}{4}n \geq -3 \) \( n \geq -12 \)

24. Three times a number subtracted from 14 is less than two. \( 14 - 3n < 2 \) \( n > 4 \)

25. Five less than twice a number is between 13 and 21. \( 13 < 2n - 5 \leq 21 \) \( n > 9 \) or \( n < 13 \)

26. **TRAVEL** Megan’s car gets between 18 and 21 miles per gallon of gasoline. If her car’s tank holds 15 gallons, what is the range of distance that Megan can drive her car on one tank of gasoline? \( \text{between 270 and 315 mi} \)

**Graph each inequality. 27–29. See pp. 365A–365D.**

27. \( y \geq 3x - 2 \)

28. \( 2x + 3y < 6 \)

29. \( x - 2y > 4 \)

**30. STANDARDIZED TEST PRACTICE** Which inequality is represented by the graph? \( B \)

\[
\begin{align*}
\text{A} & \quad |x - 2| \leq 5 \\
\text{B} & \quad |x - 2| \geq 5 \\
\text{C} & \quad |x + 2| \leq 5 \\
\text{D} & \quad |x - 1| \leq 5
\end{align*}
\]

**Portfolio Suggestion**

**Introduction** In mathematics, there is often more than one way to solve a problem. In order to solve an inequality, for example, you can write a solution set or graph the inequality.

**Ask Students** Find an inequality from your work in this chapter and show two different ways to solve it. Place your work in your portfolio.
Chapter 6 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the Chapter 6 Resource Masters.

Additional Practice

See pp. 397–398 in the Chapter 6 Resource Masters for additional standardized test practice.

ExamView® Pro

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.
10. A die is rolled. What are the odds of rolling a number less than 5? \(4:2\) or \(2:1\)

11. A car is traveling at an average speed of 54 miles per hour. How many minutes will it take the car to travel 117 miles? \(130\)

12. The price of a tape player was cut from $48 to $36. What was the percent of decrease? \(25\%\)

13. Quadrilateral \(MNOP\) has vertices \(M(0, -4), N(-2, 8), O(5, 3),\) and \(P(2, -9)\). Find the coordinates of the vertices of the image if it is reflected over the \(y\)-axis. \((Lesson\ 4-2)\)

\[M'(0, -4), N'(2, 8), O'(-5, 3), P'(-2, -9)\]

14. Write an equation in slope-intercept form that describes the graph. \((Lesson\ 5-4)\)

\[y = -x + 3\]

15. A line is parallel to the graph of the equation \(\frac{1}{2}y = \frac{2}{3}x - 1\). What is the slope of the parallel line? \((Lesson\ 5-4\ and\ 5-6)\)

\(2\)

16. Solve \(\frac{1}{4}(10x - 8) - 3(x - 1) \geq 15\) for \(x\). \((Lesson\ 6-3)\)

\(x \geq 8\)

21d. The solution set is limited to nonnegative numbers because the vendor cannot sell less than zero product.

Test-Taking Tip

- Know the slope-intercept form of linear equations: \(y = mx + b\).
- Understand the definition of slope.
- Recognize the relationships between the slopes of parallel lines and between the slopes of perpendicular lines.

www.algebra1.com/standardized_test/sol

17. Find all values of \(x\) that make the inequality \(|x - 3| > 5\) true. \((Lesson\ 6-5)\)

\[x \in (-\infty, -2) \cup (8, \infty)\]

18. Graph the equation \(y = -2x + 4\) and indicate which region represents \(y < -2x + 4\). \((Lesson\ 6-6)\)

See margin.

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

19. The Carlson family is building a house on a lot that is 91 feet long and 158 feet wide. \((Lessons\ 6-1, 6-2,\ and\ 6-4)\)

a. Town law states that the sides of a house cannot be closer than 10 feet to the edges of a lot. Write an inequality for the possible lengths of the Carlson family’s house, and solve the inequality.

\[91 - 20 \leq l \leq 71\]

b. The Carlson family wants their house to be at least 2800 square feet and no more than 3200 square feet. They also want their house to have the maximum possible length. Write an inequality for the possible widths of their house, and solve the inequality. Round your answer to the nearest whole number of feet.

\[2800 \leq w \leq 3200; 39 \leq w \leq 45\]

20. For the graph below, write an open sentence involving absolute value. \((Lesson\ 6-5)\)

Sample answer: \(|x + 2| \leq 3\)

21. A street vendor sells hot dogs for $3 each and bratwurst for $5 each. In order to cover his daily expenses, he must sell at least $400 worth of food. \((Lesson\ 6-6)\)

a. Write an inequality that represents this situation.

\[3h + 5b \geq 400\]

b. If 68 hot dogs and 38 bratwursts are sold, will the street vendor cover his costs? \(No\)

c. Find a number of hotdogs and bratwursts that could be sold and cover the daily costs. \(Sample\ answer: 100\ hot\ dogs\ and\ 21\ bratwursts\)

d. Are there any restrictions on the domain and range? Explain. \(See\ margin.\)

21d. The solution set is limited to nonnegative numbers because the vendor cannot sell less than zero products.
56. Inequalities can be used to compare the number of schools participating in certain sports, to compare the number of participating schools if sports are added or discontinued in a certain number of schools, and to determine how many schools need to add a certain sport to surpass the number participating in another sport. Answers should include the following.

