# Chapter 7

## Solving Systems of Linear Equations and Inequalities

### Chapter Overview and Pacing

Year-long and two-year pacing: pages T20–T21.

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An electronic version of this chapter is available on StudentWorks™. This backpack solution CD-ROM allows students instant access to the Student Edition, lesson worksheet pages, and web resources.
## Chapter Resource Manager

### Chapter 7 Resource Masters

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*Key to Abbreviations:* GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual

**ELL** Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.
Mathematical Connections
and Background

Continuity of Instruction

Prior Knowledge
In Chapter 2, students learned that when you add additive inverses, the sum is always 0. They used the Addition, Subtraction, Multiplication, and Division Properties of Equality to solve equations throughout Chapter 3. Students graph linear equations in Chapter 5 and graph linear inequalities in Chapter 6.

This Chapter
Chapter 7 introduces students to systems of linear equations. They first solve the systems by graphing and then classify the systems as consistent or inconsistent, and as independent or dependent. Students also learn to apply the algebraic methods to solving the systems. These methods include substitution, elimination using addition or subtraction, and elimination using multiplication first. Students must determine which method is best for different systems. The chapter ends with students solving systems of inequalities by graphing.

Future Connections
Business analysts use systems of linear equations to determine where break-even points are and to analyze trends for predicting future events. There are not only systems of linear equations and inequalities, but also systems of all types of functions including quadratic, absolute value, and sine. These systems can mix any types of functions. The solutions of these systems are not only used in business, but also in science and other fields.

Graphing Systems of Equations
A solution of a system of equations is the set of points that satisfy each equation in the system. Carefully graph each equation on the same coordinate plane. There is only one solution if the graphs of the lines intersect, since the intersection is at only one point. There is no solution if the lines are parallel. The lines never intersect, so no one point is common to both graphs. If the graphs are the same line, the system has an infinite number of solutions.

If there is one solution or infinitely many solutions, the system of equations is described as consistent. Systems with one solution are said to be independent, while those with infinitely many solutions are said to be dependent. If there is no solution, the system is described as inconsistent.

Substitution
It is sometimes difficult to determine the exact solution of a system of equations from a graph. Therefore, an algebraic method may be used to find the exact solution. Substitution is an algebraic method. First solve one equation for one variable in terms of the other. Substitute the expression into the other equation so that one variable is eliminated. Solve for the remaining variable. Substitute this value into either equation and solve for the other variable. The two values make up the solution of the system. They are written in the form \((x, y)\) to represent the point where the two lines intersect if graphed.

If a solution results in an identity, for example \(2 = 2\), the system has an infinite number of solutions. If the result is a false statement, such as \(5 = 3\), there is no solution. If you incur either of these situations during any part of the solution process, you may stop solving and write either infinitely many solutions or no solution.
Elimination Using Addition and Subtraction

Elimination is another algebraic method used to solve systems of equations. The objective is to combine the two equations to eliminate one of the variables. If the coefficients of one variable are additive inverses of each other, use addition. If the coefficients of one variable are the same, use subtraction. Because the Addition and Subtraction Properties of Equality state that equal amounts can be added to or subtracted from each side of an equation, you can add or subtract one equation with the other. This step eliminates one variable. Then solve for the other variable. Substitute this value into either original equation to find the value of the variable that was eliminated. The two values are the solution of the system.

Elimination Using Multiplication

If the coefficients of one variable are neither additive inverses nor equal, one or both equations must be changed so that the elimination method can be applied to solve the system of equations. Change either one, or both, of the equations by applying the Multiplication Property of Equality. Every term of the equation is multiplied by the same number, or both equations are multiplied by different numbers, in order to make one pair of coefficients of a variable either additive inverses or the same. Then follow the steps for solving the system using the elimination method.

Five methods for solving systems of equations have been seen in this chapter. They are graphing, substitution, elimination using addition, elimination using subtraction, and elimination using multiplication. Graphing is used only if an estimation is needed, since it is difficult to get an exact solution. Use substitution if one of the variables has a coefficient of 1 or −1. Elimination using addition is used when one of the variables has coefficients that are additive inverses of each other, and elimination using subtraction is best used when one of the variables has the same coefficient. Apply elimination using multiplication if none of the above situations occur.

Graphing Systems of Inequalities

A solution of a system of inequalities is all the points that satisfy both inequalities. Use the methods learned in Lesson 6-6 to graph each inequality. The points that are solutions of both inequalities lie in the region where the graphs overlap, or intersect. A system of inequalities has no solution if the boundary lines are parallel and the shaded regions do not overlap. Otherwise, there are infinitely many solutions since the overlapping shaded region extends on indefinitely.

Quick Review Math Handbook

Hot Words includes a glossary of terms while Hot Topics consists of explanations of key mathematical concepts with exercises to test comprehension. This valuable resource can be used as a reference in the classroom or for home study.

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GS = Getting Started, P = Preview
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Prerequisite Skills Workbook, pp. 27–28<br>
Quizzes, CRM pp. 447–448<br>
Mid-Chapter Test, CRM p. 449<br>
www.algebra1.com/self_check_quiz<br>
www.algebra1.com/extra_examples |
| Mixed Review       | pp. 374, 381, 386, 392, 398                         | Cumulative Review, CRM p. 450                                                    |                                                          |
| Error Analysis     | Find the Error, pp. 384, 396                         | Find the Error, TWE pp. 384, 396<br>
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| Standardized Test Practice | pp. 374, 381, 384, 385, 386, 392, 398, 403, 404–405 | TWE pp. 404–405<br>
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www.algebra1.com/standardized_test |
| Open-Ended Assessment | Writing in Math, pp. 374, 381, 386, 392, 398       | Modeling: TWE pp. 374<br>
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Open-Ended Assessment, CRM p. 445 |                                                          |
| Chapter Assessment | Study Guide, pp. 399–402 Practice Test, p. 403      | Multiple-Choice Tests (Forms 1, 2A, 2B), CRM pp. 433–438<br>
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Vocabulary Test/Review, CRM p. 446 | ExamView® Pro (see below)<br>
MindJogger Videoquizzes<br>
www.algebra1.com/vocabulary_review<br>
www.algebra1.com/chapter_test |

For more information on Yearly ProgressPro, see p. 188.

ExamView® Pro

Use the networkable ExamView® Pro to:
• Create multiple versions of tests.
• Create modified tests for Inclusion students.
• Edit existing questions and add your own questions.
• Use built-in state curriculum correlations to create tests aligned with state standards.
• Change English tests to Spanish and vice versa.

For more information on Intervention and Assessment, see pp. T8–T11.
Reading and Writing in Mathematics

Glencoe Algebra 1 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

**Student Edition**
- Foldables Study Organizer, p. 367
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 371, 379, 384, 390, 396)
- Reading Mathematics, p. 393
- Writing in Math questions in every lesson, pp. 374, 381, 386, 392, 398
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**Teacher Wraparound Edition**
- Foldables Study Organizer, pp. 367, 399
- Study Notebook suggestions, pp. 372, 379, 384, 390, 393, 396
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- Speaking activities, pp. 381, 392
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- **ELL** Resources, pp. 366, 373, 380, 385, 389, 391, 393, 397, 399

For more information on Reading and Writing in Mathematics, see pp. T6–T7.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Business decision makers often use systems of linear equations to model a real-world situation in order to predict future events. Being able to make an accurate prediction helps them plan and manage their businesses.

Trends in the travel industry change with time. For example, in recent years, the number of tourists traveling to South America, the Caribbean, and the Middle East is on the rise. You will use a system of linear equations to model the trends in tourism in Lesson 7-2.

**Key Vocabulary**
- system of equations (p. 369)
- substitution (p. 376)
- elimination (p. 382)
- system of inequalities (p. 394)
Prerequisite Skills To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1  
Graph Linear Equations
Graph each equation. (For review, see Lesson 4-5.)  
1. \( y = 1 \)  
2. \( y = -2x \)  
3. \( y = 4 - x \)  
4. \( y = 2x + 3 \)  
5. \( y = 5 - 2x \)  
6. \( y = \frac{1}{2}x + 2 \)

For Lesson 7-2  
Solve for a Given Variable
Solve each equation or formula for the variable specified. (For review, see Lesson 3-8.)  
7. \( 4x + a = 6x, \) for \( x = \frac{a}{2} \)  
8. \( 8a + y = 16, \) for \( a = \frac{16 - y}{8} \)  
9. \( \frac{7bc - d}{10} = 12, \) for \( b = \frac{120 + d}{7c} \)  
10. \( \frac{7m + n}{q} = 2m, \) for \( q = \frac{7m + n}{2m} \)

For Lessons 7-3 and 7-4  
Simplify Expressions
Simplify each expression. If not possible, write simplified. (For review, see Lesson 1-5.)  
11. \( (3x + y) - (2x + y) \)  
12. \( (7x - 2y) - (7x + 4y) = -6y \)  
13. \( (16x - 3y) + (11x + 3y) = 27x \)  
14. \( (8x - 4y) + (-8x + 5y) \)  
15. \( 4(2x + 3y) - (8x - y) = 13y \)  
16. \( 3(x - 4y) + (x + 12y) = 4x \)  
17. \( 2(x - 2y) + (3x + 4y) = 5x \)  
18. \( 5(2x - y) - 2(5x + 3y) = -11y \)  
19. \( 3(x + 4y) + 2(2x - 6y) = 7x \)

Solving Systems of Equations and Inequalities Make this Foldable to help you organize your notes. Begin with five sheets of grid paper.

Foldables Study Organizer

Step 1 Fold
Fold each sheet in half along the width.

Step 2 Cut
Unfold and cut four rows from the left side of each sheet, from the top to the crease.

Step 3 Stack and Staple
Stack the sheets and staple to form a booklet.

Step 4 Label
Label each page with a lesson number and title.

Tips for New Teachers
In addition to reviewing graphing as in Exercises 1-6, you may want to review slope-intercept form. These are two skills essential for students’ success in this chapter.

Organization of Data: Visualization Journal After students make their visualization journal, have them label the top of each front page with a lesson number. Under the tabs of their Foldable, students take notes and define terms presented in each lesson. At the end of each lesson, ask students to design a visual (graph, diagram, picture, chart) that presents the lesson information in a concise, easy-to-study format. Encourage students to clearly label their visuals.

For more information about Foldables, see Teaching Mathematics with Foldables.
Copying Formulas  Once the spreadsheet formulas are entered for one sales amount, those formulas can be dragged to copy for all the other sales amounts. Students must take their time creating the first formulas as any errors will be duplicated to all the other cells.

Teach

- In Excel, you create a graph by using the Chart Wizard button on the standard toolbar. If using another spreadsheet software program, consult the Help feature for instructions on creating graphs (charts).
- Remind students that spreadsheet software cannot interpret an expression such as 0.1x. Students must type the multiplication sign implied.
- Urge students not to assume that the software will apply the order of operations. Students should use parentheses to ensure that operations are performed in the correct order.
- You may wish to extend the activity by discussing the pros and cons of different sales commissions. Students who can imagine themselves as salespeople can relate personally to the amounts calculated and to the process of using equations to explore pay rates.

Assess

Exercises 1–2 Students should write the correct expressions to translate the real-world scenario into an algebraic representation.

Exercise 4 Students should be able to translate between the mathematical solution and its real-world meaning.

Example

Bill Winters is considering two job offers in telemarketing departments. The salary at the first job is $400 per week plus 10% commission on Mr. Winters’ sales. At the second job, the salary is $375 per week plus 15% commission. For what amount of sales would the weekly salary be the same at either job?

Enter different amounts for Mr. Winters’ weekly sales in column A. Then enter the formula for the salary at the first job in each cell in column B. In each cell of column C, enter the formula for the salary at the second job.

The spreadsheet shows that for sales of $500 the total weekly salary for each job is $450.

4. (500, 450); If Mr. Winters makes $500 in sales, he will make $450 for either job.
5. Sample answer: Write and graph two linear equations. Find the point where the graphs intersect.

Exercises

For Exercises 1–4, use the spreadsheet of weekly salaries above.

1. If \( x \) is the amount of Mr. Winters’ weekly sales and \( y \) is his total weekly salary, write a linear equation for the salary at the first job. \( y = 400 + 0.1x \)

2. Write a linear equation for the salary at the second job. \( y = 375 + 0.15x \)

3. Which ordered pair is a solution for both of the equations you wrote for Exercises 1 and 2? c
   - a. (100, 410)
   - b. (300, 420)
   - c. (500, 450)
   - d. (900, 510)

4. Use the graphing capability of the spreadsheet program to graph the salary data using a line graph. At what point do the two lines intersect? What is the significance of that point in the real-world situation?

5. How could you find the sales for which Mr. Winters’ salary will be equal without using a spreadsheet?
During the 1990s, sales of cassette singles decreased, and sales of CD singles increased. Assume that the sales of these singles were linear functions. If \( x \) represents the years since 1991 and \( y \) represents the sales in millions of dollars, the following equations represent the sales of these singles.

- **Cassette singles**: \( y = 69 - 6.9x \)
- **CD singles**: \( y = 5.7 + 6.3x \)

These equations are graphed at the right.

The point at which the two graphs intersect represents the time when the sales of cassette singles equaled the sales of CD singles. The ordered pair of this point is a solution of both equations.

