## Lesson Objectives

### 9-1 Factors and Greatest Common Factors (pp. 474–479)
- Find prime factorizations of integers and monomials.
- Find the greatest common factors of integers and monomials.

### 9-2 Factoring Using the Distributive Property (pp. 480–486)
*Preview:* Use algebra tiles and a product mat to factor binomials.
- Factor polynomials by using the Distributive Property.
- Solve quadratic equations of the form $ax^2 + bx = 0$.

### 9-3 Factoring Trinomials: $x^2 + bx + c$ (pp. 487–494)
*Preview:* Use algebra tiles to factor trinomials.
- Factor trinomials of the form $x^2 + bx + c$.
- Solve equations of the form $x^2 + bx + c = 0$.

### 9-4 Factoring Trinomials: $ax^2 + bx + c$ (pp. 495–500)
- Factor trinomials of the form $ax^2 + bx + c$.
- Solve equations of the form $ax^2 + bx + c = 0$.

### 9-5 Factoring Differences of Squares (pp. 501–506)
- Factor binomials that are the differences of squares.
- Solve equations involving the differences of squares.

### 9-6 Perfect Squares and Factoring (pp. 508–514)
- Factor perfect square trinomials.
- Solve equations involving perfect squares.

### Study Guide and Practice Test (pp. 515–519)
### Standardized Test Practice (pp. 520–521)

### Chapter Assessment

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### Chapter Resource Manager

#### CHAPTER 9 RESOURCE MASTERS

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*Key to Abbreviations: GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual

Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.
Factors and Greatest Common Factors

The factors of a given number are all the numbers that divide the number evenly. This includes the number itself and 1. The factors of a number can be found by determining all the pairs of numbers whose product is that number. Natural numbers greater than 1 are classified as either prime or composite. Prime numbers have exactly two factors while composite numbers have more than two factors. The number 1 is neither prime nor composite. A prime factorization is the expression of a number as the product of factors that are all prime numbers. The prime factorization of a negative integer is expressed as the product of prime numbers. Monomials can be written in factored form. A monomial in factored form is the product of prime numbers and variables. Variables, however, cannot have an exponent greater than 1. So $x^3$ must be written as $x \cdot x \cdot x$ in factored form.

Prime factorizations are used to determine the greatest common factor (GCF) of two or more integers or monomials. The GCF is the product of all the common prime factors of the two integers or monomials. If 1 is the only common factor, then they are relatively prime.

Factoring Using the Distributive Property

Many polynomials also have factors. Some polynomials are the product of a polynomial and a monomial. Reverse the process of multiplying a polynomial by a monomial to factor using the Distributive Property. First find the greatest common factor of the terms of the polynomial. If the GCF is not 1, then rewrite each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

If a polynomial has four or more terms you can factor by grouping. Group terms in pairs that have common factors. Use the Distributive Property to factor the GCF from each pair of terms. The binomials in each pair of factored terms should be identical. Use the Distributive Property to factor out the common binomial factor. The remaining factors are grouped to form a second binomial. It may appear that the factored pairs do not have identical binomials, but one may be the additive inverse of the other. Write one as the product of $-1$ and its additive inverse. Then multiply the GCF of that pair by $-1$.

If the product of two factors is 0, then at least one of them is 0 according to the Zero Product Property. If an equation has the form $ab = 0$ or can be written in this form by factoring, then the Zero Product Property can be applied to solve the equation. Set each factor equal to 0 and solve each resulting equation.
9-3 Factoring Trinomials: $x^2 + bx + c$

The FOIL method was used to multiply two binomials. Reverse the FOIL method to factor a quadratic polynomial of the form $x^2 + bx + c$ into two binomials. Find two numbers $m$ and $n$ whose product is $c$ and whose sum is $b$. The two numbers are the last terms of the two binomials $(x + m)$ and $(x + n)$.

If $b$ is negative and $c$ is positive then $m$ and $n$ must both be negative. If $c$ is negative, then $m$ and $n$ must have different signs. This is because the product of two numbers with different signs is negative.

The Zero Product Property can be used to solve some quadratic equations written in the form $x^2 + bx + c = 0$. Factor the trinomial, and then set each factor equal to 0. Solve each equation to find the solution of the quadratic equation. Be sure to check the solutions in the original equation.

9-4 Factoring Trinomials: $ax^2 + bx + c$

To factor a trinomial in which the coefficient of $x^2$ is not 1, first check to see if the terms of the polynomial have a GCF. If so, factor it out. If the coefficient of $x^2$ is still not 1, or there is no GCF, factor $ax^2 + bx + c$ by making an organized list of the factors of the product of $a$ and $c$. For example, to factor $8x^2 - 5x - 3$, make an organized list of the factors of $8 \cdot (-3)$, or $-24$. Look for a pair of factors, $m$ and $n$, whose sum is equal to $b$ in the trinomial. Then rewrite the trinomial, replacing $bx$ with $mx + nx$. The new polynomial has four terms. Use the factoring by grouping technique shown in Lesson 9-2 to factor this polynomial into two binomial factors. A polynomial that cannot be factored is a prime polynomial. Use the above method and the Zero Product Property to solve equations in the form $ax^2 + bx + c = 0$.

9-5 Factoring Differences of Squares

Lesson 8-8 discussed the pattern for the product of a sum and a difference: $(a + b)(a - b) = a^2 - b^2$. The binomial $a^2 - b^2$ is called the difference of two squares and can be factored as the product of a sum and a difference: $a^2 - b^2 = (a - b)(a + b)$. To do this, identify $a$ and $b$, the square roots of the first and last terms, respectively. Then apply the pattern.

Other aspects of factoring to watch for are factoring out a common factor, applying a technique more than once, and applying several techniques. Use any of the appropriate techniques and the Zero Product Property to solve many polynomial equations.

9-6 Perfect Squares and Factoring

Some trinomials have patterns that make their factoring easier. In Lesson 8-8, students learned about patterns for the square of a sum, $(a + b)^2 = a^2 + 2ab + b^2$, and the square of a difference, $(a - b)^2 = a^2 - 2ab + b^2$. These products, $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$, are called perfect square trinomials, because they are the result of squaring a binomial. To recognize a perfect square trinomial, first determine if the first and last terms are perfect squares. Then find the square roots of the first and last terms, checking to see if twice the product of these square roots is equal to the middle term of the trinomial. If the trinomial is a perfect square, and the middle term is positive use the pattern $(a^2 + 2ab + b^2) = (a + b)^2$ to factor it. If the middle term is negative, use the pattern $a^2 - 2ab + b^2 = (a - b)^2$. It is important to note that the last term of a perfect square trinomial cannot be negative.

If one side of an equation is a perfect square or can be written as a perfect square, then the Square Root Property can be applied to solve the equation. The Square Root Property allows you to take the square root of each side of an equation, so long as both the positive and negative square roots of a number are taken into account. So, for any number $n$ that is greater than 0, if $x^2 = n$, then $x = \pm \sqrt{n}$. Two solutions result from such equations, one using the positive square root and one using the negative square root.

Quick Review Math Handbook

Hot Words includes a glossary of terms while Hot Topics consists of explanations of key mathematical concepts with exercises to test comprehension. This valuable resource can be used as a reference in the classroom or for home study.

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GS = Getting Started, P = Preview

Additional mathematical information and teaching notes are available at www.algebra1.com/key_concepts.
### Chapter 9: Factoring

#### Key to Abbreviations:
- TWE = Teacher Wraparound Edition
- CRM = Chapter Resource Masters

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#### Yearly ProgressPro

For more information on Yearly ProgressPro, see p. 406.

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#### ExamView® Pro

Use the networkable ExamView® Pro to:
- Create multiple versions of tests.
- Create modified tests for Inclusion students.
- Edit existing questions and add your own questions.
- Use built-in state curriculum correlations to create tests aligned with state standards.
- Change English tests to Spanish and vice versa.

For more information on Intervention and Assessment, see pp. T8–T11.
Reading and Writing in Mathematics

*Glencoe Algebra 1* provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

**Student Edition**

- Foldables Study Organizer, p. 473
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 477, 484, 492, 498, 504, 512)
- Reading Mathematics, p. 507
- Writing in Math questions in every lesson, pp. 479, 485, 494, 500, 506, 514
- Reading Study Tip, pp. 489, 511
- WebQuest, p. 479

**Teacher Wraparound Edition**

- Foldables Study Organizer, pp. 473, 515
- Study Notebook suggestions, pp. 477, 480, 484, 488, 492, 498, 504, 507, 512
- Modeling activities, pp. 479, 506
- Speaking activities, pp. 486, 494
- Writing activities, pp. 500, 514
- Differentiated Instruction, (Verbal/Linguistic), p. 475
- ELL Resources, pp. 472, 475, 478, 485, 493, 499, 505, 507, 513, 515

*For more information on Reading and Writing in Mathematics, see pp. T6–T7.*

**Lesson 9-1**

**Building on Prior Knowledge**

Have groups of students find the greatest common factor (GCF) of two numbers. For example, find the GCF of 54 and 72.

\[
54 = 2 \cdot 3 \cdot 3 \\
72 = 2 \cdot 2 \cdot 3 \cdot 3
\]

Circle common prime factors.

So, \(2 \cdot 3 \cdot 3 = 18\).

Then, show students how to find the GCF of a set of monomials. Have students note similarities in the procedures.

**Lesson 9-3**

**Flexible Groups**

Give groups of four students a set of four cards with one trinomial on each card. Each group should receive a trinomial where \(b\) and \(c\) are positive, \(b\) is positive and \(c\) is negative, \(b\) is negative and \(c\) is positive, and \(b\) and \(c\) are negative. Have groups develop rules of how to find the factors of \(b\) and \(c\) and how to ultimately find the factors of the trinomial. Allow groups to share their findings with other groups.

**Lesson 9-6**

**Peer Tutoring**

Give students a set of trinomials. You can use Exercises 17–22 on page 512 or make up your own. Have students work with a partner to analyze each problem and state whether it is a perfect square trinomial. Students can then factor each trinomial and find a solution set.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Key Vocabulary

- factored form (p. 475)
- factoring by grouping (p. 482)
- prime polynomial (p. 497)
- difference of squares (p. 501)
- perfect square trinomials (p. 508)

The factoring of polynomials can be used to solve a variety of real-world problems and lays the foundation for the further study of polynomial equations. Factoring is used to solve problems involving vertical motion. For example, the height \( h \) in feet of a dolphin that jumps out of the water traveling at 20 feet per second is modeled by a polynomial equation. Factoring can be used to determine how long the dolphin is in the air. You will learn how to solve polynomial equations in Lesson 9-2.

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 9 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 9 test.
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 9.