- To find how many schools must add girls track and field to surpass the current number of schools participating in girls basketball, solve

\[
\frac{16,526}{H11021} \div \frac{14,587}{H11001} = x
\]

More than 1939 schools must add girls track and field.

Page 324, Lesson 6-2A
Algebra Activity

6. The symbols in the solutions point in the opposite direction with relationship to the variable than the symbols in the original problem.
7. \[ \begin{array}{cc|ccc} x & & 1 & 1 & 1 \\ \hline \end{array} \quad \begin{array}{cc|ccc} 1 & 1 & 1 & \geq \end{array} \]

\[ 2x \geq 6 \]

There are no negative x-tiles, so the variable remains on the left and the symbol remains \( \geq \).

Page 331, Lesson 6-2
61. Sample answer:

Page 331, Practice Quiz 1
1. \[ \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \]
2. \[ \begin{array}{cccccccc} -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 \end{array} \]
3. \[ \begin{array}{cccccccc} -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 \end{array} \]
4. \[ \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \]
5. \[ \begin{array}{cccccccc} -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \end{array} \]
6. \( \{ z | z \geq 7 \} \)
7. \( \{ v | v < 35 \} \)
8. \( \{ q | q < -35 \} \)
9. \( \{ r | r > -13 \} \)
10. \( \{ w | w \geq \frac{5}{4} \} \)

Pages 335–337, Lesson 6-3
14. \[ -5(k + 4) > 3(k - 4) \quad \text{Original inequality} \]
\[ -5k - 20 > 3k - 12 \quad \text{Distributive Property} \]
\[ -5k - 20 + 5k > 3k - 12 + 5k \quad \text{Add 5k to each side} \]
\[ -20 > 8k - 12 \quad \text{Simplify} \]
\[ -20 + 12 > 8k - 12 + 12 \quad \text{Add 12 to each side} \]
\[ -8 > 8k \quad \text{Simplify} \]
\[ \frac{-8}{8} > \frac{8k}{8} \quad \text{Divide each side by 8} \]
\[ -1 > k \quad \text{Simplify} \]
\( \{ k | k < -1 \} \)

Page 331, Practice Quiz 1

Pages 335–337, Lesson 6-3
14. \[ -5(k + 4) > 3(k - 4) \quad \text{Original inequality} \]
\[ -5k - 20 > 3k - 12 \quad \text{Distributive Property} \]
\[ -5k - 20 + 5k > 3k - 12 + 5k \quad \text{Add 5k to each side} \]
\[ -20 > 8k - 12 \quad \text{Simplify} \]
\[ -20 + 12 > 8k - 12 + 12 \quad \text{Add 12 to each side} \]
\[ -8 > 8k \quad \text{Simplify} \]
\[ \frac{-8}{8} > \frac{8k}{8} \quad \text{Divide each side by 8} \]
\[ -1 > k \quad \text{Simplify} \]
\( \{ k | k < -1 \} \)

Page 331, Practice Quiz 1

Pages 342–343, Lesson 6-4
28. \[ \begin{array}{cccccccc} 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \end{array} \]
29. \[ \begin{array}{cccccccc} -14 & -13 & -12 & -11 & -10 & -9 & -8 & -7 & -6 \end{array} \]
30. \[ \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \]
31. \[ \begin{array}{cccccccc} -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 \end{array} \]
32. \[ \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 \end{array} \]
33. \[ \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \]
34. \[ \begin{array}{cccccccc} -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 \end{array} \]
35. \[ \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \]
54. The tax table gives intervals of income and how much a taxpayer with taxable income in each interval must pay in taxes. These intervals can be expressed as compound inequalities. Answers should include the following.
- The incomes are in $50 intervals.
- $$41,100 \leq x < 41,150$$ represents the possible incomes of a head of a household paying $7024 in taxes.
Page 359, Chapter 6 Study Guide and Review

9. \( \{ c | c > -19 \} \)

10. \( \{ r | r > -12 \} \)

11. \( \{ w | w \leq 37 \} \)

12. \( \{ a | a > -4 \} \)

13. \( \{ n | n \leq -0.15 \} \)

14. \( \{ g | g > 0.2 \} \)

15. \( \{ h | h \leq -1 \} \)

16. \( \{ b | b > 5 \} \)

Page 363, Chapter 6 Practice Test

2. \( 6(a + 5) < 2a + 8 \)

Original equation

Distributive Property

\( 6a + 30 < 2a + 8 - 2a \)

Subtract 2a from each side.

\( 4a + 30 < 8 \)

Simplify.

\( 4a + 30 - 30 < 8 - 30 \)

Subtract 30 from each side.

\( 4a < -22 \)

Simplify.

\( \frac{4a}{4} < \frac{-22}{4} \)

Divide each side by 4.

\( a < -5.5 \)

Simplify.