**NUMBER OF SOLUTIONS**

Two equations, such as \( y = 69 - 6.9x \) and \( y = 5.7 + 6.3x \), together are called a **system of equations**. A solution of a system of equations is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have 0, 1, or an infinite number of solutions.

- If the graphs intersect or coincide, the system of equations is said to be **consistent**. That is, it has at least one ordered pair that satisfies both equations.
- If the graphs are parallel, the system of equations is said to be **inconsistent**. There are **no** ordered pairs that satisfy both equations.
- Consistent equations can be **independent** or **dependent**. If a system has exactly one solution, it is independent. If the system has an infinite number of solutions, it is dependent.
**Example 1** Number of Solutions

Use the graph at the right to determine whether each system has no solution, one solution, or infinitely many solutions.

a. \( y = -x + 5 \)
   \( y = x - 3 \)
   Since the graphs of \( y = -x + 5 \) and \( y = x - 3 \) are intersecting lines, there is one solution.

b. \( y = -x + 5 \)
   \( 2x + 2y = -8 \)
   Since the graphs of \( y = -x + 5 \) and \( 2x + 2y = -8 \) are parallel, there are no solutions.

c. \( 2x + 2y = -8 \)
   \( y = -x - 4 \)
   Since the graphs of \( 2x + 2y = -8 \) and \( y = -x - 4 \) coincide, there are infinitely many solutions.

**SOLVE BY GRAPHING** One method of solving systems of equations is to carefully graph the equations on the same coordinate plane.

**Example 2** Solve a System of Equations

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

a. \( y = -x + 8 \)
   \( y = 4x - 7 \)
   The graphs appear to intersect at the point with coordinates \((3, 5)\). Check this estimate by replacing \( x \) with 3 and \( y \) with 5 in each equation.

   **CHECK**
   \[ y = -x + 8 \quad y = 4x - 7 \]
   \[ 5 \neq -3 + 8 \quad 5 \neq 4(3) - 7 \]
   \[ 5 = 5 \quad 5 = 5 \]
   The solution is \((3, 5)\).

b. \( x + 2y = 5 \)
   \( 2x + 4y = 2 \)
   The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Notice that the lines have the same slope but different \( y \)-intercepts. **Recall that a system of equations that has no solution is said to be inconsistent.**
Example 3 Write and Solve a System of Equations

WORLD RECORDS Use the information on Guy Delage’s swim at the left. If Guy can swim 3 miles per hour for an extended period and the raft drifts about 1 mile per hour, how many hours did he spend swimming each day?

Words You have information about the amount of time spent swimming and floating. You also know the rates and the total distance traveled.

Variables Let $s =$ the number of hours Guy swam, and let $f =$ the number of hours he floated each day. Write a system of equations to represent the situation.

Equations

\[
\begin{align*}
\text{The number of hours swimming} & \quad + \quad \text{the number of hours floating} & \quad = \quad \text{the total number of hours in a day.} \\
3s & \quad + \quad f & \quad = \quad 24 \\
\text{The daily miles traveled swimming} & \quad + \quad \text{the daily miles traveled floating} & \quad = \quad \text{the total miles traveled in a day.} \\
3s & \quad + \quad 1f & \quad = \quad 44
\end{align*}
\]

Graph the equations $s + f = 24$ and $3s + f = 44$. The graphs appear to intersect at the point with coordinates $(10, 14)$. Check this estimate by replacing $s$ with 10 and $f$ with 14 in each equation.

CHECK $s + f = 24$ $3s + f = 44$

\[
\begin{align*}
10 + 14 & \quad \leq \quad 24 \\
30 + 14 & \quad \leq \quad 44 \\
24 & \quad = \quad 24 \checkmark \\
44 & \quad = \quad 44 \checkmark
\end{align*}
\]

Guy Delage spent about 10 hours swimming each day.

Check for Understanding

Concept Check

1–3. See pp. 405A–405D.

1. OPEN ENDED Draw the graph of a system of equations that has one solution at $(-2, 3)$.

2. Determine whether a system of equations with $(0, 0)$ and $(2, 2)$ as solutions sometimes, always, or never has other solutions. Explain.

3. Find a counterexample for the following statement.

   If the graphs of two linear equations have the same slope, then the system of equations has no solution.

Guided Practice

Use the graph at the right to determine whether each system has no solution, one solution, or infinitely many solutions.

4. $y = x - 4$ $y = \frac{1}{3}x - 2$ one solution

5. $y = \frac{1}{3}x + 2$ no solution

6. $x - y = 4$ $y = x - 4$ infinitely many

7. $x - y = 4$ $y = -\frac{1}{3}x + 4$ one solution

Lesson 7-1 Graphing Systems of Equations 371

SOLVE BY GRAPHING

In-Class Examples

2. Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

   a. $2x - y = -3$ infinitely many solutions

   $8x - 4y = -12$ solutions

   b. $x - 2y = 4$ no solution

   $x - 2y = -2$

3. BICYCLING Tyler and Pearl went on a 20-kilometer bike ride that lasted 3 hours. Because there were many steep hills on the bike ride, they had to walk for most of the trip. Their walking speed was 4 kilometers per hour. Their riding speed was 12 kilometers per hour. How much time did they spend walking?

   Let $r =$ the number of hours they rode and $w =$ the number of hours they walked.

   $r + w = 3$

   $12r + 4w = 20$

   They walked for 2 hours.
Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

8. \( y = -x \)  \( y = 2x + 1 \) one; \((0, 0)\)

9. \( x + y = 4 \)  \( x - y = 2 \) one; \((3, 1)\)

10. \( 2x + 4y = 2 \) infinitely many

11. \( x + y = 4 \)  \( x + y = 1 \) no solution

12. \( x - y = 2 \)  \( 3x + 2y = 9 \) one; \((3, 1)\)

13. \( x + y = 2 \)  \( y = 4x + 7 \) one; \((-1, 3)\)

14. RESTAURANTS A restaurant charges one price for adults and another price for children. The Rodriguez family has two adults and three children, and their bill was $40.50. The Wong family has three adults and one child. Their bill was $38. Determine the price of the buffet for an adult and the price for a child.

$10.50; $6.50

 ★ indicates increased difficulty

Practice and Apply

Use the graph at the right to determine whether each system has no solution, one solution, or infinitely many solutions.

15. \( x = -3 \)  \( y = 2x + 1 \) one

16. \( y = -x - 2 \)  \( y = 2x - 4 \) one

17. \( y + x = -2 \)  \( y = -x - 2 \) many

18. \( y = 2x + 1 \)  \( y = 2x - 4 \) no solution

19. \( y = -3x + 6 \)  \( y = 2x - 4 \) one

20. \( 2y - 4x = 2 \)  \( y = 2x - 4 \) no solution

21. \( 2y - 4x = 2 \)  \( y = 3x + 6 \) one

22. \( 2y - 4x = 2 \)  \( y = 2x + 1 \) infinitely many

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

23. \( y = -6 \)  \( 4x + y = 2 \) one; \((2, -6)\)

24. \( x = 2 \)  \( 3x - y = 8 \) one; \((2, -2)\)

25. \( y = \frac{1}{2}x \)  \( 2x + y = 10 \) one; \((4, 2)\)

26. \( y = -x \)  \( y = 2x - 6 \) one; \((2, -2)\)

27. \( y = 3x - 4 \)  \( y = -3x - 4 \) one; \((0, -4)\)

28. \( y = 2x + 6 \)  \( y = -x - 3 \) one; \((-3, 0)\)

29. \( x - 2y = 2 \)  \( 3x + y = 6 \) one; \((2, 0)\)

30. \( x + y = 2 \)  \( 2y - x = 10 \) one; \((-2, 4)\)

31. \( 3x + 2y = 12 \)  \( 3x + 2y = 6 \) no solution

32. \( 2x + 3y = 4 \) infinitely many

33. \( 2x + 3y = -4 \)  \( 5x + 3y = -6 \) one; \((-6, 8)\)

34. \( 5x - 6y = -8 \)  \(-4x - 6y = -8 \)

35. \( 3x + y = 3 \) infinitely many

36. \( y = x + 3 \)  \( y = x + 5 \) one; \((-1, 2)\)

37. \( 2x + 3y = -17 \)  \( y = x - 4 \) one; \((-1, -5)\)

38. \( y = \frac{2}{3}x - 5 \) infinitely many

39. \( 6 - \frac{3}{8}y = x \) infinitely many

40. \( \frac{1}{2}x + \frac{1}{3}y = 6 \)  \( y = \frac{2}{3}x + 2 \) one; \((8, 6)\)

41. GEOMETRY The length of the rectangle at the right is 1 meter less than twice its width. What are the dimensions of the rectangle?

13 m by 7 m

DAILY INTERVENTION

Unlocking Misconceptions

Infinitely Many Solutions Though two equations may look different because variables are on different sides of the equals sign, or because all terms are multiplied by a common factor, any two equations that have the same graph are equivalent. The easiest way to tell whether two equations have the same graph, and thus infinitely many solutions, is to rewrite both equations in slope-intercept form.
GEOMETRY For Exercises 42 and 43, use the graphs of \( y = 2x + 6 \), \( 3x + 2y = 19 \), and \( y = 2 \), which contain the sides of a triangle.

42. Find the coordinates of the vertices of the triangle. \((-2, 2), (5, 2), (1, 8)\)
43. Find the area of the triangle. \(21 \text{ units}^2\)

BALLOONING For Exercises 44 and 45, use the information in the graphic at the right.

44. In how many minutes will the balloons be at the same height? \(4 \text{ min}\)
45. How high will the balloons be at that time? \(70 \text{ m}\)

SAVINGS For Exercises 46 and 47, use the following information.
Monica and Michael Gordon both want to buy a scooter. Monica has already saved $25 and plans to save $5 per week until she can buy the scooter. Michael has $16 and plans to save $8 per week.

46. In how many weeks will Monica and Michael have saved the same amount of money? \(3 \text{ weeks}\)
47. How much will each person have saved at that time? \$40

BUSINESS For Exercises 48–50, use the graph at the right.

48. Which company had the greater profit during the ten years? Widget Company
49. Which company had a greater rate of growth? neither
50. If the profit patterns continue, will the profits of the two companies ever be equal? Explain. No; the graphs are parallel so the lines will never meet and there is no year when the profits will be equal.

POPULATION For Exercises 51–54, use the following information.

The U.S. Census Bureau divides the country into four regions. They are the Northeast, the Midwest, the South, and the West.

51. \( p = 60 + 0.4t \) ★  
52. \( p = 53 + 1t \) or \( p = 53 + t \)

51. In 1990, the population of the Midwest was about 60 million. During the 1990s, the population of this area increased an average of about 0.4 million per year. Write an equation to represent the population of the Midwest for the years since 1990.

52. The population of the West was about 53 million in 1990. The population of this area increased an average of about 1 million per year during the 1990s. Write an equation to represent the population of the West for the years since 1990.

53. Graph the population equations. See pp. 405A–405D.

54. Assume that the rate of growth of each of these areas remains the same. Estimate when the population of the West would be equal to the population of the Midwest. In about 11.7 years or sometime in 2001

55. CRITICAL THINKING The solution of the system of equations \( Ax + y = 5 \) and \( Ax + By = 20 \) is \((2, -3)\). What are the values of \( A \) and \( B? \) \( A = 4, B = -4 \)

You can graph a system of equations to predict when men’s and women’s Olympic times will be the same. Visit www.algebra1.com/webquest to continue work on your WebQuest project.
Open-Ended Assessment
Modeling  Model a line on a coordinate plane with string, spaghetti, or a similar item. Then ask volunteers to come up and model another line that would represent a system of equations with one solution. Do the same for systems of equations with no solutions and with infinitely many solutions.

Getting Ready for Lesson 7-2
PREREQUISITE SKILL  Students will learn to solve systems of equations by substitution in Lesson 7-2. The process of substitution involves solving equations for a specific variable. Use Exercises 65–68 to determine your students’ familiarity with solving equations for a specific variable.

Answer
56. Graphs can show when the sales of one item is greater than the sales of the other item and when the sales of the items are equal. Answers should include the following.
• The sales of cassette singles equaled the sales of CD singles in about 5 years or by the end of 1995.
• The graph of each equation contains all of the points whose coordinates satisfy the equation. If a point is contained in both lines, then its coordinates satisfy both equations.

Maintain Your Skills
Mixed Review  Determine which ordered pairs are part of the solution set for each inequality.
(Lesson 6-6)
59. \(y \leq 2x, \{(1, 4), (-1, 5), (5, -6), (-7, 0)\}\)
60. \(y < 8 - 3x, \{(-4, 2), (-3, 0), (1, 4), (1, 8)\}\)

61. MANUFACTURING  The inspector at a perfume manufacturer accepts a bottle if it is less than 0.05 ounce above or below 2 ounces. What are the acceptable numbers of ounces for a perfume bottle?  
(Lesson 6-5) \(a \mid 1.95 < a < 2.05\)

Write each equation in standard form.  
(Lesson 5-5)
62. \(y - 1 = 4(x - 5)\)
63. \(y + 2 = \frac{1}{3}(x + 3)\)
64. \(y - 4 = -6(x + 2)\)
65. \(4x - y = 19\)
66. \(x - 3y = 3\)
67. \(6x + y = -8\)

Getting Ready for the Next Lesson
PREREQUISITE SKILL  Solve each equation for the variable specified.
(To review solving equations for a specified variable, see Lesson 3-8.)
65. \(12x - y = 10x, \) for \(y = 2x\)
66. \(6a + b = 2a, \) for \(a = \frac{1}{4}b\)
67. \(\frac{7m - n}{q} = 10, \) for \(q = \frac{7m - n}{10}\)
68. \(\frac{5z - s}{2} = 6, \) for \(z = \frac{12 + s}{5t}\)
Systems of Equations

You can use a TI-83 Plus graphing calculator to solve a system of equations.