For Lessons 9-2 through 9-6  Distributive Property
Rewrite each expression using the Distributive Property. Then simplify.
(For review, see Lesson 1-5.)
1. $3(4 - x)$  2. $a(a + 5)$  3. $-7(n^2 - 3n + 1)$  4. $6y(-3y - 5y^2 + y^3)$
$12 - 3x$  $a^2 + 5a$  $-7n^2 + 21n - 7$  $-18y^2 - 30y^3 + 6y^4$

For Lessons 9-3 and 9-4  Multiplying Binomials
Find each product.
(For review, see Lesson 8-7.)
5. $(x + 4)(x + 7)$  6. $(3n - 4)(n + 5)$  7. $(6a - 2b)(9a + b)$  8. $(-x - 8y)(2x - 12y)$
$x^2 + 11x + 28$  $3n^2 + 11n - 20$  $54a^2 - 12ab - 2b^2$  $-2x^2 - 4xy + 96y^2$

For Lessons 9-5 and 9-6  Special Products
Find each product.
(For review, see Lesson 8-8.)
9. $(y + 9)^2$  10. $(3a - 2)^2$  11. $(n - 5)(n + 5)$  12. $(6p + 7q)(6p - 7q)$
$y^2 + 18y + 81$  $9a^2 - 12a + 4$  $n^2 - 25$  $36p^2 - 49q^2$

For Lesson 9-6  Square Roots
Find each square root.
(For review, see Lesson 2-7.)
13. $\sqrt{121}$  14. $\sqrt{0.0064}$  15. $\sqrt[3]{25} \frac{5}{6}$  16. $\sqrt[3]{\frac{8}{98}} \frac{2}{7}$

Factoring  Make this Foldable to help you organize your notes. Begin with a sheet of plain 8½" by 11" paper.

Step 1  Fold in Sixths
Fold in thirds and then in half along the width.

Step 2  Fold Again
Open. Fold lengthwise, leaving a $\frac{3}{4}$" tab on the right.

Step 3  Cut
Open. Cut the short side along the folds to make tabs.

Step 4  Label
Label each tab as shown.

Reading and Writing  As you read and study the chapter, write notes and examples for each lesson under its tab.

Organization of Data and Questioning  Before beginning each lesson, ask students to think of one question that comes to mind as they skim through the lesson. Write the question on the front of the corresponding lesson tab. As students read and work through the lesson, ask them to record the answer to their question under the tab. Students can also use their Foldables to take notes, record concepts, define terms, and record other questions that arise about factoring.
5-Minute Check

Transparency 9-1  Use as a quiz or review of Chapter 8.

Mathematical Background notes are available for this lesson on p. 472C.

How are prime numbers related to the search for extraterrestrial life?

In the search for extraterrestrial life, scientists listen to radio signals coming from faraway galaxies. How can they be sure that a particular radio signal was deliberately sent by intelligent beings instead of coming from some natural phenomenon? What if that signal began with a series of beeps in a pattern comprised of the first 30 prime numbers (“beep-beep,” “beep-beep-beep,” and so on)?

PRIME FACTORIZATION  Recall that when two or more numbers are multiplied, each number is a factor of the product. Some numbers, like 18, can be expressed as the product of different pairs of whole numbers. This can be shown geometrically. Consider all of the possible rectangles with whole number dimensions that have areas of 18 square units.

The number 18 has 6 factors, 1, 2, 3, 6, 9, and 18. Whole numbers greater than 1 can be classified by their number of factors.

Vocabulary
- prime number
- composite number
- prime factorization
- factored form
- greatest common factor (GCF)

Virginia SOL
STANDARD A.12  The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.

TEACHING TIP
An alternative definition is that a prime number is a positive integer with exactly two different factors. You may want to point out that 2 is the only even prime number.

Key Concept

<table>
<thead>
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<th>Prime and Composite Numbers</th>
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<td>Words</td>
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<tr>
<td>A whole number, greater than 1, with exactly two different factors, is called a <strong>prime number</strong>.</td>
</tr>
<tr>
<td>A whole number, greater than 1, with more than two factors is called a <strong>composite number</strong>.</td>
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0 and 1 are neither prime nor composite.

Example 1  Classify Numbers as Prime or Composite

Factor each number. Then classify each number as prime or composite.

a. 36

To find the factors of 36, list all pairs of whole numbers whose product is 36.

| 1 × 36 | 2 × 18 | 3 × 12 | 4 × 9 | 6 × 6 |

Therefore, the factors of 36, in increasing order, are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Since 36 has more than two factors, it is a composite number.
When a whole number is expressed as the product of factors that are all prime numbers, the expression is called the prime factorization of the number.

**Example 2**  
**Prime Factorization of a Positive Integer**

Find the prime factorization of 90.

**Method 1**

\[
90 = 2 \cdot 45 \\
= 2 \cdot 3 \cdot 15 \\
= 2 \cdot 3 \cdot 3 \cdot 5
\]

The least prime factor of 90 is 2. The least prime factor of 45 is 3. The least prime factor of 15 is 3.

All of the factors in the last row are prime. Thus, the prime factorization of 90 is \(2 \cdot 3 \cdot 3 \cdot 5\).

**Method 2**

Use a factor tree.

```
90
   /  \  
  9   10
     /   /  \\
   3   3  2
```

All of the factors in the last branch of the factor tree are prime. Thus, the prime factorization of 90 is \(2 \cdot 3 \cdot 3 \cdot 5\) or \(2 \cdot 3^2 \cdot 5\).  

Usually the factors are ordered from the least prime factor to the greatest.

A negative integer is factored completely when it is expressed as the product of -1 and prime numbers.

**Example 3**  
**Prime Factorization of a Negative Integer**

Find the prime factorization of -140.

\[
-140 = -1 \cdot 140 \\
= -1 \cdot 2 \cdot 70 \\
= -1 \cdot 2 \cdot 7 \cdot 10 \\
= -1 \cdot 2 \cdot 7 \cdot 2 \cdot 5 \\
\]

Thus, the prime factorization of -140 is \(-1 \cdot 2 \cdot 2 \cdot 5 \cdot 7\) or \(-1 \cdot 2^2 \cdot 5 \cdot 7\).

A monomial is in **factored form** when it is expressed as the product of prime numbers and variables and no variable has an exponent greater than 1.
GREATEST COMMON FACTOR

In-Class Examples

5 Find the GCF of each set of monomials.

a. 12 and 18 The GCF is 6
b. 27ab^2 and 15ab^2c The GCF is 3ab

6 CRAFTS Rene has crocheted 32 squares for an afghan. Each square is 1 foot square. She is not sure how she will arrange the squares but does know it will be rectangular and have a ribbon trim. What is the maximum amount of ribbon she might need to finish the afghan? 66 ft

Example 4 Prime Factorization of a Monomial

Factor each monomial completely.

a. 12a^2b^3

\[12a^2b^3 = 2 \cdot 6 \cdot a \cdot a \cdot b \cdot b \cdot b\]

Thus, \(12a^2b^3\) in factored form is \(2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b\).

b. \(-66pq^2\)

\[-66pq^2 = -1 \cdot 66 \cdot p \cdot q \cdot q\]

Express \(-66\) as \(-1\) times 66.

\[-1 \cdot 2 \cdot 33 \cdot p \cdot q \cdot q\]

Thus, \(-66pq^2\) in factored form is \(-1 \cdot 2 \cdot 3 \cdot 11 \cdot p \cdot q \cdot q\).

Key Concept Greatest Common Factor (GCF)

• The GCF of two or more integers is the product of the prime factors common to the integers.
• The GCF of two or more monomials is the product of their common factors when each monomial is in factored form.
• If two or more integers or monomials have a GCF of 1, then the integers or monomials are said to be relatively prime.

Example 5 GCF of a Set of Monomials

Find the GCF of each set of monomials.

a. 15 and 16

\[15 = 3 \cdot 5\]

\[16 = 2 \cdot 2 \cdot 2 \cdot 2\]

Circle the common prime factors, if any.

There are no common prime factors, so the GCF of 15 and 16 is 1. This means that 15 and 16 are relatively prime.

b. 36x^2y and 54xy^2z

\[36x^2y = 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot y\]

\[54xy^2z = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot y \cdot z\]

Circle the common prime factors.

The GCF of 36x^2y and 54xy^2z is 2 \cdot 3 \cdot 3 \cdot x \cdot y or 18xy.

Interactive Chalkboard

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:
• Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
• Additional, Your Turn exercises for each example
• The 5-Minute Check Transparencies
• Hot links to Glencoe Online Study Tools

Teacher to Teacher

Lisa Cook
Kaysville Jr. H.S., Kaysville, UT

“To help in identifying prime factors, I like to have my students explore the Sieve of Eratosthenes. Use a 10-by-10 grid with the numbers 1-100 on it. Cross out 1 since prime numbers are greater than 1. Circle 2 and cross out all multiples of 2. Circle 3 and cross out multiples of 3. Continue with the next odd number until all multiples have been eliminated. The circled numbers are the prime numbers less than 100.”
Example 6 Use Factors

GEOMETRY The area of a rectangle is 28 square inches. If the length and width are both whole numbers, what is the maximum perimeter of the rectangle?

Find the factors of 28, and draw rectangles with each length and width. Then find each perimeter.

The factors of 28 are 1, 2, 4, 7, 14, and 28.

The greatest perimeter is 58 inches. The rectangle with this perimeter has a length of 28 inches and a width of 1 inch.

Guided Practice

Find the factors of each number. Then classify each number as prime or composite.

1. 81, 2, 4, 8; composite
2. 49, 1, 7, 14; composite
3. 121, 1, 11; prime
4. 9, 1, 3; prime
5. 12, 1, 2, 3, 4, 6, 12; composite
6. 25, 1, 5; prime
7. 32, 2, 4, 8; composite
8. 16, 2, 4, 8; composite
9. 100, 1, 2, 4, 5, 10, 20, 25, 50, 100; composite
10. 45, 3² · 5
11. 36, 2² · 3²
12. 100, 2² · 5²

Factor each monomial completely.

11. 8x² · 3y³ · z
12. 27x³ · y² · z²

Find the GCF of each set of monomials.

13. 12, 3, 4, 6; 1, 2, 3, 6; relatively prime
14. 18xy, 36y²; 18, 36y²; composite
15. 12a²b, 90a³b²c; 6a²b; composite
16. 35x³, 70x⁵; 35x³; composite

Application

19. GARDENING Ashley is planting 120 tomato plants in her garden. In what ways can she arrange them so that she has the same number of plants in each row, at least 5 rows of plants, and at least 5 plants in each row?

Find the factors of each number. Then classify each number as prime or composite.

20. 19
21. 25
22. 80
23. 61
24. 91
25. 119
26. 126
27. 304

Answers

20. 1, 19; prime
21. 1, 5, 25; composite
22. 1, 2, 4, 5, 8, 10, 16, 20, 40, 80; composite
23. 1, 61; prime
24. 1, 7, 13, 91; composite
25. 1, 7, 17, 119; composite
26. 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126; composite
27. 1, 2, 4, 8, 16, 19, 38, 76, 152, 304; composite

www.algebra1.com/self_check_quiz/sol

Lesson 9-1 Factors and Greatest Common Factors 477
How many tiles of this size will she need?

What is the maximum perimeter of the rectangle?

32. 39 \cdot 13

33. \begin{align*}
-98 &\quad -1 \cdot 2 \cdot 7^2 \\
34. &\quad 117 \cdot 2^2 \cdot 13
\end{align*}

35. 102 \cdot 2 \cdot 3 \cdot 17

36. \begin{align*}
-115 &\quad -1 \cdot 5 \cdot 23 \\
37. &\quad 180 \cdot 2^2 \cdot 3^2
\end{align*}

38. 360 \cdot 2^3 \cdot 3^2 \cdot 5

39. \begin{align*}
&\quad -1 \cdot 2 \cdot 3 \cdot 7 \cdot 11
\end{align*}

Factor each monomial completely. 40. 46d^4

41. 85x^2y^2

42. 49a^2b^2

43. 50gh

44. 128p^2q^2

45. 243m^6n^4

46. -183xy^3

47. -169a^2bc^2

Find the GCF of each set of monomials.

48. 27, 2, 9

49. 18, 35, 1

50. 32, 48, 16

51. 84, 70

52. 16, 20, 64

53. 42, 63, 105

21

54. 15x, 28x^2

55. 55, 24x^2, 30x^2d

6d

56. 20y^3, 36y^3c^2

4gh

57. 21p^2q, 32r^2t

58. 18x, 30xy, 54y

6

59. 28a^2, 63a^2b^2, 91b^7

7

60. 14m^2n^2, 18mn, 2m^2n^2

2mn

61. 80r^2b, 96r^2b^3, 128r^2b^2

16a^2b

62. NUMBER THEORY Twin primes are two consecutive odd numbers that are prime. The first pair of twin primes is 3 and 5. List the next five pairs of twin primes. 5, 7; 11, 13; 17, 19; 29, 31; 41, 43

MARCHING BANDS

For Exercises 63 and 64, use the following information.

Central High’s marching band has 75 members, and the band from Northeast High has 90 members. During the halftime show, the bands plan to march into the stadium from opposite ends using formations with the same number of rows.

63. If the bands want to match up in the center of the field, what is the maximum number of rows? 15

64. How many band members will be in each row after the bands are combined? 11

NUMBER THEORY

For Exercises 65 and 66, use the following information.

One way of generating prime numbers is to use the formula \(2^n - 1\), where \(p\) is a prime number. Primes found using this method are called Mersenne primes. For example, when \(p = 2, 2^2 - 1 = 3\). The first Mersenne prime is 3.

65. Find the next two Mersenne primes. 7, 31

66. Will this formula generate all possible prime numbers? Explain your reasoning. No, it does not generate the first prime number, 2.

Answers

28. The factors of 96 whose sum when doubled is the least are 12 and 18.

29. The factors of 96 whose sum when doubled is the greatest are 1 and 96.
Finding the GCF of distances will help you make a scale model of the solar system. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

68a. false; counterexample: \( a = 3, b = 4 \)

69. **Writing in Math** Answer the question that was posed at the beginning of the lesson. See p. 521A.

How are prime numbers related to the search for extraterrestrial life?

Include the following in your answer:

- a list of the first 30 prime numbers and an explanation of how you found them, and
- an explanation of why a signal of this kind might indicate that an extraterrestrial message is to follow.

70. Miko claims that there are at least four ways to design a 120-square-foot rectangular space that can be tiled with 1-foot by 1-foot tiles. Which statement best describes this claim? D

A. Her claim is false because 120 is a prime number.
B. Her claim is false because 120 is not a perfect square.
C. Her claim is true because 240 is a multiple of 120.
D. Her claim is true because 120 has at least eight factors.

71. Suppose \( \Psi_x \) is defined as the largest prime factor of \( x \). For which of the following values of \( x \) would \( \Psi_x \) have the greatest value? A

A. 53
B. 74
C. 99
D. 117

---

**Maintain Your Skills**

**Mixed Review** Find each product. (Lessons 8-7 and 8-8) 73. \( 9a^2 - 25 \) 74. \( 49p^4 + 56p^2 + 16 \)

72. \( (2x - 1)^2 \) 73. \( (3a + 5)(3a - 5) \)

75. \( (6r + 7)(2r - 5) \) 76. \( (10k + k)(2h + 5k) \)

77. \( (b + 4)(b^2 + 3b - 18) \) 78. \( 12r^2 - 16r - 35 \)

79. \( 20h^2 + 52hk + 5k^2 \) 80. \( 12^2 - 16r - 35 \)

Find the value of \( r \) so that the line that passes through the given points has the given slope. (Lesson 5-1)

81. \( (1, 2), (-2, r) \), \( m = 3 \)

79. \( (-5, 9), (r, 6) \), \( m = -\frac{3}{5} \)

---

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Use the Distributive Property to rewrite each expression.

81. \( 5(2x + 8) \) 82. \( a(3a + 1) \)

83. \( 2g(3g - 4) \) 84. \( -12y^2 + 24y \)

85. \( 7b + 7c \) 86. \( 2x + 3x(2 + 3)x \)

---

**Answers**

40. \( 2 \cdot 3 \cdot 11 \cdot d \cdot d \cdot d \)

41. \( 5 \cdot 17 \cdot x \cdot x \cdot y \cdot y \)

42. \( 7 \cdot 7 \cdot a \cdot a \cdot b \cdot b \)

43. \( 2 \cdot 5 \cdot 5 \cdot g \cdot h \)

44. \( 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot p \cdot q \cdot q \)

45. \( 3 \cdot 3 \cdot 3 \cdot 3 \cdot n \cdot n \cdot n \cdot m \)

46. \( -1 \cdot 3 \cdot 61 \cdot x \cdot y \cdot z \cdot z \cdot z \)

47. \( -1 \cdot 13 \cdot 13 \cdot a \cdot b \cdot c \cdot c \)

67. base 1 cm, height 40 cm; base 2 cm, height 20 cm; base 4 cm, height 10 cm; base 5 cm, height 8 cm; base 8 cm, height 5 cm; base 10 cm, height 4 cm; base 20 cm, height 2 cm; base 40 cm, height 1 cm

68b. If \( 6 \) is a factor of \( ab \), then the prime factorization of \( ab \) must contain \( 2 \cdot 3 \cdot 3 \). \( 3 \) must be a factor of either \( a \) or \( b \).
Factoring Using the Distributive Property

Sometimes you know the product of binomials and are asked to find the factors. This is called factoring. You can use algebra tiles and a product mat to factor binomials.

**Activity 1** Use algebra tiles to factor $3x + 6$.

*Step 1* Model the polynomial $3x + 6$.

*Step 2* Arrange the tiles into a rectangle. The total area of the rectangle represents the product, and its length and width represent the factors.

The rectangle has a width of 3 and a length of $x + 2$. So, $3x + 6 = 3(x + 2)$.

**Activity 2** Use algebra tiles to factor $x^2 - 4x$.

*Step 1* Model the polynomial $x^2 - 4x$.

*Step 2* Arrange the tiles into a rectangle.

The rectangle has a width of $x$ and a length of $x - 4$. So, $x^2 - 4x = x(x - 4)$.

9. Binomials can be factored if they can be represented by a rectangle.

Examples: $2x + 2$ can be factored and $2x + 1$ cannot be factored.

**Model and Analyze**

Use algebra tiles to factor each binomial.

1. $2x + 10$ (master for algebra tiles)  2. $2(x + 5)$  3. $5x^2 + 2x$  4. $3x - 3x - x$  5. $2x + 2$ (master for product mat)  6. $6x - 8$  7. $2x - 2x$  8. $2x + 3$ (student recording sheet)

Tell whether each binomial can be factored. Justify your answer with a drawing.

5. $4x - 10$ yes  6. $3x - 7$ no  7. $x^2 + 2x$ yes  8. $2x^2 + 3$ no


9. **MAKE A CONJECTURE** Write a paragraph that explains how you can use algebra tiles to determine whether a binomial can be factored. Include an example of one binomial that can be factored and one that cannot.
Factor each number.
Circle the common prime factors.
In Chapter 8, you
Factor polynomials by using the Distributive Property.
Simplify remaining factors.
Solve quadratic equations of the form $ax^2 + bx = 0$.

FACTOR BY USING THE DISTRIBUTIVE PROPERTY In Chapter 8, you used the Distributive Property to multiply a polynomial by a monomial.

$$2a(6a + 8) = 2a(6a) + 2a(8)$$
$$= 12a^2 + 16a$$

You can reverse this process to express a polynomial as the product of a monomial factor and a polynomial factor.

$$12a^2 + 16a = 2a(6a) + 2a(8)$$
$$= 2a(6a + 8)$$

Thus, a factored form of $12a^2 + 16a$ is $2a(6a + 8)$.

Factoring a polynomial means to find its completely factored form. The expression $2a(6a + 8)$ is not completely factored since $6a + 8$ can be factored as $2(3a + 4)$.

**Example 1 Use the Distributive Property**

Use the Distributive Property to factor each polynomial.

a. $12a^2 + 16a$

First, find the GCF of $12a^2$ and $16a$.

$12a^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a$

Factor each number.

$16a = 2 \cdot 2 \cdot 2 \cdot 2 \cdot a$

Circle the common prime factors.

GCF: $2 \cdot 2 \cdot a = 4a$

Write each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

$12a^2 + 16a = 4a(3a) + 4a(4)$

Rewrite each term using the GCF.

Simplify remaining factors.

$= 4a(3a + 4)$

Distributive Property

Thus, the completely factored form of $12a^2 + 16a$ is $4a(3a + 4)$.

Lesson 9-2 Factoring Using the Distributive Property 481

**5-Minute Check Transparency 9-2** Use as a quiz or review of Lesson 9-1.

**Mathematical Background** notes are available for this lesson on p. 472C.

**Building on Prior Knowledge**

In Chapter 1, students were introduced to the Distributive Property and learned how to use it to simplify expressions. In Chapter 8, students learned to multiply a polynomial by a monomial using the Distributive Property. In this lesson, students will reverse that process to factor polynomials.

**How can you determine how long a baseball will remain in the air?**

Ask students:

- What is the greatest common factor (GCF) of two numbers? The GCF is the greatest number that is a factor of both original numbers.
- What is the GCF of $15t$ and $16t^2$? $t$
- What is the height of the ball when $t = 0$? $0$ ft
- What is the height of the ball when $t = 1$? $135$ ft

**Workbook and Reproducible Masters**

- **Chapter 9 Resource Masters**
  - Study Guide and Intervention, pp. 529–530
  - Skills Practice, p. 531
  - Practice, p. 532
  - Reading to Learn Mathematics, p. 533
  - Enrichment, p. 534
  - Assessment, p. 573

- **Parent and Student Study Guide Workbook**, p. 69
- **Prerequisite Skills Workbook**, pp. 13–14
- **School-to-Career Masters**, p. 17

**Transparencies**

- 5-Minute Check Transparency 9-2
- Answer Key Transparencies

**Technology**

- Interactive Chalkboard
2 Teach

FACTOR BY USING THE DISTRIBUTIVE PROPERTY

1 Use the Distributive Property to factor each polynomial.

a. $15x + 25x^2 = 5x(3 + 5x)$

b. $12xy + 24xy^2 - 30x^2y^3 = 6xy(2 + 4y - 5xy^2)$

2 Factor $2xy + 7x - 2y - 7 = (x - 1)(2y + 7)$

3 Factor $15a - 3ab + 4b - 20 = (-3a + 4)(b - 5)$

Study Tip

Factoring by Grouping

Sometimes you can group terms in more than one way when factoring a polynomial. For example, the polynomial in Example 2 could have been factored in the following way.