**Example**

Solve the system of equations. State the decimal solution to the nearest hundredth.

\[ 2.93x + y = 6.08 \]
\[ 8.32x - y = 4.11 \]

**Step 1** Solve each equation for \( y \) to enter them into the calculator.

\[
\begin{align*}
2.93x + y &= 6.08 \\
2.93x + y - 2.93x &= 6.08 - 2.93x \\
y &= 6.08 - 2.93x \\
8.32x - y &= 4.11 \\
8.32x - y - 8.32x &= 4.11 - 8.32x \\
- y &= 4.11 - 8.32x \\
(-1) (-y) &= (-1)(4.11 - 8.32x) \\
y &= -4.11 + 8.32x
\end{align*}
\]

**Step 2** Enter these equations in the \( Y= \) list and graph.

**KEYSTROKES:** Review on pages 224–225.

**Step 3** Use the CALC menu to find the point of intersection.

**KEYSTROKES:** \[ \text{2nd} \] [CALC] 5 ENTER ENTER ENTER

The solution is approximately \((0.91, 3.43)\).

**Exercises**

Use a graphing calculator to solve each system of equations. Write decimal solutions to the nearest hundredth.

1. \( y = 3x - 4 \)
   \( y = -0.5x + 6 \) \((2.86, 4.57)\)
2. \( y = 2x + 5 \)
   \( y = -0.2x - 4 \) \((-4.09, -3.18)\)
3. \( x + y = 5.35 \)
   \( 3x - y = 3.75 \) \((2.28, 3.08)\)
4. \( 0.35x - y = 1.12 \)
   \( 2.25x + y = -4.05 \) \((-1.13, -1.51)\)
5. \( 1.5x + y = 6.7 \)
   \( 5.2x - y = 4.1 \) \((1.61, 4.28)\)
6. \( 5.4x - y = 1.8 \)
   \( 6.2x + y = -3.8 \) \((-0.17, -2.73)\)
7. \( 5x - 4y = 26 \)
   \( 4x + 2y = 53.3 \) \((10.2, 6.25)\)
8. \( 2x + 3y = 11 \)
   \( 4x + y = -6 \) \((-2.9, 5.6)\)
9. \( 0.22x + 0.15y = 0.30 \)
   \(-0.33x + y = 6.22 \) \((-2.35, 5.44)\)
10. \( 125x - 200y = 800 \)
    \( 65x - 20y = 140 \) \((1.14, -3.29)\)

www.algebra1.com/other_calculator_keystrokes
1 Focus

5-Minute Check Transparency 7-2 Use as a quiz or review of Lesson 7-1.

Mathematical Background notes are available for this lesson on p. 366C.

**How can a system of equations be used to predict media use?**

Ask students:

- Which is changing at a greater rate: the number of newspaper readers, or the number of people online? the number of people online

- According to the graph, in about what year will the number of hours online per person equal time spent reading newspapers? in about 2003

- If the two equations weren’t labeled, how would you know which was which? The one with negative slope represents newspaper readers because the time spent reading newspapers is declining.

**SUBSTITUTION** The exact solution of a system of equations can be found by using algebraic methods. One such method is called substitution.

**Algebra Activity**

Using Substitution

Use algebra tiles and an equation mat to solve the system of equations.

3x + y = 8 and y = x – 4

**Model and Analyze**

Since y = x – 4, use 1 positive x tile and 4 negative 1 tiles to represent y. Use algebra tiles to represent 3x + y = 8.

1. Use what you know about equation mats to solve for x. What is the value of x? 3

2. Use y = x – 4 to solve for y. −1

3. What is the solution of the system of equations? (3, −1)

**Make a Conjecture**

4. Explain how to solve the following system of equations using algebra tiles.

4x + 3y = 10 and y = x + 1

5. Why do you think this method is called substitution?
**Example 1** Solve Using Substitution

Use substitution to solve the system of equations.

\[ y = 3x \]
\[ x + 2y = -21 \]

Since \( y = 3x \), substitute \( 3x \) for \( y \) in the second equation.

\[ x + 2(3x) = -21 \quad \text{Second equation} \]
\[ x + 6x = -21 \quad \text{Simplify.} \]
\[ 7x = -21 \quad \text{Combine like terms.} \]
\[ \frac{7x}{7} = \frac{-21}{7} \quad \text{Divide each side by 7.} \]
\[ x = -3 \quad \text{Simplify.} \]

Use \( y = 3x \) to find the value of \( y \).

\[ y = 3x \quad \text{First equation} \]
\[ y = 3(-3) \quad x = -3 \]
\[ y = -9 \quad \text{The solution is} \ (-3, -9) \].

**Example 2** Solve for One Variable, Then Substitute

Use substitution to solve the system of equations.

\[ x + 5y = -3 \]
\[ 3x - 2y = 8 \]

Solve the first equation for \( x \) since the coefficient of \( x \) is 1.

\[ x + 5y = -3 \quad \text{First equation} \]
\[ x + 5y - 5y = -3 - 5y \quad \text{Subtract 5y from each side.} \]
\[ x = -3 - 5y \quad \text{Simplify.} \]

Find the value of \( y \) by substituting \(-3 - 5y\) for \( x \) in the second equation.

\[ 3x - 2y = 8 \quad \text{Second equation} \]
\[ 3(-3 - 5y) - 2y = 8 \]
\[ -9 - 15y - 2y = 8 \quad \text{Distributive Property} \]
\[ -9 - 17y = 8 \quad \text{Combine like terms.} \]
\[ -9 - 17y + 9 = 8 + 9 \quad \text{Add 9 to each side.} \]
\[ -17y = 17 \quad \text{Simplify.} \]
\[ \frac{-17y}{-17} = \frac{17}{-17} \quad \text{Divide each side by} \ -17. \]
\[ y = -1 \quad \text{Simplify.} \]

Substitute \(-1\) for \( y \) in either equation to find the value of \( x \).

Choose the equation that is easier to solve.

\[ x + 5y = -3 \quad \text{First equation} \]
\[ x + 5(-1) = -3 \]
\[ x - 5 = -3 \quad \text{Simplify.} \]
\[ x = 2 \quad \text{Add 5 to each side.} \]

The solution is \((2, -1)\). The graph verifies the solution.

---

**Algebra Activity**

**Materials:** algebra tiles, equation mat

- Remind students that anything they add to one side of the equation mat must also be added to the other side of the mat.
- Remind students that the purpose is to eliminate zero pairs.
- Once a value is found for \( x \), have students use that value to find \( y \).
In-Class Example

Teaching Tip If there are infinitely many solutions, then the two equations represent the same line. If you solve both equations for y, then you will see this is true. If there are no solutions, then the equations represent two parallel lines. If you solve both equations for y, the equations will have the same slope but a different y-intercept.

3 Use substitution to solve the system of equations.

\[2x + 2y = 8\]
\[x + y = -2\]

no solution

REAL-WORLD PROBLEMS

In-Class Example

4 GOLD Gold is alloyed with different metals to make it hard enough to be used in jewelry. The amount of gold present in a gold alloy is measured in 24ths called karats.

24-karat gold is \(\frac{24}{24}\) or 100% gold. Similarly, 18-karat gold is \(\frac{18}{24}\) or 75% gold. How many ounces of 18-karat gold should be added to an amount of 12-karat gold to make 4 ounces of 14-karat gold?

1 \(\frac{1}{3}\) ounces of 18-karat gold and 2 \(\frac{2}{3}\) ounces of 12-karat gold

Example 3 Dependent System

Use substitution to solve the system of equations.

\[6x - 2y = -4\]
\[y = 3x + 2\]

Since \(y = 3x + 2\), substitute \(3x + 2\) for \(y\) in the first equation.

\[6x - 2(3x + 2) = -4\]
\[6x - 6x - 4 = -4\]

Distributive Property
\[-4 = -4\] Simplify.

The statement \(-4 = -4\) is true. This means that there are infinitely many solutions of the system of equations. This is true because the slope-intercept form of both equations is \(y = 3x + 2\). That is, the equations are equivalent, and they have the same graph.

In general, if you solve a system of linear equations and the result is a true statement (an identity such as \(-4 = -4\)), the system has an infinite number of solutions. However, if the result is a false statement (for example, \(-4 = 5\)), the system has no solution.

REAL-WORLD PROBLEMS Sometimes it is helpful to organize data before solving a problem. Some ways to organize data are to use tables, charts, different types of graphs, or diagrams.

Example 4 Write and Solve a System of Equations

METAL ALLOYS A metal alloy is 25% copper. Another metal alloy is 50% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper?

Let \(a\) = the number of grams of the 25% copper alloy and \(b\) = the number of grams of the 50% copper alloy. Use a table to organize the information.

<table>
<thead>
<tr>
<th>Grams of Copper</th>
<th>25% Copper</th>
<th>50% Copper</th>
<th>45% Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Grams</td>
<td>(a)</td>
<td>(b)</td>
<td>1000</td>
</tr>
<tr>
<td>Grams of Copper</td>
<td>0.25(a)</td>
<td>0.50(b)</td>
<td>0.45(1000)</td>
</tr>
</tbody>
</table>

The system of equations is \(a + b = 1000\) and \(0.25a + 0.50b = 0.45(1000)\). Use substitution to solve this system.

\[a + b = 1000\] First equation
\[a + b - b = 1000 - b\] Subtract \(b\) from each side.
\[a = 1000 - b\] Simplify.

\[0.25a + 0.50b = 0.45(1000)\] Second equation
\[0.25(1000 - b) + 0.50b = 0.45(1000)\]
\[250 - 0.25b + 0.50b = 450\] Distributive Property
\[250 + 0.25b = 450\] Combine like terms.
\[250 + 0.25b - 250 = 450 - 250\] Subtract 250 from each side.
\[0.25b = 200\] Simplify.
\[\frac{0.25b}{0.25} = \frac{200}{0.25}\] Divide each side by 0.25.
\[b = 800\] Simplify.
Check for Understanding

1. Explain why you might choose to use substitution rather than graphing to solve a system of equations. \( \text{Substitution may result in a more accurate solution.} \)

2. Describe the graphs of two equations if solving the system of equations yields the equation \( 4 = 2 \). \( \text{They are parallel lines.} \)

3. OPEN-ENDED Write a system of equations that has infinitely many solutions. 
Sample answer: \( y = x + 3, 2y = 2x + 6 \)

Guided Practice

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.

<table>
<thead>
<tr>
<th>GUIDED PRACTICE KEY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercises</td>
</tr>
<tr>
<td>4–9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

4. \( x = 2y \)
   \( 4x + 2y = 15 \)
   \( 5. y = 3x - 8 \)
   \( y = 4 - x \)
   \( 6. 2x + 7y = 3 \)
   \( x = 1 - 4y \)
   \( (3, 15) \)
   \( (3, 1) \)
   \( (5, -1) \)

7. \( 6x - 2y = -4 \)
   \( 8. x + 3y = 12 \)
   \( y = 3x + 2 \)
   \( \text{infinitely many} \)
   \( x - y = 8 \)
   \( (9, 1) \)
   \( 3x - 3y = 15 \)

Application

10. TRANSPORTATION The Thrust SSC is the world’s fastest land vehicle. Suppose the driver of a car whose top speed is 200 miles per hour requests a race against the SSC. The car gets a head start of one-half hour. If there is unlimited space to race, at what distance will the SSC pass the car? \( \text{about 135.5 mi} \)

About the Exercises...

Organization by Objective
- Substitution: 11–28
- Real-World Problems: 29–37

Odd/Even Assignments
Exercises 11–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 38 requires the Internet or other research materials.

Assignment Guide
- Basic: 11–33 odd, 34, 35, 39–53
- Average: 11–33 odd, 34, 35, 37, 39–53
- Advanced: 12–32 even, 36–49 (optional: 50–53)
- All: Practice Quiz 1 (1–5)

www.algebra1.com/self_check_quiz/sol
29. GEOMETRY The base of the triangle is 4 inches longer than the length of one of the other sides. Use a system of equations to find the length of each side of the triangle. 14 in., 14 in., 18 in.

30. FUND-RAISING The Future Teachers of America Club at Paint Branch High School is making a healthy trail mix to sell to students during lunch. The mix will have three pounds of raisins as sunflower seeds. Sunflower seeds cost $4.00 per pound, and raisins cost $1.50 per pound. If the group has $34.00 to spend on the raisins and sunflower seeds, how many pounds of each should they buy? 4 lb of sunflower seeds, 12 lb of raisins

31. CHEMISTRY MX Labs needs to make 500 gallons of a 34% acid solution. The only solutions available are a 25% acid solution and a 50% acid solution. How many gallons of each should they buy to make the 34% solution? 320 gal of 25% acid, 180 gal of 50% acid

32. GEOMETRY Supplementary angles are two angles whose measures sum to 180 degrees. Angles X and Y are supplementary, and the measure of angle X is 24 degrees greater than the measure of angle Y. Find the measures of angles X and Y. m \( \angle X = 102 \), m \( \angle Y = 78 \)

33. SPORTS At the end of the 2000 baseball season, the New York Yankees and the Cincinnati Reds had won a total of 31 World Series. The Yankees had won 5.2 times as many World Series as the Reds. How many World Series did each team win? Yankees: 26, Reds: 5

38. RESEARCH Use the Internet or other resources to find the pricing plans for various call phones. Determine the number of minutes you would need to use the phone for two plans to cost the same amount of money. Support your answer with a table, a graph, and/or an equation. See students' work.
39. CRITICAL THINKING  Solve the system of equations. Write the solution as an ordered triple of the form \((x, y, z)\).

\[
\begin{align*}
2x + 3y - z &= 17 \\
y &= -3z - 7 \\
2x &= z + 2 \quad (\text{-1}, 5, -4)
\end{align*}
\]

40. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson. See margin.