$4ab + 8b + 3a + 6$

$= (4ab + 8b) + (3a + 6)$

$= 4b(a + 2) + 3(a + 2)$

$= (a + 2)(4b + 3)$

Notice that this result is the same as in Example 2.

Concept Check

Factoring Using the Distributive Property

Malcolm and Fatima each factored the polynomial $2ax + 6cx + ab + 3bc$. Malcolm’s answer was $(2x + b)(a + 3c)$ and Fatima’s was $(a + 3c)(2x + b)$. Which is correct? Explain your answer.

Both are correct. The order in which factors are multiplied does not affect the product.

The Distributive Property can also be used to factor some polynomials having four or more terms. This method is called factoring by grouping because pairs of terms are grouped together and factored. The Distributive Property is then applied a second time to factor a common binomial factor.

Example 2 Use Grouping

Factor $4ab + 8b + 3a + 6$.

$4ab + 8b + 3a + 6 = (4ab + 8b) + (3a + 6)$

$= 4b(a + 2) + 3(a + 2)$

$= (a + 2)(4b + 3)$

CHECK

Use the FOIL method.

$F \quad O \quad I \quad L$

$(a + 2)(4b + 3) = (a)(4b) + (a)(3) + (2)(4b) + (2)(3)$

$= 4ab + 3a + 8b + 6 \checkmark$

Recognizing binomials that are additive inverses is often helpful when factoring by grouping. For example, $7 - y$ and $y - 7$ are additive inverses. By rewriting $7 - y$ as $-1(y - 7)$, factoring by grouping is possible in the following example.

Example 3 Use the Additive Inverse Property

Factor $35x - 5xy + 3y - 21$.

$35x - 5xy + 3y - 21 = (35x - 5xy) + (3y - 21)$

$= 5x(7 - y) + 3(y - 7)$

$= 5x(-1)(y - 7) + 3(y - 7)$

$= -5x(y - 7) + 3(y - 7)$

$= (y - 7)(-5x + 3)$

Concept Summary

Factoring by Grouping

- **Words**
  - A polynomial can be factored by grouping if all of the following situations exist.
  - There are four or more terms.
  - Terms with common factors can be grouped together.
  - The two common factors are identical or are additive inverses of each other.

- **Symbols**
  - $ax + bx + ay + by = x(a + b) + y(a + b)$
  - $(a + b)(x + y)$
SOLVE EQUATIONS BY FACTORING Some equations can be solved by factoring. Consider the following products.

\[
6(0) = 0 \\
0(-3) = 0 \\
(5 - 5)(0) = 0 \\
-2(-3 + 3) = 0
\]

Notice that in each case, at least one of the factors is zero. These examples illustrate the Zero Product Property.

Key Concept Zero Product Property

- **Words**: If the product of two factors is 0, then at least one of the factors must be 0.
- **Symbols**: For any real numbers \(a\) and \(b\), if \(ab = 0\), then either \(a = 0\), \(b = 0\), or both \(a\) and \(b\) equal zero.

**Example 4** Solve an Equation in Factored Form

Solve \((d - 5)(3d + 4) = 0\). Then check the solutions.

If \((d - 5)(3d + 4) = 0\), then according to the Zero Product Property either \(d - 5 = 0\) or \(3d + 4 = 0\).

\[
\begin{align*}
(d - 5)(3d + 4) &= 0 \\
(d - 5) &= 0 \quad \text{or} \\
3d + 4 &= 0
\end{align*}
\]

\[
\begin{align*}
d &= 5 \\
3d &= -4 \\
d &= \frac{-4}{3}
\end{align*}
\]

The solution set is \(\left\{5, \frac{-4}{3}\right\}\).

**CHECK** Substitute 5 and \(\frac{-4}{3}\) for \(d\) in the original equation.

\[
\begin{align*}
(d - 5)(3d + 4) &= 0 \\
(5 - 5)(3(5) + 4) &= 0 \\
(0)(19) &= 0 \\
0 &= 0
\end{align*}
\]

If an equation can be written in the form \(ab = 0\), then the Zero Product Property can be applied to solve that equation.

**Example 5** Solve an Equation by Factoring

Solve \(x^2 = 7x\). Then check the solutions.

Write the equation so that it is of the form \(ab = 0\).

\[
\begin{align*}
x^2 &= 7x \\
x^2 - 7x &= 0 \\
(x)(x - 7) &= 0 \\
x &= 0 \quad \text{or} \\
x - 7 &= 0
\end{align*}
\]

\[
\begin{align*}
x &= 0 \\
x &= 7
\end{align*}
\]

The solution set is \(\{0, 7\}\). Check by substituting 0 and 7 for \(x\) in the original equation.

SOLVE EQUATIONS BY FACTORING

**In-Class Examples**

**Teaching Tip** If students think using the Zero Product Property to set factors equal to zero is somewhat arbitrary, have them multiply the two factors to obtain a polynomial, and set the polynomial equal to zero. Then have students substitute the two solutions into the polynomial to see that they produce a true sentence.

4. Solve \((x - 2)(4x - 1) = 0\). Then check the solutions. \(\left\{2, \frac{1}{4}\right\}\)

**Teaching Tip** Remind students that for the Zero Product Property to work, one of two factors is equal to zero. Therefore, students must factor \(x^2 - 7x\) before they can assume that one term is equal to zero.

5. Solve \(4y = 12y^2\). Then check the solutions. \(\{0, \frac{1}{3}\}\)

Common Misconception You may be tempted to try to solve the equation in Example 5 by dividing each side of the equation by \(x\). Remember, however, that \(x\) is an unknown quantity. If you divide by \(x\), you may actually be dividing by zero, which is undefined.

www.algebra1.com/extra_examples/sol

Lesson 9-2 Factoring Using the Distributive Property 483

**DAILY INTERVENTION**

**Differentiated Instruction**

**Visual/Spatial** When students solve factored equations, have them write each factor on a separate sheet of scrap paper, followed by "= 0" on a third sheet of scrap paper. Then, have students remove one of the factors and solve the remaining equation. Once the first solution is found, have students place the other factor equal to zero and solve for the second solution.
Check for Understanding

Concept Check

2. an equation that can be written as a product of factors that equal 0

Guided Practice

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Application

PHYSICAL SCIENCE For Exercises 13–15, use the information below and in the graphic.

A flare is launched from a life raft. The height $h$ of the flare in feet above the sea is modeled by the formula $h = 100t - 16t^2$, where $t$ is the time in seconds after the flare is launched.

13. At what height is the flare when it returns to the sea? 0 ft
14. Let $h = 0$ in the equation $h = 100t - 16t^2$ and solve for $t$. 0, 6.25
15. How many seconds will it take for the flare to return to the sea? Explain your reasoning. 6.25 s; The answer 0 is not reasonable since it represents the time at which the flare is launched.

Factor each polynomial. 16–39. See p. 521A.

16. $5x + 30y$
17. $16a + 4b$
18. $a^5b - a$
19. $x^3y^2 + x$
20. $21cd - 3d$
21. $14gh - 18h$
22. $15a^2y - 30xy$
23. $8bc^2 + 24bc$
24. $12x^2y^2 + 40xy^2z^2$
25. $a + a^2b + a^3b^3$
26. $15x^2y^2 + 25xy + x$
27. $3p^3q - 9pq^2 + 36pq$
28. $12x^2y^2 + 36xy$
29. $x^2 + 2x + 3 + 6$
30. $3x^2 + 5x + 7x + 35$
31. $4x^2 + 14x + 6x + 21$
32. $12y^2 + 9y + 8y + 6$
33. $6a^2 - 15a - 8a + 20$
34. $18x^2 - 30x - 3x + 5$
35. $2xy + 7x + 7m + 2xy$
36. $4ax + 3ay + 4bx + 3by$
37. $10x^2 - 14xy - 15x + 21y$
38. $8ax - 6x - 12a + 9$
39. $12a^2 - 3a - 2a^2$
40. Write this expression in factored form. $\frac{1}{2}n(a - 3)$
41. Find the number of diagonals in a decagon (10-sided polygon). 35

Odd/Even Assignments
Exercises 16–39 and 44–59 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 17–39 odd, 40–43, 47–59 odd, 62–81
Average: 17–39 odd, 42, 43, 45, 47–61 odd, 62–81
Advanced: 16–38 even, 44, 45, 46–60 even, 62–75 (optional: 76–81)
All: Practice Quiz 1 (1–10)

Answers
1. Sample answers: $4(x^2 + 3x)$, $x(4x + 12)$, or $4x(x + 3)$; $4x(x + 3)$; $4x$ is the GCF of $4x^2$ and $12x$.
63. Answers should include the following.
   • Let $h = 0$ in the equation $h = 151t - 16t^2$. To solve $0 = 151t - 16t^2$, factor the right-hand side as $t(151 - 16t)$. Then, since $t(151 - 16t) = 0$, either $t = 0$ or $151 - 16t = 0$. Solving each equation for $t$, we find that $t = 0$ or $t = 9.44$.
   • The solution $t = 0$ represents the point at which the ball was initially thrown into the air. The solution $t = 9.44$ represents how long it took after the ball was thrown for it to return to the same height at which it was thrown.

About the Exercises...

Organization by Objective
• Factor by Using the Distributive Property: 16–39
• Solve Equations by Factoring: 48–59
**GEOMETRY**

Write an expression in factored form for the area of each shaded region.

44. [Diagram of shaded region]

\[ 4(a + b + 4) \]

45. [Diagram of shaded region]

\[ 2r^2(4 - \pi) \]

**GEOMETRY**

Find an expression for the area of a square with the given perimeter.

46. \[ P = 12x + 20y \text{ in.} \]

\[ 9x^2 + 30xy + 25y^2 \text{ in}^2 \]

Solve each equation. Check your solutions.