How can a system of equations be used to predict media use?

Include the following in your answer:

- an explanation of solving a system of equations by using substitution, and
- the year when the number of hours spent reading daily newspapers is the same as the hours spent online.

41. When solving the following system, which expression could be substituted for \(x\)?

\[
\begin{align*}
x + 4y &= 1 \\
2x - 3y &= -9
\end{align*}
\]

\(\text{A} 4y - 1 \quad \text{B} 1 - 4y \quad \text{C} 3y - 9 \quad \text{D} -9 - 3y\)

42. If \(x - 3y = -9\) and \(5x - 2y = 7\), what is the value of \(x\)?

\(\text{A} 1 \quad \text{B} 2 \quad \text{C} 3 \quad \text{D} 4\)

Maintain Your Skills

Mixed Review

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.  (Lesson 7-1)  43–45. See pp. 405A–405D for graphs.

43. \(x + y = 3\)  \(2x - 3y = -9\)  \(\text{no solution}\)

44. \(x + 2y = 1\)  \(2x + y = 5\)  \(\text{one; (3, -1)}\)

45. \(x + y = 4\)  \(2x + y = 6\)  \(\text{many}\)

Graph each inequality.  (Lesson 6-6)  46–48. See pp. 405A–405D.

46. \(y < -5\)

47. \(x \geq 4\)

48. \(2x + y > 6\)

49. RECYCLING  When a pair of blue jeans is made, the leftover denim scraps can be recycled. One pound of denim is left after making every five pair of jeans. How many pounds of denim would be left from 250 pairs of jeans?  (Lesson 3-6)  50 lb

Getting Ready for the Next Lesson

PREREQUISITE SKILL  Simplify each expression.

(To review simplifying expressions, see Lesson 1-5.)

50. \(6a - 9a - 3a\)

51. \(8t + 4t\)

52. \(-7g - 8g - 15g\)

53. \(7d - (2d + b)\)

54. \(5d - b\)

Practice Quiz 1  Lessons 7-1 and 7-2

1–2. See margin for graphs.

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.  (Lesson 7-1)

1. \(x + y = 3\)

\(x - y = 1\)  \(\text{one; (2, 1)}\)

2. \(3x - 2y = -6\)

\(3x - 2y = 6\)  \(\text{no solution}\)

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.  (Lesson 7-2)

3. \(x + y = 0\)

\(3x + y = -8\)  \(\text{(-4, 4)}\)

4. \(x - 2y = 5\)

\(3x - 5y = 8\)  \(\text{(-9, -7)}\)

5. \(x + y = 2\)

\(y = 2 - x\)  \(\text{infinitely many}\)

Answer

40. When problems about technology involve a system of equations, the problem can be solved by substitution. Answers should include the following.

- To solve a system of equations using substitution, solve one equation for one unknown. Substitute this value for the unknown in the other equation and solve the equation. Use this number to find the other unknown.

- The number of hours will be the same about 9.8 years after 1993. That represents the year 2002.
Focus

5-Minute Check Transparency 7-3  Use as a quiz or review of Lesson 7-2.

Mathematical Background notes are available for this lesson on p. 366D.

How can you use a system of equations to solve problems about weather?

Ask students:

• If \(2n = 36\), then how many hours of darkness are there in Seward, AK, on the winter solstice? 18 hours

• Since you know the value of \(n\) is 18, how can you find the value of \(d\)? Use substitution.

• How many hours of daylight are there in Seward, AK, on the winter solstice? 6

• Geography The Arctic Circle is at latitude 66.5° North and passes through northern Alaska. Because of the Earth’s tilt on its axis, the sun does not rise at the Arctic Circle on the winter solstice. Use this fact to write a system of equations for the number of hours of daylight at the Arctic Circle on the winter solstice. Use the problem in the text as an example. \(n + d = 24\)

ELIMINATION USING ADDITION  Sometimes adding two equations together will eliminate one variable. Using this step to solve a system of equations is called elimination.

Example 1  Elimination Using Addition

Use elimination to solve each system of equations.

\[
\begin{align*}
3x - 5y &= -16 \\
2x + 5y &= 31
\end{align*}
\]

Since the coefficients of the \(y\) terms, \(-5\) and \(5\), are additive inverses, you can eliminate the \(y\) terms by adding the equations.

\[
\begin{align*}
5x &= 15 \\
\frac{5x}{5} &= \frac{15}{5} \\
x &= 3
\end{align*}
\]

Now substitute 3 for \(x\) in either equation to find the value of \(y\).

\[
\begin{align*}
3(3) - 5y &= -16 \\
9 - 5y &= -16 \\
9 - 5y - 9 &= -16 - 9 \\
-5y &= -25 \\
\frac{-5y}{-5} &= \frac{-25}{-5} \\
y &= 5
\end{align*}
\]

The solution is \((3, 5)\).
Example 2 Write and Solve a System of Equations

Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.

Let $x$ represent the first number and $y$ represent the second number.

\[
\begin{align*}
\text{Twice one number} & \quad \text{added to} \quad \text{another number} \quad \text{is} \quad 18. \\
2x & + y = 18 \\
\text{Four times the first number} & \quad \text{minus} \quad \text{the other number} \quad \text{is} \quad 12. \\
4x & - y = 12
\end{align*}
\]

Use elimination to solve the system.

\[
\begin{align*}
2x + y & = 18 \quad \text{Write the equations in column form and add.} \\
(+) \; 4x - y & = 12 \\
6x & = 30 \quad \text{Notice that the variable $y$ is eliminated.} \\
\frac{6x}{6} & = \frac{30}{6} \quad \text{Divide each side by 6.} \\
x & = 5 \quad \text{Simplify.}
\end{align*}
\]

Now substitute 5 for $x$ in either equation to find the value of $y$.

\[
\begin{align*}
4x - y & = 12 \quad \text{Second equation} \\
4(5) - y & = 12 \quad \text{Replace $x$ with 5.} \\
20 - y & = 12 \quad \text{Simplify.} \\
20 - y - 20 & = 12 - 20 \quad \text{Subtract 20 from each side.} \\
-y & = -8 \quad \text{Simplify.} \\
\frac{-y}{-1} & = \frac{-8}{-1} \quad \text{Divide each side by $-1$.} \\
y & = 8 \quad \text{The numbers are 5 and 8.}
\end{align*}
\]

ELIMINATION USING SUBTRACTION Sometimes subtracting one equation from another will eliminate one variable.

Example 3 Elimination Using Subtraction

Use elimination to solve the system of equations.

\[
\begin{align*}
5s + 2t & = 6 \\
9s + 2t & = 22
\end{align*}
\]

Since the coefficients of the $t$ terms, 2 and 2, are the same, you can eliminate the $t$ terms by subtracting the equations.

\[
\begin{align*}
5s + 2t & = 6 \quad \text{Write the equations in column form and subtract.} \\
( - ) \; 9s + 2t & = 22 \\
-4s & = -16 \quad \text{Notice that the variable $t$ is eliminated.} \\
\frac{-4s}{-4} & = \frac{-16}{-4} \quad \text{Divide each side by $-4$.} \\
s & = 4 \quad \text{Simplify.}
\end{align*}
\]

Now substitute 4 for $s$ in either equation to find the value of $t$.

\[
\begin{align*}
5s + 2t & = 6 \quad \text{First equation} \\
5(4) + 2t & = 6 \quad s = 4 \\
20 + 2t & = 6 \quad \text{Simplify.} \\
20 + 2t - 20 & = 6 - 20 \quad \text{Subtract 20 from each side.} \\
2t & = -14 \quad \text{Simplify.} \\
\frac{2t}{2} & = \frac{-14}{2} \quad \text{Divide each side by 2.} \\
t & = -7 \quad \text{The solution is (4, -7).}
\end{align*}
\]

Building on Prior Knowledge

In Lesson 7-2 students learned to solve a system of equations by using substitution. Even though they are learning different methods for solving systems of equations in this lesson, they still must use substitution as part of those methods.

ELIMINATION USING ADDITION

In-Class Examples

Teaching Tip Explain that, when using the elimination method, either the $x$ or $y$ coefficients must be the same.

1 Use elimination to solve the system of equations.

\[
\begin{align*}
-3x + 4y & = 12 \\
3x - 6y & = 18 \quad (-24, -15)
\end{align*}
\]

2 Four times one number minus three times another number is 12. Two times the first number added to three times the second number is 6. Find the numbers. The numbers are 3 and 0.

Concept Check

Ask students to explain why the system, $2x + y = 7$ and $7x + y = 32$, cannot be solved with elimination using addition. Adding these two equations will not eliminate a variable.

ELIMINATION USING SUBTRACTION

In-Class Example

3 Use elimination to solve the system of equations.

\[
\begin{align*}
4x + 2y & = 28 \\
4x - 3y & = 18 \quad (6, 2)
\end{align*}
\]
Example 4  Elimination Using Subtraction

Multiple-Choice Test Item

If \( x - 3y = 7 \) and \( x + 2y = 2 \), what is the value of \( x \)?

\[
\begin{array}{llll}
\text{A} & 4 & \text{B} & -1 \\
\text{C} & (-1, 4) & \text{D} & (4, -1)
\end{array}
\]

Read the Test Item
You are given a system of equations, and you are asked to find the value of \( x \).

Solve the Test Item
You can eliminate the \( x \) terms by subtracting one equation from the other.

\[
\begin{align*}
-5y &= 5 \\
y &= -1
\end{align*}
\]

Notice the variable is eliminated.

Now substitute \(-1\) for \( y \) in either equation to find the value of \( x \).

\[
\begin{align*}
x &+ 2(2) = 2 \\
x &+ 2(-1) = 2 \\
x &- 2 = 2
\end{align*}
\]

Simplify.

The solution is \((4, 3)\).

Test-Taking Tip
Always read the question carefully. Ask yourself, “What does the question ask?” Then answer that question.

FIND THE ERROR
Tell students to look at the original system of equations before they evaluate the students’ work. Do they need to use addition or subtraction to eliminate the variable?

Michael and Yoomee are solving a system of equations.

- **Michael**
  \[
  \begin{align*}
  2r + s &= 5 \\
  3r &= 6 \\
  r &= 2 \\
  2r + s &= 5 \\
  2(2) + s &= 5 \\
  4 + s &= 5 \\
  s &= 1
  \end{align*}
  \]

- **Yoomee**
  \[
  \begin{align*}
  2r + s &= 5 \\
  (-)r - s &= 0 \\
  r &= 4 \\
  r - s &= 1 \\
  4 - s &= 1 \\
  -s &= -3 \\
  s &= 3
  \end{align*}
  \]

Who is correct? Explain your reasoning.
Use elimination to solve each system of equations. 9. \((-\frac{1}{2}, -2)\)
4. \(x - y = 14\) \(\text{Exercises} 1 - 3\)
5. \(2x - 3b = 11\)
6. \(4x + y = -9\) \((-2, -1)\)
\(x + y = 20\) \((17, 3)\)
\(a + 3b = 8\) \((-1, 3)\)
\(4x + 2y = -10\)
7. \(6x + 2y = 10\) \(30.6, 4.25\)
8. \(2x + 4b = 30\) \((-4m + 2n = 6\)
\(2x + 2y = -10\) \((0, -5)\)
\(-2a - 2b = -21.5\)
\(-4m + n = 8\)
10. The sum of two numbers is 24. First find the number and the other number is 12. What are the two numbers? \(8, 16\)

11. If \(2x + 7y = 17\) and \(2x + 5y = 11\), what is the value of \(2y\)? \(D\)
(A) \(-4\) (B) \(-2\) (C) \(3\) (D) \(6\)

Use elimination to solve each system of equations. 26. \((1.75, 2.5)\) 27. \((15.8, 3.4)\)
12. \(x + y = -3\) \(13. s - t = 4\)
14. \(3m - 2n = 13\)
\(x - y = 1\) \((-1, -2)\)
\(s + t = 2\) \((3, -1)\)
\(\text{Exercises} 30 - 39\) \(2\)
15. \(-4x + 2y = 8\) \(16. 3a + b = 5\)
17. \(2m - 5n = -6\)
\(4x - 3y = -10\) \((-1, 2)\)
\(2a + b = 10\) \((-5, 20)\)
\(20. 3x - 5y = 15\)
\(2x - 5z = 30\) \((-5, 4)\)
\(-3x + 2y = -10(2, -2)\)
21. \(6s + 5t = 1\) \(22. 4x - 3y = 12\)
\(6s - 5t = 11\) \((-1, -1)\)
\(24. 4x + 5y = 7\)
\(25. 8x + b = 1\)
\(8x + 5y = 9\) \(\frac{1}{2}, 1\)
\(26. 1.44x - 3.24y = -5.58\)
\(1.08x + 3.24y = 9.99\)
28. \(\frac{c + d}{5} = 9\)
29. \(\frac{3c - 1}{2y} = 14\)
\(7.22 + 6.27n = 136.54\)
\(\frac{7c + 5d}{11} = 15\)
30. The sum of two numbers is 48, and their difference is 24. What are the numbers? \(36, 12\)
31. Find the numbers whose sum is 55 and whose difference is 9. \(32, 19\)
32. Three times one number added to another number is 18. The first number minus the other number is 12. Find the numbers. \(6, 0\)
33. One number added to twice another number is 23. Four times the first number added to twice the other number is 38. What are the numbers? \(5, 9\)

34. **BUSINESS** In 1999, the United States produced about 2 million more motor vehicles than Japan. Together, the two countries produced about 22 million motor vehicles. How many vehicles were produced in each country? 