48. \( x(x - 24) = 0 \)

49. \( a(a + 16) = 0 \)

50. \( (g + 4)(3g - 15) = 0 \)

51. \( (3y + 9)(y - 7) = 0 \)

52. \( (2b - 3)(3b - 8) = 0 \)

53. \( (4n + 5)(3n - 7) = 0 \)

54. \( 4c^2 + 12c = 0 \)

55. \( 7d^2 - 35d = 0 \)

56. \( 2x^2 = 5x \)

57. \( 7x^2 = 6x \)

58. \( 6x^2 = -4x \)

**MARINE BIOLOGY**

In a pool at a water park, a dolphin jumps out of the water traveling at 20 feet per second. Its height, \( h \), in feet, above the water after \( t \) seconds is given by the formula \( h = 20 - 16t^2 \). How long is the dolphin in the air before returning to the water? **1.25 s**

**BASEBALL**

Malik popped a ball straight up with an initial upward velocity of 45 feet per second. The height, \( h \), in feet, of the ball above the ground is modeled by the equation \( h = 2 + 45t - 16t^2 \). How long was the ball in the air if the catcher catches the ball when it is 2 feet above the ground? **about 2.8 s**

**CRITICAL THINKING**

Factor \( a^2 + y + a^2b - a^2b^2 - a^2 + y \). \( (a^2 - b)(a^2b - 1) \)

**WRITING IN MATH**

Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you determine how long a baseball will remain in the air?
Include the following in your answer:

- An explanation of how to use factoring and the Zero Product Property to find how long the ball would be in the air, and
- An interpretation of each solution in the context of the problem.

---

**Enrichment, p. 534**

**Perfect, Excessive, Defective, and Amicable Numbers**

A perfect number is a sum of all of its factors except itself.

- Ex: 6
  - Factors: 1, 2, 3, 6
  - Sum: 1 + 2 + 3 = 6

A prime number is either excessive or defective.

- 4
  - Factors: 1, 2
  - Sum: 1 + 2 = 3

A defective number is less than its factors.

- 8
  - Factors: 1, 2, 4
  - Sum: 1 + 2 + 4 = 7

**Solve each problem.**

1. Write the perfect numbers between 6 and 31.

2. 6
4 Assess

Open-Ended Assessment

Speaking  Ask a volunteer to describe the similarities and differences between factoring using grouping, and factoring using the additive inverse property. Encourage other students to ask questions.

Getting Ready for Lesson 9-3

PREREQUISITE SKILL  Students will learn to factor trinomials in Lesson 9-3. It is important that students recall how to multiply binomials to check that they correctly factored trinomials. Use Exercises 76–81 to determine your students’ familiarity with multiplying polynomials.

Assessment Options

Practice Quiz 1  The quiz provides students with a brief review of the concepts and skills in Lessons 9-1 and 9-2. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 9-1 and 9-2) is available on p. 573 of the Chapter 9 Resource Masters.

Answer

1. 1, 3, 5, 9, 15, 25, 45, 75, 225; composite

Maintain Your Skills

Mixed Review

Find the factors of each number. Then classify each number as prime or composite.  

(Exercise 9-1)

66. 123; composite
67. 300; composite
68. 67; prime

Find each product.  (Exercise 8-8)

69. 16s^2 + 24s^3 + 9
70. 4x^2 – 25y^2

Simplify. Assume that no denominator is equal to zero.  (Exercise 8-2)

72. \(\frac{s^4}{s^2} \cdot s^5\)
73. \(\frac{18s^3}{12s^2} \cdot \frac{3x}{2y^2}\)
74. \(\frac{3p^2q^3r^5}{2p^2}\)

75. FINANCE  Michael uses at most 60% of his annual FlynnCo stock dividend to purchase more shares of FlynnCo stock. If his dividend last year was $885 and FlynnCo stock is selling for $14 per share, what is the greatest number of shares that he can purchase?  (Lesson 6-2) 37 shares

Getting Ready for the Next Lesson

PREREQUISITE SKILL  Find each product.  

(To review multiplying polynomials, see Lesson 8-7)

76. (n + 8)(n + 3)
77. (x – 4)(x – 5)
78. (b – 10)(b + 7)

79. (3a + 1)(6a – 4)
80. (5p + 2)(9p – 3)
81. (2y – 5)(4y + 3)

82. \(a^2 – 6a – 4\)
83. \(45p^2 – 33p + 6\)
84. \(8y^2 – 14y – 15\)

Practice Quiz 1

1. Find the factors of 225. Then classify the number as prime or composite.  (Lesson 9-1) See margin.
2. Find the prime factorization of –320.  (Lesson 9-1) \(-1 \cdot 2^6 \cdot 5\)
3. Factor 78a^2b^3 completely.  (Lesson 9-1) \(2 \cdot 3 \cdot 13 \cdot a \cdot a \cdot b \cdot c \cdot c \cdot c\)
4. Find the GCF of 54x^3, 42x^2y, and 30xy^2.  (Lesson 9-1) 6x

Factor each polynomial.  

(Exercise 9-2)

5. \(4xy^2 – xy\)
6. \(32a^2 + 40b^3 – 8xy^2\)
7. \(6xy + 16y – 15y – 40\)

Solve each equation. Check your solutions.  

(Exercise 9-2)

8. \((8n + 5)(n – 4) = 0\)
9. \(9x^2 – 27x = 0\)
10. \(10x^2 = –3x\)

See margin.
Factoring Trinomials

You can use algebra tiles to factor trinomials. If a polynomial represents the area of a rectangle formed by algebra tiles, then the rectangle’s length and width are factors of the area.

Activity 1
Use algebra tiles to factor \(x^2 + 6x + 5\).

Step 1
Model the polynomial \(x^2 + 6x + 5\).

Step 2
Place the \(x^2\) tile at the corner of the product mat. Arrange the 1 tiles into a rectangular array. Because 5 is prime, the 5 tiles can be arranged in a rectangle in one way, a 1-by-5 rectangle.

Step 3
Complete the rectangle with the \(x\) tiles. The rectangle has a width of \(x + 1\) and a length of \(x + 5\). Therefore, \(x^2 + 6x + 5 = (x + 1)(x + 5)\).

Activity 2
Use algebra tiles to factor \(x^2 + 7x + 6\).

Step 1
Model the polynomial \(x^2 + 7x + 6\).

Step 2
Place the \(x^2\) tile at the corner of the product mat. Arrange the 1 tiles into a rectangular array. Since 6 = 2 \times 3, try a 2-by-3 rectangle. Try to complete the rectangle. Notice that there are two extra \(x\) tiles.

(continued on the next page)
• **Activity 3** Remind students to pay close attention to the sign of the tiles. It would be very easy to mistake the factors as \((x + 3)(x + 3)\) if they do not pay attention to the signs.

• Students may need to be reminded that adding a zero pair is similar to adding the same number to both sides of an equation. Be sure that students are careful to add one \(x\) tile and one \(-x\) tile when they add a zero pair.

**Assess**

• After students complete Exercises 1–8, ask them whether they notice a correlation between the need to use zero pairs to factor the trinomial, and the appearance of the resulting factors. **Sample answer:** When zero pairs are used, the signs of the factors are opposite. When zero pairs are not used, the signs of the factors are the same.

**Study Notebook**

You may wish to have students summarize this activity and what they learned from it.

**Activity 3** Use algebra tiles to factor \(x^2 - 2x - 3\).

**Step 1** Model the polynomial \(x^2 - 2x - 3\).

**Step 2** Place the \(x^2\) tile at the corner of the product mat. Arrange the 1 tiles into a 1-by-3 rectangular array as shown.

**Step 3** Place the \(x\) tile as shown. Recall that you can add zero-pairs without changing the value of the polynomial. In this case, add a zero pair of \(x\) tiles.

The rectangle has a width of \(x + 1\) and a length of \(x - 3\). Therefore, \(x^2 - 2x - 3 = (x + 1)(x - 3)\).

**Model** 1. \((x + 3)(x + 1)\) 2. \((x + 4)(x + 1)\) 3. \((x - 3)(x + 2)\) 4. \((x - 2)(x - 1)\)

Use algebra tiles to factor each trinomial.

1. \(x^2 + 4x + 3\) 2. \(x^2 + 5x + 4\) 3. \(x^2 - x - 6\) 4. \(x^2 - 3x + 2\)

5. \(x^2 + 7x + 12\) 6. \(x^2 - 4x + 4\) 7. \(x^2 - x - 2\) 8. \(x^2 - 6x + 8\)

488 Chapter 9 Factoring
**Interest Math**
A trinomial of degree 2.
This means that the
variable is 2.

**How can factoring be used to find the dimensions of a garden?**

Tamika has enough bricks to make a 30-foot
border around the rectangular vegetable garden
she is planting. The booklet she got from
the nursery says that the plants will need a space
of 54 square feet to grow. What should the
dimensions of her garden be? To solve this
problem, you need to find two numbers whose
product is 54 and whose sum is 15, half the
perimeter of the garden.

**FACTOR** $x^2 + bx + c$ In Lesson 9-1, you learned that when two numbers are
multiplied, each number is a factor of the product. Similarly, when two binomials
are multiplied, each binomial is a factor of the product.

To factor some trinomials, you will use the pattern for multiplying two binomials.
Study the following example.

- **L** &nbsp; Use the FOIL method.
- **S** &nbsp; Simplify.
- **D** &nbsp; Distributive Property
- **S** &nbsp; Simplify.

$$\begin{align*}
(x + 2)(x + 3) &= (x \cdot x) + (x \cdot 3) + (2 \cdot 2) + (2 \cdot 3) \\
&= x^2 + 3x + 2x + 6 \\
&= x^2 + (3 + 2)x + 6 \\
&= x^2 + 5x + 6
\end{align*}$$

Observe the following pattern in this multiplication.

$$\begin{align*}
(x + 2)(x + 3) &= x^2 + (3 + 2)x + (2 \cdot 3) \\
(x + m)(x + n) &= x^2 + (n + m)x + mn \\
&= x^2 + (m + n)x + mn \\
&= x^2 + \frac{bx}{2} + c
\end{align*}$$

Observe the following pattern in this multiplication.

$$\begin{align*}
(x + 2)(x + 3) &= x^2 + (3 + 2)x + (2 \cdot 3) \\
(x + m)(x + n) &= x^2 + (n + m)x + mn \\
&= x^2 + (m + n)x + mn \\
&= x^2 + \frac{bx}{2} + c
\end{align*}$$

Notice that the coefficient of the middle term is the sum of $m$ and $n$ and the last term
is the product of $m$ and $n$. This pattern can be used to factor quadratic trinomials of
the form $x^2 + bx + c$.

**Key Concept**

**Words** To factor quadratic trinomials of the form $x^2 + bx + c$, find
two integers, $m$ and $n$, whose sum is equal to $b$ and whose product
is equal to $c$. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

**Symbols**

$$x^2 + bx + c = (x + m)(x + n)$$

when $m + n = b$ and $mn = c$.

**Example**

$$x^2 + 5x + 6 = (x + 2)(x + 3), \text{ since } 2 + 3 = 5 \text{ and } 2 \cdot 3 = 6.$$
The concept of factoring trinomials as introduced in this lesson may seem somewhat abstract to some students. Whenever you introduce abstract concepts, it is good to reinforce them with a concrete example. After introducing factoring trinomials, refer students back to the lesson opener problem. Ask students to describe any similarities they notice between finding the dimensions of the garden and factoring a trinomial.

### In-Class Examples

#### Teaching Tip
Tell students that the order in which they record the factors does not matter. So, \((x + 4)(x + 2)\) is also correct.

1. Factor \(x^2 + 7x + 12\).
\[(x + 3)(x + 4)\]

#### Teaching Tip
If students use the graphing calculator to check their factoring, make sure they clear all other functions from the Y= list, and clear all other drawings from the draw menu.

2. Factor \(x^2 - 12x + 27\).
\[(x - 3)(x - 9)\]

### Study Tip

#### Teaching Tip
Tell students to test any other factors. Once you find the correct factors, there is no need to test any other factors. Therefore, it is not necessary to test \(-4\) and \(-4\) in Example 2.

#### Teaching Tip
Caution students that two graphs may appear to coincide in the standard viewing window, but they do not. Have them use the TABLE feature to verify the identical \(y\) values.

### Example 1 \(b\) and \(c\) Are Positive

#### Factor \(x^2 + 6x + 8\).

In this trinomial, \(b = 6\) and \(c = 8\). You need to find two numbers whose sum is 6 and whose product is 8. Make an organized list of the factors of 8, and look for the pair of factors whose sum is 6.

<table>
<thead>
<tr>
<th>Factors of 8</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 8</td>
<td>9</td>
</tr>
<tr>
<td>2, 4</td>
<td>6</td>
</tr>
</tbody>
</table>

The correct factors are 2 and 4.

\[
x^2 + 6x + 8 = (x + m)(x + n)
\]

Write the pattern.

\[
= (x + 2)(x + 4)
\]

\(m = 2\) and \(n = 4\)

**CHECK** You can check this result by multiplying the two factors.

\[
(x + 2)(x + 4) = x^2 + 4x + 2x + 8
\]

\[
= x^2 + 6x + 8
\]

Simplify.

When factoring a trinomial where \(b\) is negative and \(c\) is positive, you can use what you know about the product of binomials to help narrow the list of possible factors.

#### Example 2 \(b\) Is Negative and \(c\) Is Positive

#### Factor \(x^2 - 10x + 16\).

In this trinomial, \(b = -10\) and \(c = 16\). This means that \(m + n\) is negative and \(mn\) is positive. So \(m\) and \(n\) must both be negative. Therefore, make a list of the negative factors of 16, and look for the pair of factors whose sum is \(-10\).

<table>
<thead>
<tr>
<th>Factors of 16</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1, -16)</td>
<td>(-17)</td>
</tr>
<tr>
<td>(-2, -8)</td>
<td>(-10)</td>
</tr>
<tr>
<td>(-4, -4)</td>
<td>(-8)</td>
</tr>
</tbody>
</table>

The correct factors are \(-2\) and \(-8\).

\[
x^2 - 10x + 16 = (x + m)(x + n)
\]

Write the pattern.

\[
= (x - 2)(x - 8)
\]

\(m = -2\) and \(n = -8\)

**CHECK** You can check this result by using a graphing calculator. Graph \(y = x^2 - 10x + 16\) and \(y = (x - 2)(x - 8)\) on the same screen.

Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly.

You will find that keeping an organized list of the factors you have tested is particularly important when factoring a trinomial like \(x^2 + x - 12\), where the value of \(c\) is negative.

### Differentiated Instruction

#### Kinesthetic
As students are learning the rules for factoring trinomials, encourage them to use algebra tiles to confirm their results. Students should soon realize that the greater the values of \(b\) and \(c\) in the trinomials, the more cumbersome algebra tiles become, which should reinforce the importance of learning to factor using the method in the text.
Some equations of the form

The correct factors are

Solve each equation.

Zero Product Property

Rewrite the equation so that one side equals 0.

Original equation

Write the pattern.

Study Tip

Try factor pairs of −12 until the sum of the products of the inner and outer terms is x.

Example 3  b Is Positive and c Is Negative

Factor \( x^2 + x - 12 \).

In this trinomial, \( b = 1 \) and \( c = -12 \). This means that \( m + n \) is positive and \( mn \) is negative. So either \( m \) or \( n \) is negative, but not both. Therefore, make a list of the factors of −12, where one factor of each pair is negative. Look for the pair of factors whose sum is 1.

<table>
<thead>
<tr>
<th>Factors of −12</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, −12</td>
<td>−11</td>
</tr>
<tr>
<td>−1, 12</td>
<td>11</td>
</tr>
<tr>
<td>2, −6</td>
<td>−4</td>
</tr>
<tr>
<td>−2, 6</td>
<td>4</td>
</tr>
<tr>
<td>3, −4</td>
<td>−1</td>
</tr>
<tr>
<td>−3, 4</td>
<td>1</td>
</tr>
</tbody>
</table>

The correct factors are −3 and 4.

\[ x^2 + x - 12 = (x + m)(x + n) \] Write the pattern.

\[ = (x - 3)(x + 4) \] \( m = -3 \) and \( n = 4 \)

Example 4  b Is Negative and c Is Negative

Factor \( x^2 - 7x - 18 \).

Since \( b = -7 \) and \( c = -18 \), \( m + n \) is negative and \( mn \) is negative. So either \( m \) or \( n \) is negative, but not both.

<table>
<thead>
<tr>
<th>Factors of −18</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, −18</td>
<td>−17</td>
</tr>
<tr>
<td>−1, 18</td>
<td>17</td>
</tr>
<tr>
<td>2, −9</td>
<td>−7</td>
</tr>
</tbody>
</table>

The correct factors are 2 and −9.

\[ x^2 - 7x - 18 = (x + m)(x + n) \] Write the pattern.

\[ = (x + 2)(x - 9) \] \( m = 2 \) and \( n = -9 \)

SOLVE EQUATIONS BY FACTORING  Some equations of the form \( x^2 + bx + c = 0 \) can be solved by factoring and then using the Zero Product Property.

Example 5  Solve an Equation by Factoring

Solve \( x^2 + 5x = 6 \). Check your solutions.

\[ x^2 + 5x = 6 \] Original equation

\[ x^2 + 5x - 6 = 0 \] Rewrite the equation so that one side equals 0.

\( (x - 1)(x + 6) = 0 \) Factor.

\[ x - 1 = 0 \text{ or } x + 6 = 0 \] Zero Product Property

\[ x = 1 \text{ or } x = -6 \] Solve each equation.

The solution set is \( \{1, -6\} \).

CHECK  Substitute 1 and −6 for \( x \) in the original equation.

\[ x^2 + 5x = 6 \]

\[ (1)^2 + 5(1) \geq 6 \]

\[ 6 = 6 \checkmark \]

\[ (-6)^2 + 5(-6) \geq 6 \]

\[ 36 = 6 \checkmark \]

www.algebra1.com/extra_examples/sol

Lesson 9-3  Factoring Trinomials: \( x^2 + bx + c \) 491

In-Class Examples

**Teaching Tip** Point out to students that whenever \( c \) is negative, either \( m \) or \( n \) must be negative, because if a product is negative, then one of the two factors must be negative.

3. Factor \( x^2 + 3x - 18 \).

\( (x + 6)(x - 3) \)

4. Factor \( x^2 - x - 20 \).

\( (x - 5)(x + 4) \)

**SOLVE EQUATIONS BY FACTORING**

5. Solve \( x^2 + 2x = 15 \). Check your solutions. \( \{-5, 3\} \)

6. **ARCHITECTURE** Marion has a small art studio in her backyard. She wants to build a new studio that has three times the area of the old studio by increasing the length and width by the same amount. What are the dimensions of the new studio?

The dimensions of the new studio should be 18 ft by 20 ft.
492 Chapter 9

Factoring

One page of a document containing text on factoring trinomials and solving equations by factoring. The page includes exercises with solutions and explanations on how to factor trinomials.

Example 6: Solve a Real-World Problem by Factoring

YEARBOOK DESIGN: A sponsor for the school yearbook has asked that the length and width of a photo in their ad be increased by the same amount in order to double the area of the photo. If the photo was originally 12 centimeters wide by 8 centimeters long, what should the new dimensions of the enlarged photo be?

Explore

Begin by making a diagram like the one shown above, labeling the appropriate dimensions.

Plan

Let \( x \) = the amount added to each dimension of the photo.

\[
\begin{align*}
\text{The new length} & \quad \times \quad \text{the new width} & = & \quad \text{the new area,} \\
 x + 12 & \quad \times \quad x + 8 & = & \quad 2(8)(12) \\
\text{old area} & & & \\
\end{align*}
\]

Solve

\[
\begin{align*}
(x + 12)(x + 8) & = 2(8)(12) \quad \text{Write the equation.} \\
x^2 + 20x + 96 & = 192 \quad \text{Multiply.} \\
x^2 + 20x - 96 & = 0 \quad \text{Subtract 192 from each side.} \\
(x + 24)(x - 4) & = 0 \quad \text{Factor.} \\
\end{align*}
\]

\[
\begin{align*}
x + 24 & = 0 \quad \text{or} \quad x - 4 = 0 \quad \text{Zero Product Property} \\
x = -24 & \quad \text{or} \quad x = 4 \quad \text{Solve each equation.}
\end{align*}
\]

Examine

The solution set is \{-24, 4\}. Only 4 is a valid solution, since dimensions cannot be negative. Thus, the new length of the photo should be \(4 + 12\) or \(16\) centimeters, and the new width should be \(4 + 8\) or \(12\) centimeters.

Check for Understanding

1. Explain why, when factoring \(x^2 + 6x + 9\), it is not necessary to check the sum of the factor pairs \(-1\) and \(-9\) or \(-3\) and \(-3\). See margin.

2. OPEN ENDED Give an example of an equation that can be solved using the factoring techniques presented in this lesson. Then, solve your equation.

3. FIND THE ERROR Peter and Aleta are solving \(x^2 + 2x = 15\).

Peter

\[
\begin{align*}
x^2 + 2x + 15 & = 15 \\
x(x + 2) & = 15 \\
x = 15 \quad \text{or} \quad x + 2 = 15 \\
x = 13 \\
\end{align*}
\]

Aleta

\[
\begin{align*}
x^2 + 2x + 15 & = 15 \\
x^2 + 2x - 15 & = 0 \\
(x + 3)(x - 5) & = 0 \\
x + 3 & = 0 \quad \text{or} \quad x - 5 = 0 \\
x = -3 & \quad \text{or} \quad x = 5
\end{align*}
\]

Who is correct? Explain your reasoning. Aleta; to use the Zero Product Property, one side of the equation must equal zero.

Guided Practice

Factor each trinomial.

4. \((x + 3)(x + 8)\)\(5. \((c - 1)(c - 2)\)
6. \((n - 3)(n + 16)\)\(7. \(x^2 + 11x + 24\)\)
8. \(62 + 27a + a^2\)\(9. \(x^2 - 4xy + 3y^2\)\)

\((p + 5)(p - 7)\)\((a + 3)(a + 24)\)\((x - 3y)(x - y)\)

Answers

1. In this trinomial, \(b = 6\) and \(c = 9\). This means that \(m + n\) is positive and \(mn\) is positive. Only two positive numbers have both a positive sum and product. Therefore, negative factors of 9 need not be considered.