**U.S.: about 12 million vehicles, Japan: about 10 million vehicles**

35. **PARKS** A youth group and their leaders visited Mammoth Cave. Two adults and 5 students in one van paid $77 for the Grand Avenue Tour of the Cave. Two adults and 7 students in a second van paid $95 for the same tour. Find the adult price and the student price of the tour. adult: $16, student: $9

36. **FOOTBALL** During the National Football League’s 1999 season, Troy Aikman, the quarterback for the Dallas Cowboys, earned $6.467 million more than Deion Sanders, the Cowboys’ cornerback. Together they cost the Cowboys $12.867 million. How much did each player make? Aikman: $6.687 million, Sanders: $6.200 million

**Enrichment, p. 420**

Rosza Peter

Born Peter (1886–1977) was a Hungarian mathematician dedicated to studying certain aspects of mathematics. He is best known for his work on the field of number theory. In his early years, Peter was a student of the world-renowned mathematician Paul Erdős, and he later taught at several universities in Europe. He made significant contributions to number theory, particularly in the study of number sequences and their properties. Peter's work helped advance the understanding of certain mathematical concepts and was influential in the development of modern mathematics. His research on the distribution of prime numbers and the properties of integers was particularly noteworthy. Peter was also known for his collaborative efforts, working closely with other mathematicians to solve complex problems. His work continues to be studied and built upon by mathematicians around the world.
About the Exercises...

Organization by Objective
- Elimination Using Addition: 12–15, 20–22, 26, 28, 30–32, 34, 36
- Elimination Using Subtraction: 16–19, 23–25, 27, 29, 33, 35

Odd/Even Assignments
Exercises 12–35 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 13–25 odd, 29–35 odd, 40–54
Average: 13–35 odd, 40–54
Advanced: 12–36 even, 37–50 (optional: 51–54)

Open-Ended Assessment

Writing Have students write a real-world problem that can be solved with elimination using addition and subtraction.

Getting Ready for Lesson 7-4

PREREQUISITE SKILL Students will learn to solve systems of equations by elimination using multiplication in Lesson 7-4. This method will include multiplying whole equations and simplifying them using the Distributive Property. Use Exercises 51–54 to determine your students’ familiarity with rewriting expressions using the Distributive Property.

Assessment Options
Quiz (Lesson 7-3) is available on p. 447 of the Chapter 7 Resource Masters.
Mid-Chapter Test (Lessons 7-1 through 7-3) is available on p. 449 of the Chapter 7 Resource Masters.

38. \( y = 0.0104x + 1.01 \)

USA TODAY Snapshots®

India’s exploding population

India is expected to pass China as the world’s most populous nation within 50 years. Largest populations today vs. predicted in 2050 (in billions):

| Country | 2000 | 2050
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>1.28</td>
<td>1.52</td>
</tr>
<tr>
<td>India</td>
<td>0.28</td>
<td>1.53</td>
</tr>
<tr>
<td>USA</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.21</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Source: Population Reference Bureau, United Nations Population Division

By Anne R. Carey and Kay Worthington, USA TODAY

40. CRITICAL THINKING The graphs of \( Ax + By = 15 \) and \( Ax - By = 9 \) intersect at (2, 1). Find \( A \) and \( B \).

41. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See pp. 405A–405D.

How can you use a system of equations to solve problems about weather?

Include the following in your answer:
- an explanation of how to use elimination to solve a system of equations, and

42. If \( 2x - 3y = -9 \) and \( 3x - 3y = -12 \), what is the value of \( y \)?

43. What is the solution of \( 4x + 2y = 8 \) and \( 2x + 2y = 2? \)

44. \( y = 5x \)

45. \( x = 2y + 3 \)

46. \( 2y - x = -5 \)

47. \( x - y = 3 \) one; \( (1, -2) \)

48. \( 2x - 3y = 7 \) no solution

49. \( 4x + y = 12 \)

50. Write an equation of a line that is parallel to the graph of \( y = \frac{5}{4}x - 3 \) and passes through the origin.

51. \( 2\left(3x + 4y\right) \)

52. \( 6\left(2a - 5b\right) \)

53. \(-3\left(-2m + 3n\right) \)

54. \(-5\left(4t - 2c\right) \)

Mixed Review

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 7-2)

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it. (Lesson 7-3) 47–49. See pp. 405A–405D for graphs.

47. \( x - y = 3 \) one; \( (1, -2) \)
48. \( 2x - 3y = 7 \) no solution
49. \( 4x + y = 12 \)

50. Write an equation of a line that is parallel to the graph of \( y = \frac{5}{4}x - 3 \) and passes through the origin.

51. \( 2\left(3x + 4y\right) \)
52. \( 6\left(2a - 5b\right) \)
53. \(-3\left(-2m + 3n\right) \)
54. \(-5\left(4t - 2c\right) \)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to rewrite each expression.

51. \( 2\left(3x + 4y\right) \)
52. \( 6\left(2a - 5b\right) \)
53. \(-3\left(-2m + 3n\right) \)
54. \(-5\left(4t - 2c\right) \)

386 Chapter 7 Solving Systems of Linear Equations and Inequalities

Teacher to Teacher

Lou Jane Tynan
Sacred Heart Model School, Louisville, KY

“Many students will forget to distribute the negative sign over the entire equation being subtracted in Example 3. I require my students to change the signs of the equation being subtracted and then add the two equations.”

\[
\begin{align*}
5s + 2t &= 6 \\
(-) 4s + 2t &= 22 \\
\Rightarrow (+) -9s - 2t &= -22
\end{align*}
\]
**What You’ll Learn**

- Solve systems of equations by using elimination with multiplication.
- Determine the best method for solving systems of equations.

**How can a manager use a system of equations to plan employee time?**

The Finneytown Bakery is making peanut butter cookies and loaves of quick bread. The preparation and baking times for each are given in the table below.

For these two items, the management has allotted 800 minutes of employee time and 900 minutes of oven time. If \( c \) represents the number of batches of cookies and \( b \) represents the number of loaves of bread, the following system of equations can be used to determine how many of each to bake.

\[
20c + 10b = 800 \\
10c + 30b = 900
\]

**ELIMINATION USING MULTIPLICATION** Neither variable in the system above can be eliminated by simply adding or subtracting the equations. However, you can use the Multiplication Property of Equality so that adding or subtracting eliminates one of the variables.

**Example 1 Multiply One Equation to Eliminate**

Use elimination to solve the system of equations.

\[
3x + 4y = 6 \\
5x + 2y = -4
\]

Multiply the second equation by \(-2\) so the coefficients of the \( y \) terms are additive inverses. Then add the equations.

\[
\begin{align*}
3x + 4y &= 6 \\
5x + 2y &= -4 \\
\text{Multiply by } -2 & \quad 3x + 4y = 6 \\
\text{Multiply by } -2 & \quad 5x + 2y = -4 \quad (+) = -10x - 4y = 8 \\
\text{Add the equations.} & \quad -7x = 14 \\
\text{Divide each side by } -7 & \quad x = -2
\end{align*}
\]

Now substitute \(-2\) for \( x \) in either equation to find the value of \( y \).

\[
\begin{align*}
3x + 4y &= 6 \\
3(-2) + 4y &= 6 \\
-6 + 4y &= 6 \\
-6 + 4y + 6 &= 6 + 6 \\
4y &= 12 \\
\frac{4y}{4} &= \frac{12}{4} \\
y &= 3
\end{align*}
\]

The solution is \((-2, 3)\).

**Lesson 7-4 Elimination Using Multiplication 387**

**Resource Manager**

**Workbook and Reproducible Masters**

- **Chapter 7 Resource Masters**
  - Study Guide and Intervention, pp. 421–422
  - Skills Practice, p. 423
  - Practice, p. 424
  - Reading to Learn Mathematics, p. 425
  - Enrichment, p. 426
  - Assessment, p. 448

- **Parent and Student Study Guide Workbook**, p. 56
- **School-to-Career Masters**, p. 14

**Transparencies**

5-Minute Check Transparency 7-4

Answer Key Transparencies

**Technology**

Interactive Chalkboard
For some systems of equations, it is necessary to multiply each equation by a different number in order to solve the system by elimination. You can choose to eliminate either variable.

**Example 2  Multiply Both Equations to Eliminate**

Use elimination to solve the system of equations.

\[3x + 4y = -25\]
\[2x - 3y = 6\]

**Method 1  Eliminate \(x\).**

\[
\begin{align*}
3x + 4y &= -25 \\
2x - 3y &= 6
\end{align*}
\]

Multiply by 2. \(\text{Multiply by } -3.\)

\[
\begin{align*}
6x + 8y &= -50 \\
(-) -6x + 9y &= -18
\end{align*}
\]

\[17y = -68\]

\[\frac{17y}{17} = \frac{-68}{17}\]

\[y = -4\]

Add the equations. Simplify.

Now substitute \(-4\) for \(y\) in either equation to find the value of \(x\).

\[
\begin{align*}
2x - 3y &= 6 & \text{Second equation} \\
2x - 3(-4) &= 6 & y = -4 \\
2x + 12 &= 6 & \text{Simplify.} \\
2x &= -6 & \text{Subtract 12 from each side.} \\
\frac{2x}{2} &= \frac{-6}{2} & \text{Divide each side by } 2. \\
x &= -3 & \text{Simplify.}
\end{align*}
\]

The solution is \((-3, -4)\).

**Method 2  Eliminate \(y\).**

\[
\begin{align*}
3x + 4y &= -25 \\
2x - 3y &= 6
\end{align*}
\]

Multiply by 3. \(\text{Multiply by } 4.\)

\[
\begin{align*}
9x + 12y &= -75 \\
(+) 8x - 12y &= 24
\end{align*}
\]

\[17x = -51\]

\[\frac{17x}{17} = \frac{-51}{17}\]

\[x = -3\]

Add the equations. Simplify.

Now substitute \(-3\) for \(x\) in either equation to find the value of \(y\).

\[
\begin{align*}
2x - 3y &= 6 & \text{Second equation} \\
2(-3) - 3y &= 6 & x = -3 \\
-6 - 3y &= 6 & \text{Simplify.} \\
-3y &= 12 & \text{Add 6 to each side.} \\
\frac{-3y}{-3} &= \frac{12}{-3} & \text{Divide each side by } -3. \\
y &= -4 & \text{Simplify.}
\end{align*}
\]

The solution is \((-3, -4)\), which matches the result obtained with Method 1.
You have learned five methods for solving systems of linear equations.

**Concept Summary**

<table>
<thead>
<tr>
<th>Method</th>
<th>The Best Time to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphing</td>
<td>to estimate the solution, since graphing usually does not give an exact solution</td>
</tr>
<tr>
<td>Substitution</td>
<td>if one of the variables in either equation has a coefficient of 1 or −1</td>
</tr>
<tr>
<td>Elimination Using Addition</td>
<td>if one of the variables has opposite coefficients in the two equations</td>
</tr>
<tr>
<td>Elimination Using Subtraction</td>
<td>if one of the variables has the same coefficient in the two equations</td>
</tr>
<tr>
<td>Elimination Using Multiplication</td>
<td>if none of the coefficients are 1 or −1 and neither of the variables can be eliminated by simply adding or subtracting the equations</td>
</tr>
</tbody>
</table>

**Example 3 Determine the Best Method**

Determine the best method to solve the system of equations. Then solve the system.

\[
\begin{align*}
4x - 3y &= 12 \\
x + 2y &= 14
\end{align*}
\]

- For an exact solution, an algebraic method is best.
- Since neither the coefficients of \(x\) nor the coefficients of \(y\) are the same or additive inverses, you cannot use elimination using addition or subtraction.
- Since the coefficient of \(x\) in the second equation is 1, you can use the substitution method. You could also use elimination using multiplication.

The following solution uses substitution. Which method would you prefer?

\[
\begin{align*}
x + 2y &= 14 & \text{Second equation} \\
x + 2y - 2y &= 14 - 2y & \text{Subtract } 2y \text{ from each side.} \\
x &= 14 - 2y & \text{Simplify.} \\
4x - 3y &= 12 & \text{First equation} \\
4(14 - 2y) - 3y &= 12 & x = 14 - 2y \\
56 - 8y - 3y &= 12 & \text{Distributive Property} \\
56 - 11y &= 12 & \text{Combine like terms.} \\
56 - 11y - 56 &= 12 - 56 & \text{Subtract } 56 \text{ from each side.} \\
-11y &= -44 & \text{Simplify.} \\
\frac{-11y}{-11} &= \frac{-44}{-11} & \text{Divide each side by } -11. \\
y &= 4 & \text{Simplify.} \\
x + 2y &= 14 & \text{Second equation} \\
x + 2(4) &= 14 & y = 4 \\
x + 8 &= 14 & \text{Simplify.} \\
x + 8 - 8 &= 14 - 8 & \text{Subtract } 8 \text{ from each side.} \\
x &= 6 & \text{Simplify.}
\end{align*}
\]

The solution is \((6, 4)\).