2. Sample answer: \(x^2 - 14x + 40 = 0; (4, 10)\)

3. (4, 10)
Solve each equation. Check your solutions. 10–15. See margin.
10. \( n^2 + 7n + 6 = 0 \)
11. \( a^2 + 5a - 36 = 0 \)
12. \( p^2 - 19p - 42 = 0 \)
13. \( y^2 + 9 = -10y \)
14. \( 9x + x^2 = 22 \)
15. \( d^2 - 3d = 70 \)

**Application**

**16. NUMBER THEORY** Find two consecutive integers whose product is 156. 12 and 13 or 13 and 12

**Practice and Apply**

**Factor each trinomial. 17–34. See margin.**
17. \( a^2 + 8a + 15 \)
18. \( x^2 + 12x + 27 \)
19. \( c^2 + 12c + 35 \)
20. \( y^2 + 13y + 30 \)
21. \( m^2 + 22m + 21 \)
22. \( d^2 - 7d - 10 \)
23. \( p^2 - 17p + 72 \)
24. \( g^2 - 19g + 60 \)
25. \( x^2 - 6x - 7 \)
26. \( b^2 + b - 20 \)
27. \( h^2 + 3h - 40 \)
28. \( n^2 + 3n - 54 \)
29. \( y^2 - y - 42 \)
30. \( s^2 - 18s - 40 \)
31. \( -72 + 6w + w^2 \)
32. \(-30 + 13x + x^2 \)
33. \( a^2 + 5ab + 4b^2 \)
34. \( x^2 - 13xy + 36y^2 \)

**GEOMETRY** Find an expression for the perimeter of a rectangle with the given area.
35. \( \text{area} = x^2 + 24x - 81 \)
36. \( \text{area} = x^2 + 13x - 90 \)

**Solve each equation. Check your solutions.**
37. \( x^2 + 16x + 28 = 0 \)
38. \( b^2 + 20b + 36 = 0 \)
39. \( y^2 + 4y - 12 = 0 \)
40. \( a^2 + 2d - 8 = 0 \)
41. \( a^2 - 3a - 28 = 0 \)
42. \( g^2 - 4g - 45 = 0 \)
43. \( m^2 - 19m + 48 = 0 \)
44. \( n^2 - 22n + 72 = 0 \)
45. \( z^2 = 18 - 7z \)
46. \( h^2 + 15 = -16h \)
47. \( 24 + k^2 = 10k \)
48. \( x^2 - 20 = x \)
49. \( c^2 = -50 = -23c - 2\) \( (2150) \)
50. \( y^2 - 29y = -54 \)
51. \( 14p + p^2 = 51 \)
52. \( x^2 - 2x - 6 = 74 \)

**54. SUPREME COURT** When the Justices of the Supreme Court assemble to go on the Bench each day, each Justice shakes hands with each of the other Justices for a total of 36 handshakes. The total number of handshakes \( H \) possible for \( n \) people is given by \( H = \frac{n^2 - n}{2} \). Write and solve an equation to determine the number of Justices on the Supreme Court. \( 36 = \frac{n^2 - n}{2} \)

**55. NUMBER THEORY** Find two consecutive even integers whose product is 168. -14 and -12 or 12 and 14

**56. GEOMETRY** The area of an triangle has 40 square centimeters. Find the height \( h \) of the triangle. \( 5 \text{ cm} \)

**Critical Thinking** Find all values of \( k \) so that each trinomial can be factored using integers.
57. \( x^2 + kx - 19 \)
58. \( x^2 + kx + 14 \)
59. \( x^2 - 8x + k \)
60. \( x^2 - 5x + k \)

**RUGBY** For Exercises 61 and 62, use the following information.
The length of a Rugby League field is 52 meters longer than its width \( w \).
61. Write an expression for the area of the rectangular field. \( [w(w+52)] \text{ m}^2 \)
62. The area of a Rugby League field is 8160 square meters. Find the dimensions of the field. \( 120 \text{ m by 68 m} \)

**Lesson 9-3 Factoring Trinomials: \( x^2 + bx + c \) 493**

**Enrichment, p. 540**

**Puzzling Primes**
A prime number has only two factors, itself and 1. The number 1 is not a prime because it has only one factor, 1, and 2 are not prime. The number 2 is not a prime because it is not divisible by any number other than 1 and 2. The factors of the number 2 are 1 and 2. The factors of the number 2 are 1 and 2.

**Prize numbers have interesting mathematical properties. For example, there are 10,000 prize numbers between 1 and 100,000. Each number is the sum of its digits. The number 1 is the sum of its digits.**

For example, the sum of its digits is 1. Each number is the sum of its digits. The number 2 is the sum of its digits. The number 2 is the sum of its digits.

**Find the prime numbers generated by Euler's formula \( x = 4n + 1 \) through 10.**
41, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

**Helping You Remember**
If you are using the pattern \( x = 4n + 1 \) to factor a trinomial of the form \( x^2 + bx + c \), you need to have a good understanding of multiples and factors to help you remember which \( c \) are positive or negative factors. The sum of its digits is 1. Each number is the sum of its digits.

For example, the sum of its digits is 1. Each number is the sum of its digits. The number 2 is the sum of its digits. The number 2 is the sum of its digits.

To have a product of positive 32, the numbers must both be positive or both negative. If both were positive, one sum would be positive instead of negative.

**Rereading Reading Mathematics, p. 539**

**ELL**

**Pre-Activity** How can factoring be used to find the dimensions of a garden? Read the introduction to Lesson 9-3 of the top page 493 in your textbook.

**Why do you need to find the sum of numbers that are positive?**
This provides a way to find the length and width of the garden. You know the area is 54 \( m^2 \). You can use the factored dimensions to find the sum of numbers whose product is 54.

**Why is the sum of these two numbers the product of the garden?**
You need to find the rectangles that are on the double line graph. Since you used the factored dimensions, you need to find the sum of numbers whose product is 54. The sum of numbers whose product is 54 is the length of the garden. The sum of numbers whose product is 54 is the length of the garden.

**Reading the Lesson**
Tell the whole and product you want \( x \) and \( c \) in order to use the pattern \( x^2 + bx + c \) to factor the given trinomial:
1. \( x^2 + 3x + 20 \)
2. \( x^2 - 3x - 27 \)
3. \( x^2 + 2x - 5 \)
4. \( x^2 - 6x + 12 \)

**Helping You Remember**
If you are using the pattern \( x = 4n + 1 \) to factor a trinomial of the form \( x^2 + bx + c \), you need to have a good understanding of multiples and factors to help you remember which \( c \) are positive or negative factors. The sum of its digits is 1. Each number is the sum of its digits.

For example, the sum of its digits is 1. Each number is the sum of its digits. The number 2 is the sum of its digits. The number 2 is the sum of its digits.

To have a product of positive 32, the numbers must both be positive or both negative. If both were positive, one sum would be positive instead of negative.
63. **Writing in Math** Answer the question that was posed at the beginning of the lesson. See margin.

How can factoring be used to find the dimensions of a garden?

Include the following in your answer:

- a description of how you would find the dimensions of the garden, and
- an explanation of how the process you used is related to the process used to factor trinomials of the form $x^2 + bx + c$.

64. Which is the factored form of $x^2 - 17x + 42$?  
   C  \((x - 1)(y - 42)\)  \((x - 2)(y - 21)\)  \((x - 3)(y - 14)\)  \((x - 6)(y - 7)\)

65. **Grid In** What is the positive solution of $p^2 - 13p - 30 = 0$? 15

Use a graphing calculator to determine whether each factorization is correct. Write yes or no. If no, state the correct factorization.

- 66. $x^2 - 14x + 48 = (x + 6)(x + 8)$  
  \text{yes}

- 67. $x^2 - 16x - 105 = (x + 5)(x - 21)$  
  \text{yes}

- 68. $x^2 + 25x + 66 = (x + 33)(x + 2)$  
  \text{no; } (x + 6)(x - 8)

- 69. $x^2 + 11x - 210 = (x + 10)(x - 21)$  
  \text{no; } (x + 22)(x + 3)

- 70. no; \((x - 6)(x - 8)\)

- 71. no; \((x + 22)(x + 3)\)

- 72. no; \((x - 10)(x + 21)\)

Read the lesson on pages 575-576 of the Chapter 9 Resource Masters.

**Assessment Options**

- Quiz (Lesson 9-3) is available on p. 573 of the Chapter 9 Resource Masters.
- Mid-Chapter Test (Lessons 9-1 through 9-3) is available on p. 575 of the Chapter 9 Resource Masters.

**Maintain Your Skills**

**Mixed Review** Solve each equation. Check your solutions. (Lesson 9-2)

- 70. \((x + 3)(2x - 5) = 0\)
- 71. \(b(7b - 4) = 0\)
- 72. \(5y^2 = -9y\) \(\{-9, 0\}\)

Find the GCF of each set of monomials. (Lesson 9-1)

- 73. 24, 36, 72  \text{12}
- 74. \(9p^3q^2, 21p^3q^3\)  \text{3p}^2q^2
- 75. \(30x^4y^2, 20x^2y^7, 75x^3y^4\)  \text{5x}^2y^4

**Internet** For Exercises 76 and 77, use the graph at the right. (Lessons 3-7 and 8-3)

- 76. Find the percent increase in the number of domain registrations from 1997 to 2000. 1731%  
- 77. Use your answer from Exercise 76 to verify the claim that registrations grew more than 18-fold from 1997 to 2000.

\[
1.54 \times 1.54 = 1731 \\text{or 18.31 (1.54)}
\]

**Getting Ready for the Next Lesson**

**Prerequisite Skill** Factor each polynomial. (To review factoring by grouping, see Lesson 9-2)

- 78. \(3y^2 + 2y + 9y + 6\)
- 79. \(3x^2 + 2a + 12a + 8\)
- 80. \(4x^2 + 3x + 8x + 6\)

- 81. \(2p^2 - 6p + 7p - 21\)
- 82. \(3b^2 + 7b - 12b - 28\)
- 83. \(8y^2 - 2y - 6x + 3\)

\[
(2p + 7)(p - 3) \\
(b - 4)(3b + 7) \\
(2g - 3)(2g - 1)
\]

**Usa Today Snapshots®**

USA Today Snapshots® are photographs of USA Today graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.
For trinomials of the form \(x^3\), the correct factors are 2 and 15.

Write the pattern.

\[ (2x + 5)(3x + 1) = 6x^2 + 2x + 15x + 5 \]

Use the FOIL method.

\[ 2 \cdot 15 = 30 \]

\[ 6 \cdot 5 = 30 \]

The factors of \(2x^2 + 7x + 6\) are the dimensions of the rectangle formed by the algebra tiles shown below.