**Differentiated Instruction**

**Verbal/Linguistic** Place students in pairs or small groups and assign systems of equations for them to solve. Tell students to use the Concept Summary on p. 389 to discuss which method is the best to use to solve the system of equations they have been assigned. Make sure all group members participate in the discussion.
In-Class Example

4 TRANSPORTATION  A fishing boat travels 10 miles downstream in 30 minutes. The return trip takes the boat 40 minutes. Find the rate of the boat in still water. \(17.5\) mi/h

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include examples of how to solve a system of equations by using elimination with multiplication.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- Elimination Using Multiplication: 13–26
- Determine the Best Method: 27–43

Odd/Even Assignments
Exercises 13–38 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 13–39 odd, 40, 44–57
Average: 13–39 odd, 40, 41, 43–57
Advanced: 14–42 even, 43–53 (optional: 54–57)
All: Practice Quiz 2 (1–5)

Check for Understanding

1. Explain why multiplication is sometimes needed to solve a system of equations by elimination. See margin.

2. OPEN ENDED  Write a system of equations that could be solved by multiplying one equation by 5 and then adding the two equations together to eliminate one variable. Sample answer: \(3x + 2y = 5\), \(4x - 10y = -6\)

3. Describe two methods that could be used to solve the following system of equations. Which method do you prefer? Explain. See pp. 405A–405D.

   \[
   \begin{align*}
   a - b & = 5 \\
   2a + 3b & = 15
   \end{align*}
   \]

   Use elimination to solve each system of equations.

   4. \(2x - y = 6\)
   \(3x + 4y = -2\) \((2, -2)\)

   5. \(x + 5y = 4\)
   \(3x - 7y = -10\) \((-1, 1)\)

   6. \(4x + 7y = 6\)
   \(6x + 5y = 20\) \((5, -2)\)

   7. \(4x + 2y = 10.5\)
   \(2x + 3y = 10.75\) \((1.25, 2.75)\)

   Determine the best method to solve each system of equations. Then solve the system.

   8. \(4x + 3y = 19\)
   \(3x - 4y = 8\) elimination \((\times); (4, 1)\)

   9. \(3x - 7y = 6\)
   \(2x + 7y = 4\) elimination \((+); (2, 0)\)

   10. \(y = 4x + 11\)
   \(3x - 2y = -7\) substitution \((-3, -1)\)

   11. \(5x - 2y = 12\)
   \(3x - 2y = -2\) elimination \((-); (7, 11.5)\)

Answer

1. If one of the variables cannot be eliminated by adding or subtracting the equations, you must multiply one or both of the equations by numbers so that a variable will be eliminated when the equations are added or subtracted.
Use elimination to solve each system of equations.

13. \(-5x + 3y = 6\)  
   \(x - y = 4\)

14. \(x + y = 3\)
   \(2x - 3y = 16\)

15. \(2x + y = 5\)
   \(3x - 2y = 4\)

16. \(4x - 3y = 12\)
   \(5x - 2y = 15\)

17. \(x + 2y = 14\)
   \(3x + 8y = 37\)

18. \(8x - 3y = -11\)
   \(2x - 5y = 27\)

19. \(4x - 7y = 10\)
   \(2x - 3y = 2\)

20. \(1.8x - 0.3y = 14.4\)
   \(3x + 4y = 28\)

21. \(x - 0.6y = 2.8\)
   \(4x + 7y = 53\)

22. \(0.4x + 0.5y = 2.5\)
   \(3x - \frac{1}{2}y = 10\)

23. \(24x + 2\frac{1}{3}y = 4\)

24. \(5x + \frac{1}{4}y = 8\)
   \(7 - \frac{1}{2}y = 7\)

25. Seven times a number plus three times another number equals negative one. The sum of the two numbers is negative three. What are the numbers? \(-2, -5\)

26. Five times a number minus twice another number equals twenty-two. The sum of the numbers is three. Find the numbers. \(4, -1\)

Determine the best method to solve each system of equations. Then solve the system.

27. Elimination \((\times); (-2, -1)\)

28. Elimination \((+); (2, -3)\)

29. Substitution; \((2, 6)\)

30. Substitution; no solution

31. Elimination; \((+); (8, 3)\)

32. Elimination \((\times)\) or Substitution; \((3, 1)\)

33. Substitution; infinitely many solutions

34. Elimination; \((-); no solution

35. Elimination; \(3, a)\), Find the values of a and b. \(a = -2, b = 22\)

41. Careers

Mrs. Henderson discovered that she had accidentally reversed the digits of a test and shorted a student 36 points. Mrs. Henderson told the student that the sum of the digits was 14 and agreed to give the student his correct score plus extra credit if he could determine his actual score without looking at his test. What was his actual score on the test? 95
4 Assess

Open-Ended Assessment

Speaking Using the Concept Summary from p. 389, write on the board a list of the different methods for solving systems of equations. For each method, have a volunteer explain in his or her own words when it is appropriate to use the method.

Getting Ready for Lesson 7-5

PREREQUISITE SKILL Students will learn to graph systems of inequalities in Lesson 7-5. Students must be comfortable graphing inequalities before beginning this lesson because otherwise they will find it very confusing to have different shaded regions. Use Exercises 54–57 to determine your students’ familiarity with graphing inequalities.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 7-3 and 7-4. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lesson 7-4) is available on p. 448 of the Chapter 7 Resource Masters.

Answers

42. NUMBER THEORY The sum of the digits of a two-digit number is 14. If the digits are reversed, the new number is 18 less than the original number. Find the original number. 86

43. TRANSPORTATION Traveling against the wind, a plane flies 2100 miles from Chicago to San Diego in 4 hours and 40 minutes. The return trip, traveling with a wind that is twice as fast, takes 4 hours. Find the rate of the plane in still air. 475 mph

44. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See pp. 405A–405D.

How can a manager use a system of equations to plan employee time? Include the following in your answer:

• a demonstration of how to solve the system of equations concerning the cookies and bread, and
• an explanation of how a restaurant manager would schedule oven and employee time.

45. If $5x + 3y = 12$ and $4x - 5y = 17$, what is the value of $y$? A

46. Determine the number of solutions of the system $x + 2y = -1$ and $2x + 4y = -2$. D

47. Use elimination to solve each system of equations. (Lesson 7-3)

48. Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions. (Lesson 7-2)

49. Use substitution to solve each system of equations. (Lesson 6-3)

50. 2x + 3y = 3

51. x + y = 0

52. x - 2y = 7

53. CAREERS A store manager is paid $32,000 a year plus 4% of the revenue the store makes above quota. What is the amount of revenue above quota needed for the manager to have an annual income greater than $45,000? (Lesson 6-3)

54. More than $325,000

55. $y \geq x - 7$

56. $x + 3y \geq 9$

57. $-3x + y \geq -1$

Use elimination to solve each system of equations. (Lessons 7-3 and 7-4)

1. $5x + 4y = 2$

2. $2x - 3y = 13$

3. $6x - 2y = 24$

4. $5x + 2y = 4$

3x - 4y = 14

2x + 2y = -2

3x + 4y = 27

10x + 4y = 9

(2, -2)

(2, -3)

(5, 3)

(2, -3)

(5, 3)

5. The price of a cellular telephone plan is based on peak and nonpeak service. Kelsey used 45 peak minutes and 50 nonpeak minutes and was charged $27.75. That same month, Mitch used 70 peak minutes and 30 nonpeak minutes for a total charge of $36. What are the rates per minute for peak and nonpeak time? (Lesson 7-4) $0.45; $0.15

392 Chapter 7 Solving Systems of Linear Equations and Inequalities
Making Concept Maps

After completing a chapter, it is wise to review each lesson’s main topics and vocabulary. In Lesson 7-1, the new vocabulary words were system of equations, consistent, inconsistent, independent, and dependent. They are all related in that they explain how many and what kind of solutions a system of equations has.

A graphic organizer called a concept map is a convenient way to show these relationships. A concept map is shown below for the vocabulary words for Lesson 7-1. The main ideas are placed in boxes. Any information that describes how to move from one box to the next is placed along the arrows.

Concept maps are used to organize information. They clearly show how ideas are related to one another. They also show the flow of mental processes needed to solve problems.

Reading to Learn

Review Lessons 7-2, 7-3, and 7-4. 1–4. See margin.

1. Write a couple of sentences describing the information in the concept map above.
2. How do you decide whether to use substitution or elimination? Give an example of a system that you would solve using each method.
3. How do you decide whether to multiply an equation by a factor?
4. How do you decide whether to add or subtract two equations?
5. Copy and complete the concept map below for solving systems of equations by using either substitution or elimination.

Answers

1. There are two types of systems of equations, consistent and inconsistent. Consistent systems have one or more solutions and inconsistent systems have no solutions. If consistent systems have one solution, they are called independent. If consistent systems have infinite solutions, they are called dependent.

2. Use substitution if an expression for one variable is given or if the coefficient of a variable is ±1. Otherwise, use elimination.

Sample answers:

- system to solve using substitution
  - \( y = 3x + 3 \)
  - \( 5x + 2y = 6 \)
- system to solve using elimination
  - \( 4x + 3y = 9 \)
  - \( 6x - y = 10 \)

3. Multiply by a factor if neither variable has the same or opposite coefficients in the two equations.

4. Add if one of the variables has opposite coefficients in the two equations. Subtract if one of the variables has the same coefficient in the two equations.
To solve a system of inequalities by graphing.

Suppose Joshua eats 2600 Calories one day, but only 55 grams of fat. Is this an appropriate intake of Calories and fat? How do you know? It is not appropriate because it does not fall within the green section of the graph.

Vocabulary
- system of inequalities

How can you use a system of inequalities to plan a sensible diet?

Joshua watches what he eats. His doctor told him to eat between 2000 and 2400 Calories per day. The doctor also wants him to keep his daily fat intake between 60 and 75 grams. The graph indicates the appropriate amounts of Calories and fat for Joshua. The graph is of a system of inequalities. He should try to keep his Calorie and fat intake to amounts represented in the green section.

Systems of Inequalities
To solve a system of inequalities, you need to find the ordered pairs that satisfy all the inequalities involved. One way to do this is to graph the inequalities on the same coordinate plane. The solution set is represented by the intersection, or overlap, of the graphs.

Example 1 Solve by Graphing

Solve the system of inequalities by graphing.

\( y < -x + 1 \)
\( y \leq 2x + 3 \)

The solution includes the ordered pairs in the intersection of the graphs of \( y < -x + 1 \) and \( y \leq 2x + 3 \). This region is shaded in green at the right. The graph of \( y = -x + 1 \) is dashed and is not included in the graph of \( y < -x + 1 \). The graph of \( y = 2x + 3 \) is included in the graph of \( y \leq 2x + 3 \).

Example 2 No Solution

Solve the system of inequalities by graphing.

\( x - y < -1 \)
\( x - y > 3 \)

The graphs of \( x - y = -1 \) and \( x - y = 3 \) are parallel lines. Because the two regions have no points in common, the system of inequalities has no solution.
REAL-WORLD PROBLEMS In real-life problems involving systems of inequalities, sometimes only whole-number solutions make sense.

Example 3 Use a System of Inequalities to Solve a Problem

**Example: College** The middle 50% of first-year students attending Florida State University score between 520 and 620, inclusive, on the verbal portion of the SAT and between 530 and 630, inclusive, on the math portion. Graph the scores that a student would need to be in the middle 50% of FSU freshmen.

**Words** The verbal score is between 520 and 620, inclusive. The math score is between 530 and 630, inclusive.

**Variables** If \( v \) = the verbal score and \( m \) = the math score, the following inequalities represent the middle 50% of Florida State University freshmen.

**Inequalities**

The verbal score is between 520 and 620, inclusive.

\[ 520 \leq v \leq 620 \]

The math score is between 530 and 630, inclusive.

\[ 530 \leq m \leq 630 \]

The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. However, since SAT scores are whole numbers, only whole-number solutions make sense in this problem.

### Systems of Inequalities

You can use a TI-83 Plus to solve systems of inequalities.

**Graphing Calculator Investigation**

**Graphing Systems of Inequalities** Extra applications can be downloaded from the Internet for the TI-83 Plus calculator, including an inequality application that automates and simplifies inequality graphing.
Example 4 Use a System of Inequalities

AGRICULTURE To ensure a growing season of sufficient length, Mr. Hobson has at most 16 days left to plant his corn and soybean crops. He can plant corn at a rate of 250 acres per day and soybeans at a rate of 200 acres per day. If he has at most 3500 acres available, how many acres of each type of crop can he plant?

Let \(c\) = the number of days that corn will be planted and \(s\) = the number of days that soybeans will be planted. Since both \(c\) and \(s\) represent a number of days, neither can be a negative number. The following system of inequalities can be used to represent the conditions of this problem.

\[

c \geq 0 \\
\begin{align*}
s \geq 0 \\
c + s & \leq 16 \\
250c + 200s & \leq 3500
\end{align*}
\]

The solution is the set of all ordered pairs whose graphs are in the intersection of the graphs of these inequalities. This region is shown in green at the right. Only the portion of the region in the first quadrant is used since \(c \geq 0\) and \(s \geq 0\).

Any point in this region is a possible solution. For example, since \((7, 8)\) is a point in the region, Mr. Hobson could plant corn for 7 days and soybeans for 8 days. In this case, he would use 15 days to plant 250(7) or 1750 acres of corn and 200(8) or 1600 acres of soybeans.

Check for Understanding

Concept Check

1. See margin for sample answer.

2. Determine which of the following ordered pairs represent a solution of the system of inequalities graphed at the right.
   - a. \((3, 1)\) yes
   - b. \((-1, -3)\) no
   - c. \((2, 3)\) yes
   - d. \((4, -2)\) yes
   - e. \((3, -2)\) no
   - f. \((1, 4)\) no

3. Kayla; the graph of \(x + 2y \geq -2\) is the region representing \(x + 2y = -2\) and the half-plane above it.

Kayla

Sonia

Who is correct? Explain your reasoning.

Differentiated Instruction

Interpersonal If students have difficulty graphing systems of inequalities, place them in pairs. Have one student graph one inequality while the other student graphs the other inequality on a separate piece of paper. Then, when each student has finished, have them together trace each graph onto a third piece of paper for the final graph.
Health. For Exercises 10 and 11, use the following information.

Natasha walks and jogs at least 3 miles every day. Natasha walks 4 miles per hour and jogs 8 miles per hour. She only has a half-hour to exercise. 10. See pp. 405A–405D. 10. Draw a graph of the possible amounts of time she can spend walking and jogging. 11. List three possible solutions. Sample answers: walk: 15 min, jog: 15 min; walk: 10 min, jog: 20 min; walk: 5 min, jog: 25 min

\[ x \geq 5 \quad y \geq 4 \]
\[ 2x + y \geq 4 \quad 2y + x < 6 \]
\[ y \leq -2x + 1 \quad 3x - y \geq 4 \]
\[ x \geq 0 \quad x \geq -2 \]

Write a system of inequalities for each graph.

27. \[ y \leq x, \quad y > x - 3 \]

Extra Practice See page 836.
33. Make a graph showing the number of desks and the number of tables that can be made in a week. See margin.

34. List three possible solutions. Sample answers: 8 desks, 10 tables; 6 desks, 12 tables; 4 desks, 14 tables

35. Writing in Math Answer the question that was posed at the beginning of the lesson. See pp. 405A–405D.

How can you use a system of inequalities to plan a sensible diet?

Include the following in your answer:
- two appropriate Calorie and fat intakes for a day, and
- the system of inequalities that is represented by the graph.

36. Which ordered pair does not satisfy the system $x + 2y > 5$ and $3x - y < -2$? D

37. Which system of inequalities is represented by the graph? A

- $x + 2y > 3$
- $2x - y < 1$
- $y < 2x + 2$

38. Use elimination to solve each system of equations.

41. $2x + 3y = 1$
42. $5x - 2y = -3$
43. $-3x + 2y = 12$
44. $4x - 5y = 13$
45. $3x + 6y = -9$
46. $2x - 3y = -13$
47. $6x - 2y = 4$
48. $5x - 3y = -18$
49. $5x + 2y = 15$

40. Which system of inequalities is represented by the graph? A

- $x + 2y > 3$
- $2x - y < 1$
- $y < 2x + 2$

47. Write an equation of the line that passes through each point with the given slope.

48. $y - 2x = 9$
49. $y - 6x + 6$
50. $y = \frac{1}{2}x - \frac{11}{3}$
51. $(4, -1), m = 2$
52. $(1, 0), m = -6$
53. $(5, -2), m = \frac{1}{3}$

For Exercises 33 and 34, use the following information.

The Natural Wood Company has machines that sand and varnish desks and tables. The table gives the time requirements of the machines.

<table>
<thead>
<tr>
<th>Machine</th>
<th>Hours per Desk</th>
<th>Hours per Table</th>
<th>Total Hours Available Each Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sanding</td>
<td>2</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td>Varnishing</td>
<td>1.5</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

33–35 12–28

36. Which system of inequalities is represented by the graph?

37. Which system of inequalities is represented by the graph?

38. Which system of inequalities is represented by the graph?

40. Which system of inequalities is represented by the graph?

The Spirit of the Games

It’s time to complete your project. Use the information and data you have gathered about the Olympics to prepare a portfolio or Web page. Be sure to include graphs and/or tables in your project.
Choose the correct term to complete each statement.

1. If a system of equations has exactly one solution, it is (dependent, independent).
2. If the graph of a system of equations is parallel lines, the system is (consistent, inconsistent).
3. A system of equations that has infinitely many solutions is (dependent, independent).
4. If the graphs of the equations in a system have the same slope and different y-intercepts, the graph of the system is a pair of (intersecting lines, parallel lines).
5. If the graphs of the equations in a system have the same slope and y-intercept(s), the system has (exactly one, infinitely many) solution(s).
6. The solution of a system of equations is (3, -5). The system is (consistent, inconsistent).

Graphing Systems of Inequalities

Concept Summary

<table>
<thead>
<tr>
<th>Graph of a System</th>
<th>Intersecting Lines</th>
<th>Same Line</th>
<th>Parallel Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Solutions</td>
<td>exactly one solution</td>
<td>infinitely many</td>
<td>no solutions</td>
</tr>
<tr>
<td>Terminology</td>
<td>consistent and independent</td>
<td>consistent and dependent</td>
<td>inconsistent</td>
</tr>
</tbody>
</table>

Example

Graph the system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it.

3x + y = -4
6x + 2y = -8

When the lines are graphed, they coincide. There are infinitely many solutions.

Exercises

Graph each system of equations. Then determine whether the system of equations has one solution, no solution, or infinitely many solutions. If the system has one solution, name it. See Example 2 on page 370. 7–10. See margin for graphs.

7. x - y = 9  
   x + y = 11
8. 9x + 2 = 3y
   y - 3x = 8
9. 2x - 3y = 4
   6y = 4x - 8
10. 3x - y = 8
    3x = 4 - y

one; (10, 1)  

no solution  

infinitely many  

one; (2, -2)

www.algebra1.com/vocabulary_review

For each lesson,
• the main ideas are summarized,
• additional examples review concepts, and
• practice exercises are provided.

Vocabulary PuzzleMaker

The Vocabulary PuzzleMaker software improves students’ mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

See p. 400 for the graphs for Exercises 7–10.
**Study Guide and Review**

### 7-2 Substitution

**Concept Summary**
- In a system of equations, solve one equation for a variable, and then substitute that expression into the second equation to solve.

**Example**
Use substitution to solve the system of equations.
\[
\begin{align*}
y &= x - 1 \\
4x - y &= 19
\end{align*}
\]
Since \( y = x - 1 \), substitute \( x - 1 \) for \( y \) in the second equation.
\[
\begin{align*}
4x - (x - 1) &= 19 \\
3x + 1 &= 19
\end{align*}
\]
Subtract 1 from each side.
\[
x = 6
\]
Divide each side by 3.
Use \( y = x - 1 \) to find the value of \( y \).
\[
\begin{align*}
y &= x - 1 \\
y &= 6 - 1 \\
y &= 5
\end{align*}
\]
The solution is \((6, 5)\).

**Exercises**
Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solutions or infinitely many solutions. See Examples 1–3 on pages 377 and 378.

11. \[
\begin{align*}
2m + n &= 1 \\
m - n &= 8
\end{align*}
\]
(3, -5)

12. \[
\begin{align*}
x &= 3 - 2y \\
2x + 4y &= 6
\end{align*}
\]
infinitely many solutions

13. \[
\begin{align*}
3x - y &= 1 \\
2x + 4y &= 3
\end{align*}
\]
(1, 1)

14. \[
\begin{align*}
0.6m - 0.2n &= 0.9 \\
4.5 - 3m &= 0.9
\end{align*}
\]
(1.5, 0)

### 7-3 Elimination Using Addition and Subtraction

**Concept Summary**
- Sometimes adding or subtracting two equations will eliminate one variable.

**Example**
Use elimination to solve the system of equations.
\[
\begin{align*}
2m - n &= 4 \\
m + n &= 2
\end{align*}
\]
You can eliminate the \( n \) terms by adding the equations.
\[
\begin{align*}
(+) m + n &= 2 \\
3m &= 6
\end{align*}
\]
Notice the variable \( n \) is eliminated.
\[
m = 2
\]
Divide each side by 3.
Now substitute 2 for \( m \) in either equation to find \( n \).

\[
\begin{align*}
m + n &= 2 \\
2 + n &= 2
\end{align*}
\]

Second equation

\[
\begin{align*}
2 + n - 2 &= 2 - 2 \\
n &= 0
\end{align*}
\]

Subtract 2 from each side.

The solution is \((2, 0)\).

**Exercises** Use elimination to solve each system of equations.

See Examples 1–3 on pages 382 and 383.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + 2y = 6 )</td>
<td>( x - 3y = -4 )</td>
<td>((2, 2))</td>
</tr>
<tr>
<td>( 2m - n = 5 )</td>
<td>( 2m + n = 3 )</td>
<td>((2, -1))</td>
</tr>
<tr>
<td>( 3x - y = 11 )</td>
<td>( x + y = 5 )</td>
<td>((4, 1))</td>
</tr>
<tr>
<td>( 6x + 7y = 0 )</td>
<td>( 7x - y = 0 )</td>
<td>((3, -2))</td>
</tr>
</tbody>
</table>

**Elimination Using Multiplication**

**Concept Summary**

- Multiplying one equation by a number or multiplying each equation by a different number is a strategy that can be used to solve a system of equations by elimination.
- There are five methods for solving systems of equations.

<table>
<thead>
<tr>
<th>Method</th>
<th>The Best Time to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphing</td>
<td>to estimate the solution, since graphing usually does not give an exact solution</td>
</tr>
<tr>
<td>Substitution</td>
<td>if one of the variables in either equation has a coefficient of 1 or (-1)</td>
</tr>
<tr>
<td>Elimination Using Addition</td>
<td>if one of the variables has opposite coefficients in the two equations</td>
</tr>
<tr>
<td>Elimination Using Subtraction</td>
<td>if one of the variables has the same coefficient in the two equations</td>
</tr>
<tr>
<td>Elimination Using Multiplication</td>
<td>if none of the coefficients are 1 or (-1) and neither of the variables can be eliminated by simply adding or subtracting the equations</td>
</tr>
</tbody>
</table>

**Example**

Use elimination to solve the system of equations.

\[
\begin{align*}
x + 2y &= 8 \\
3x + y &= 1.5
\end{align*}
\]

Multiply the second equation by \(-2\) so the coefficients of the \( y \) terms are additive inverses. Then add the equations.

\[
\begin{align*}
x + 2y &= 8 \\
3x + y &= 1.5 \\
\text{Multiply by } -2. \\
-x - 4y &= -16 \\
(+)
6x - 2y &= -3 \\
-5x &= 5 \\
\text{Add the equations.} \\
-5x &= 5 \\
-5 &= -5 \\
x &= -1 \\
\text{Divide each side by } -5. \\
\text{Simplify.}
\end{align*}
\]

(continued on the next page)
Solving Systems of Linear Equations and Inequalities

**Exercises** Use elimination to solve each system of equations. See Examples 1 and 2 on pages 387 and 388.

19. \[ x - 5y = 0 \]
   \[ 2x - 3y = 7 \]
   \( \text{(5, 1)} \)

20. \[ x - 2y = 5 \]
   \[ 3x - 5y = 8 \]
   \( \text{(-9, -7)} \)

21. \[ 2x + 3y = 8 \]
   \[ x - y = 2 \]
   \( \text{\(\frac{2}{5}, \frac{4}{5}\)} \)

22. \[ -5x + 8y = 21 \]
   \[ 10x + 3y = 15 \]
   \( \text{(3, 3)} \)

Determine the best method to solve each system of equations. Then solve the system. See Example 3 on page 389.

23. \[ y = 2x \]
   \[ x + 2y = 8 \]
   \( \text{substitution; \(\frac{3}{5}, \frac{3}{5}\)} \)

24. \[ 9x + 8y = 7 \]
   \[ 18x - 15y = 14 \]
   \( \text{\(\frac{7}{6}, \frac{1}{6}\)} \)

25. \[ 3x + 5y = 2x \]
   \[ x + 3y = y \]
   \( \text{substitution; \(0, 0\)} \)

26. \[ 2x + y = 3x - 15 \]
   \[ x + 5 = 4y + 2x \]
   \( \text{substitution; \(13, -2\)} \)

**7-5 Graphing Systems of Inequalities**

**Concept Summary**
- Graph each inequality on a coordinate plane to determine the intersection of the graphs.

**Example**

Solve the system of inequalities.

\[ x \geq -3 \]
\[ y \leq x + 2 \]

The solution includes the ordered pairs in the intersection of the graphs \( x \geq -3 \) and \( y \leq x + 2 \). This region is shaded in green. The graphs of \( x \geq -3 \) and \( y \leq x + 2 \) are boundaries of this region.

**Exercises** Solve each system of inequalities by graphing. See Examples 1 and 2 on page 394. **27–30. See margin.**

27. \[ y < 3x \]
   \[ x + 2y \geq -21 \]

28. \[ y > -x - 1 \]
   \[ y \leq 2x + 1 \]

29. \[ 2x + y < 9 \]
   \[ x + 11y < -6 \]

30. \[ x \geq 1 \]
   \[ y + x \leq 3 \]

Answers (page 403)
Choose the letter that best matches each description.

1. a system of equations with two parallel lines c
2. a system of equations with at least one ordered pair that satisfies both equations a
3. a system of equations may be solved using this method b

Graph each system of equations. Then determine whether the system has no solution, one solution, or infinitely many solutions. If the system has one solution, name it. 4–6. See margin for graphs.

4. \( y = x + 2 \)
   \( y = 2x + 7 \) one; \((-5, -3)\)
5. \( x + 2y = 11 \)
6. \( 3x + y = 5 \)
   \( 2y - 10 = -6x \) infinitely many

Use substitution or elimination to solve each system of equations.

7. \( 2x + 5y = 16 \)
   \( 5x - 2y = 11 \) (3, 2)
8. \( y + 2x = -1 \)
   \( y - 4 = -2x \) no solution
9. \( 2x + y = -4 \)
   \( 5x + 3y = -6 \) \((-6, 8)\)
10. \( y = 7 - x \)
    \( x - y = -3 \) (2, 5)
11. \( x = 2y - 7 \)
    \( y - 3x = -9 \) (5, 6)
12. \( x + y = 10 \)
    \( x - y = 2 \) (6, 4)
13. \( 3x - y = 11 \)
    \( x + 2y = -36 \) \((-2, -17)\)
14. \( 3x + y = 10 \)
    \( 3x - 2y = 16 \) (4, -2)
15. \( 5x - 3y = 12 \)
    \(-2x + 3y = -3 \) (3, 1)
16. \( 2x + 5y = 12 \)
    \( x - 6y = -11 \) (1, 2)
17. \( x + y = 6 \)
    \( 3x - 3y = 13 \) \((\frac{5}{6}, \frac{5}{6})\)
18. \( 3x + \frac{1}{3}y = 10 \)
    \( 2x - \frac{5}{3}y = 35 \) \((5, -15)\)

19. **NUMBER THEORY** The units digit of a two-digit number exceeds twice the tens digit by 1. Find the number if the sum of its digits is 10. \( \boxed{37} \)
20. **GEOMETRY** The difference between the length and width of a rectangle is 7 centimeters. Find the dimensions of the rectangle if its perimeter is 50 centimeters. \( 16 \text{ cm by } 9 \text{ cm} \)

Solve each system of inequalities by graphing. 21–23. See pp. 405A–405D.

21. \( y > -4 \)
    \( y < -1 \)
22. \( y \leq 3 \)
    \( y > -x + 2 \)
23. \( x \leq 2y \)
    \( 2x + 3y \leq 7 \)

24. **FINANCE** Last year, Jodi invested $10,000, part at 6% annual interest and the rest at 8% annual interest. If she received $760 in interest at the end of the year, how much did she invest at each rate? $2000 \text{ at } 6\%, \$8000 \text{ at } 8\%

25. **STANDARDIZED TEST PRACTICE** Which graph represents the system of inequalities \( y > 2x + 1 \) and \( y < -x - 2 \)? D

**Portfolio Suggestion**

**Introduction** Have you ever found that your preference for completing a task may be different from those of others?

**Ask Students** Select one of the systems of equations from this chapter that could be solved by various methods. Demonstrate how to solve it using some of the methods you learned in this chapter. Write some pros and cons for using each method. Which method do you prefer, and why?
Part 1  Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. What is the solution of \(4x - 2(x - 2) - 8 = 0\)? (Lesson 3-4)  B
   \[\begin{align*}
   &A \quad -2 \\
   &B \quad 2 \\
   &C \quad 5 \\
   &D \quad 6
   \end{align*}\]

2. Noah paid $17.11 for a CD, including tax. If the tax rate is 7%, then what was the price of the CD before tax? (Lesson 3-5)  C
   \[\begin{align*}
   &A \quad $10.06 \\
   &B \quad $11.98 \\
   &C \quad $15.99 \\
   &D \quad $17.04
   \end{align*}\]

3. What is the range of \(f(x) = 2x - 3\) when the domain is \([3, 4, 5]\)? (Lesson 4-3)  B
   \[\begin{align*}
   &A \quad [0, 1, 2] \\
   &B \quad [3, 5, 7] \\
   &C \quad [6, 8, 10] \\
   &D \quad [9, 11, 13]
   \end{align*}\]

4. Jolene kept a log of the numbers of birds that visited a bird feeder over periods of several hours. In the table below, she recorded the number of hours she watched and the cumulative number of birds that she saw each session. Which equation best represents the data set shown in the table? (Lesson 4-8)  D

<table>
<thead>
<tr>
<th>Number of hours, (x)</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of birds, (y)</td>
<td>6</td>
<td>14</td>
<td>18</td>
<td>26</td>
</tr>
</tbody>
</table>

   \[\begin{align*}
   &A \quad y = x + 5 \\
   &B \quad y = 3x + 3 \\
   &C \quad y = 3x + 5 \\
   &D \quad y = 4x + 2
   \end{align*}\]

5. Which equation describes the graph? (Lesson 5-3)  C
   \[\begin{align*}
   &A \quad 3y - 4x = -12 \\
   &B \quad 4y + 3x = -16 \\
   &C \quad 3y + 4x = -12 \\
   &D \quad 3y + 4x = -9
   \end{align*}\]

6. Which equation represents a line parallel to the line given by \(y = -3x + 6\)? (Lesson 5-6)  B
   \[\begin{align*}
   &A \quad y = -3x + 4 \\
   &B \quad y = 3x - 2 \\
   &C \quad y = \frac{1}{3}x + 6 \\
   &D \quad y = -\frac{1}{3}x + 4
   \end{align*}\]

7. Tamika has $185 in her bank account. She needs to deposit enough money so that she will still have at least $200 left in the account. Which inequality describes \(d\), the amount she needs to deposit? (Lesson 6-1)  D
   \[\begin{align*}
   &A \quad d(185 - 230) \geq 200 \\
   &B \quad 185 - 230d \geq 200 \\
   &C \quad 185 + 230 + d \geq 200 \\
   &D \quad 185 + d - 230 \geq 200
   \end{align*}\]

8. The perimeter of a rectangular garden is 68 feet. The length of the garden is 4 more than twice the width. Which system of equations will determine the length \(\ell\) and the width \(w\) of the garden? (Lesson 6-2)  B
   \[\begin{align*}
   &A \quad 2\ell + 2w = 68 \\
   &B \quad 2\ell + 2w = 68 \\
   &C \quad 2 + 2w = 68 \\
   &D \quad 2\ell + 2w = 68 \\
   &       \ell = w = 4
   \end{align*}\]

9. Ernesto spent a total of $64 for a pair of jeans and a shirt. The jeans cost $6 more than the shirt. What was the cost of the jeans? (Lesson 7-2)  C
   \[\begin{align*}
   &A \quad \$26 \\
   &B \quad \$29 \\
   &C \quad \$35 \\
   &D \quad \$58
   \end{align*}\]

10. What is the value of \(y\) in the following system of equations? (Lesson 7-3)  C
    \[\begin{align*}
    &3x + 4y = 8 \\
    &3x + 2y = -2
    \end{align*}\]
    \[\begin{align*}
    &A \quad -2 \\
    &B \quad 4 \\
    &C \quad 5 \\
    &D \quad 6
    \end{align*}\]
11. The diagram shows the dimensions of the cargo area of a delivery truck.

What is the maximum volume of cargo, in cubic feet, that can fit in the truck? (Prerequisite Skill) 1644

12. The perimeter of the square below is 204 feet. What is the value of x? (Lesson 3-4) 9

13. What is the x-intercept of the graph of 4x + 3y = 12? (Lesson 4-5) 3

14. What are the slope and the y-intercept of the graph of the equation 4x – 2y = 5? (Lesson 5-4) m = 2, b = \(-\frac{5}{2}\)

15. Solve the following system of equations. (Lesson 7-2) (3, 5)

\[
\begin{align*}
5x - y &= 10 \\
7x - 2y &= 11
\end{align*}
\]

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

16. Two times one number minus three times another number is \(-11\). The sum of the first number and three times the second number is 8. What are the two numbers? (Lesson 7-4) \(-1, 3\)

17. Write a system of inequalities for the graph. (Lesson 7-5)

\[
y > \frac{3}{2}x - 1, y \leq -x + 4
\]

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

18. During a work-out session, Mark either ran at a speed of 7 miles per hour or walked at a speed of 3 miles per hour. He completed 20 miles in 3 hours. (Lesson 7-1)

a. Let \(r\) represent the number of miles Mark ran and \(w\) represent the number of miles Mark walked. Write a system of equations that represents the situation.

\[
r + w = 3, 7r + 3w = 20
\]

b. Solve the system of equations to find how much time Mark spent running. 2.75 h or 2 h 45 min

19. The manager of a movie theater found that Saturday’s sales were $3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost $7.50, and children’s tickets cost $4.50. (Lesson 7-2) a–b. See margin.

a. Write equations to represent the number of tickets sold and the amount of money collected.

b. How many of each kind of ticket were sold? Show your work. Include all steps.

Answers

19a. \(A + C = 650,\)

\[7.5A + 4.5C = 3675\]

19b. \(A + C = 650\)

\[
A + C - C = 650 - C \\
A = 650 - C \\
7.5A + 4.5C = 3675 \\
7.5(650 - C) + 4.5C = 3675 \\
4875 - 7.5C + 4.5C = 3675 \\
4875 - 3C = 3675 \\
4875 - 3C - 4875 = 3675 - 4875 \\
-3C = -1200 \\
-3C = -1200 \\
-3 = -3 \\
C = 400
\]

\(A = 650 - C\)

\(A = 650 - 400\) or \(A = 250\)

250 adult tickets and 400 child tickets

Preparing for Standardized Tests

For test-taking strategies and more practice, see pages 867–884.

Evaluating Extended Response Questions

Extended Response questions are graded by using a multilevel rubric that guides you in assessing a student’s knowledge of a particular concept.

Goal: Use a system of equations to find ticket sales and money earned from sales.

Sample Scoring Rubric: The following rubric is a sample scoring device. You may wish to add more detail to this sample to meet your individual scoring needs.
2. Always; if the system of linear equations has 2 solutions, their graphs are the same line and there are infinitely many solutions.

3. Sample answer: The graphs of the equations $x + y = 3$ and $2x + 2y = 6$ have a slope of $-1$. Since the graphs of the equations coincide, there are infinitely many solutions.
Elimination can be used to solve problems about meteorology if the coefficients of one variable are the same or are additive inverses. Answers should include the following.

- The two equations in the system of equations are added or subtracted so that one of the variables is eliminated. You then solve for the remaining variable. This number is substituted into one of the original equations, and that equation is solved for the other variable.

\[
\begin{align*}
n + d &= 24 \\
(+) n - d &= 12 \\
2n &= 36 \\
\frac{2n}{2} &= 18 \\
n &= 18 \\
n + d &= 24 \\
18 + d &= 24 \\
18 &= 6 \\
d &= 6 \\
\end{align*}
\]

- Write the equations in column form and add.
- Notice the \( d \) variable is eliminated.
- Divide each side by 2.
- Simplify.
- First equation
- Second equation
- Subtract 18 from each side.
- Simplify.

On the winter solstice, Seward, Alaska, has 18 hours of nighttime and 6 hours of daylight.
3. Sample answer: (1) You could solve the first equation for $a$ and substitute the resulting expression for $a$ in the second equation. Then find the value of $b$. Use this value for $b$ and one of the original equations to find the value of $a$. (2) You could multiply the first equation by 3 and add this new equation to the second equation. This will eliminate the $b$ term. Find the value of $a$. Use this value for $a$ and one of the original equations to find the value of $b$. See student’s work for their preference and explanation.

44. By having two equations that represent the time restraints, a manager can determine the best use of employee time. Answers should include the following.

- $20c + 10b = 800$ → $20c + 10b = 800$
  $10c + 30b = 900$ → $-20c + 60b = -1800$
  $-50b = -1000$
  $b = 20$
  $20c + 10b = 800$
  $20c + 10(20) = 800$
  $20c + 200 = 800$
  $20c + 200 - 200 = 800 - 200$
  $20c = 600$
  $20c = 600$
  $20 = 20$
  $c = 30$

- In order to make the most of the employee and oven time, the manager should make assignments to bake 30 batches of cookies and 20 loaves of bread.

Page 395, Graphing Calculator Investigation

2. $[-10, 10] \text{ scl: 1}$

4. $[-10, 10] \text{ scl: 1}$
By graphing a system of equations, you can see the appropriate range of Calories and fat intake. Answers should include the following.

- Two sample appropriate Calorie and fat intakes are 2200 Calories and 60 g of fat and 2300 Calories and 65 g of fat.
- The graph represents $2000 \leq c \leq 2400$ and $60 \leq f \leq 75$. 

35. By graphing a system of equations, you can see the appropriate range of Calories and fat intake. Answers should include the following.

- Two sample appropriate Calorie and fat intakes are 2200 Calories and 60 g of fat and 2300 Calories and 65 g of fat.
- The graph represents $2000 \leq c \leq 2400$ and $60 \leq f \leq 75$. 