The process you use to form the rectangle is the same mental process you can use to factor this trinomial algebraically.

\[ \text{FACTOR } ax^2 + bx + c \]

For trinomials of the form \(x^2 + bx + c\), the coefficient of \(x^2\) is 1. To factor trinomials of this form, you find the factors of \(c\) whose sum is \(b\). We can modify this approach to factor trinomials whose leading coefficient is not 1.

Observe the following pattern in this product.

\[
\begin{align*}
6x^2 + 2x + 15x + 5 & \quad ax^2 + mx + nx + c \\
6x^2 + 17x + 5 & \quad ax^2 + bx + c \\
2 + 15 = 17 \text{ and } 2 \cdot 15 = 6 \cdot 5 & \quad m + n = b \text{ and } mn = ac
\end{align*}
\]

You can use this pattern and the method of factoring by grouping to factor \(6x^2 + 17x + 5\). Find two numbers, \(m\) and \(n\), whose product is 6 \cdot 5 or 30 and whose sum is 17.

<table>
<thead>
<tr>
<th>Factors of 30</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 30</td>
<td>31</td>
</tr>
<tr>
<td>2, 15</td>
<td>17</td>
</tr>
</tbody>
</table>

The correct factors are 2 and 15.

\[
\begin{align*}
6x^2 + 17x + 5 & = 6x^2 + mx + nx + 5 \\
& = 6x^2 + 2x + 15x + 5 \\
& = (6x^2 + 2x) + (15x + 5) \\
& = 2x(3x + 1) + 5(3x + 1) \\
& = (3x + 1)(2x + 5)
\end{align*}
\]

Write the pattern.

\[
\begin{align*}
m & = 2 \text{ and } n = 15 \\
\text{Group terms with common factors.} \\
3x + 1 & \text{ is the common factor.}
\end{align*}
\]

Therefore, \(6x^2 + 17x + 5 = (3x + 1)(2x + 5)\).
**Example 1**  Factor \( ax^2 + bx + c \)

a. Factor \( 7x^2 + 22x + 3 \).

In this trinomial, \( a = 7, b = 22 \) and \( c = 3 \). You need to find two numbers whose sum is 22 and whose product is \( 7 \cdot 3 = 21 \). Make an organized list of the factors of 21 and look for the pair of factors whose product is 22.

<table>
<thead>
<tr>
<th>Factors of 21</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 21</td>
<td>22</td>
</tr>
</tbody>
</table>

\[
7x^2 + 22x + 3 = 7x^2 + mx + nx + 3
\]

Write the pattern.

\[
= 7x^2 + 1x + 21x + 3
\]

\[
= (7x^2 + 1x) + (21x + 3)
\]

Group terms with common factors.

\[
= x(7x + 1) + 3(7x + 1)
\]

Factor the GCF from each grouping.

\[
= (7x + 1)(x + 3)
\]

Distributive Property

CHECK You can check this result by multiplying the two factors.

\[
(7x + 1)(x + 3) = 7x^2 + 22x + 3
\]

Simplify.

b. Factor \( 10x^2 - 43x + 28 \).

In this trinomial, \( a = 10, b = -43 \) and \( c = 28 \). Since \( b \) is negative, \( m + n \) is negative. Since \( c \) is positive, \( mn \) is positive. Therefore, make a list of the negative factors of 10 \( \cdot \) 28, and look for the pair of factors whose sum is \(-43\).

<table>
<thead>
<tr>
<th>Factors of 280</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1, -280</td>
<td>-281</td>
</tr>
<tr>
<td>-2, -140</td>
<td>-142</td>
</tr>
<tr>
<td>-4, -70</td>
<td>-74</td>
</tr>
<tr>
<td>-5, -56</td>
<td>-61</td>
</tr>
<tr>
<td>-7, -40</td>
<td>-47</td>
</tr>
<tr>
<td>-8, -35</td>
<td>-43</td>
</tr>
</tbody>
</table>

The correct factors are \(-8\) and \(-35\).

\[
10x^2 - 43x + 28 = 10x^2 + mx + nx + 28
\]

Write the pattern.

\[
= 10x^2 + (-8)x + (-35)x + 28
\]

\[
= (10x^2 - 8x) + (-35x + 28)
\]

Group terms with common factors.

\[
= 2x(5x - 4) + 7(-5x + 4)
\]

Factor the GCF from each grouping.

\[
= 2x(5x - 4) + 7(-1)(5x - 4)
\]

\[
= 2x(5x - 4) - (-7)(5x - 4)
\]

\[
= 2x(5x - 4) - 7(5x - 4)
\]

\[
= (5x - 4)(2x - 7)
\]

Distributive Property

Sometimes the terms of a trinomial will contain a common factor. In these cases, first use the Distributive Property to factor out the common factor. Then factor the trinomial.

**Example 2**  Factor When \( a, b, \) and \( c \) Have a Common Factor

Factor \( 3x^2 + 24x + 45 \).

Notice that the GCF of the terms \( 3x^2, 24x, \) and \( 45 \) is 3. When the GCF of the terms of a trinomial is an integer other than 1, you should first factor out this GCF.

\[
3x^2 + 24x + 45 = 3(x^2 + 8x + 15)
\]

Distributive Property

---

**Differentiated Instruction**

**Interpersonal** Place students in groups to factor polynomials such as those in Examples 1 and 2. Have each group member find one or two factors for \( mn \), depending on the number of factors and number of students in the group. By dividing the labor, students should be able to quickly find the factors for \( mn \) that sum to \( m + n \). Once they find the factors, have students complete the factoring as a group.
Now factor $x^2 + 8x + 15$. Since the lead coefficient is 1, find two factors of 15 whose sum is 8.

<table>
<thead>
<tr>
<th>Factors of 15</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 15</td>
<td>16</td>
</tr>
<tr>
<td>3, 5</td>
<td>8</td>
</tr>
</tbody>
</table>

The correct factors are 3 and 5.

So, $x^2 + 8x + 15 = (x + 3)(x + 5)$. Thus, the complete factorization of $3x^2 + 24x + 45$ is $3(x + 3)(x + 5)$.

A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a prime polynomial.

Example 3 Determine Whether a Polynomial Is Prime

Factor $2x^2 + 5x - 2$.

In this trinomial, $a = 2$, $b = 5$ and $c = -2$. Since $b$ is positive, $m + n$ is positive. Since $c$ is negative, $mn$ is negative. So either $m$ or $n$ is negative, but not both. Therefore, make a list of the factors of $2 \cdot -2$ or $-4$, where one factor in each pair is negative. Look for a pair of factors whose sum is 5.

<table>
<thead>
<tr>
<th>Factors of $-4$</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, $-4$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$-1$, 4</td>
<td>3</td>
</tr>
<tr>
<td>$-2$, 2</td>
<td>0</td>
</tr>
</tbody>
</table>

There are no factors whose sum is 5. Therefore, $2x^2 + 5x - 2$ cannot be factored using integers. Thus, $2x^2 + 5x - 2$ is a prime polynomial.

Solve Equations by Factoring

Some equations of the form $ax^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

Example 4 Solve Equations by Factoring

Solve $8a^2 - 9a - 5 = 4 - 3a$. Check your solutions.

$8a^2 - 9a - 5 = 4 - 3a$ Original equation

$8a^2 - 6a - 9 = 0$ Rewrite so that one side equals 0.

$(4a + 3)(2a - 3) = 0$ Factor the left side.

$4a + 3 = 0$ or $2a - 3 = 0$ Zero Product Property

$4a = -3$ or $2a = 3$ Solve each equation.

$a = -\frac{3}{4}$ or $a = \frac{3}{2}$

The solution set is $\left\{-\frac{3}{4}, \frac{3}{2}\right\}$.

CHECK Check each solution in the original equation.

$8\left(-\frac{3}{4}\right)^2 - 9\left(-\frac{3}{4}\right) - 5 = 4 - 3\left(-\frac{3}{4}\right)$

$8\left(\frac{9}{16}\right) - 9\left(-\frac{3}{4}\right) - 5 = 4 - 3\left(-\frac{3}{4}\right)$

$\frac{9}{2} + \frac{27}{4} - 5 = 4 + \frac{9}{4}$

$\frac{25}{4} = \frac{25}{4}$

$\frac{1}{2} = \frac{1}{2}$

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Unlocking Misconceptions

Dividing Students may assume that because they cannot divide each side of $x(x + 2) = 0$ by $x$, they may never divide both sides of an equation set equal to zero by anything. Make sure they realize that they can divide each side by a known number, as in Example 5, but they cannot divide each side by a variable that may equal zero.
Students should notice that he challenge students to find which student is correct. Then, exercise for which Dasan and Craig are factoring 2 \( x^2 \) out of 2.

### Concept Check

1. **m** and **n** are the factors of **ac** that add to **b**.
2. **OPEN ENDED** Write a trinomial that can be factored using a pair of numbers whose sum is 9 and whose product is 14. Sample answer: \( 2x^2 + 9x + 7 \)
3. **FIND THE ERROR** Dasan and Craig are factoring \( 2x^2 + 11x + 18 \).

### Example 5 Solve Real-World Problems by Factoring

**PEP RALLY** At a pep rally, small foam footballs are launched by cheerleaders using a sling-shot. How long is a football in the air if a student in the stands catches it on its way down 26 feet above the gym floor?

Use the model for vertical motion.

\[
\begin{align*}
    h &= -16t^2 + vt + s \\
    26 &= -16t^2 + 42t + 6 \\
    0 &= -16t^2 + 42t - 20 \\
    0 &= -2(8t^2 - 21t + 10) \\
    0 &= 8t^2 - 21t + 10 \\
    8t - 5 &= 0 \\
    t &= \frac{5}{8}
\end{align*}
\]

The solutions are \( \frac{5}{8} \) second and 2 seconds. The first time represents how long it takes the football to reach a height of 26 feet on its way up. The later time represents how long it takes the ball to reach a height of 26 feet again on its way down. Thus, the football will be in the air for 2 seconds before the student catches it.

### Check for Understanding

1. Explain how to determine which values should be chosen for **m** and **n** when factoring a polynomial of the form \( ax^2 + bx + c \).
2. **OPEN ENDED** Write a trinomial that can be factored using a pair of numbers whose sum is 9 and whose product is 14. Sample answer: \( 2x^2 + 9x + 7 \)
3. **FIND THE ERROR** Dasan and Craig are factoring \( 2x^2 + 11x + 18 \).

### Guided Practice

Find each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.

4. \( 3x^2 + 8x + 4 \)
5. \( 2x^2 - 11x + 7 \) prime
6. \( 2p^2 + 14p + 24 \)
7. \( 2x^2 + 13x + 20 \)
8. \( 6x^2 + 15x - 9 \)
9. \( 4x^2 - 4x - 35 \)

\( (x + 4)(2x + 5) \)
\( 3(2x - 1)(x + 3) \)
\( (2n + 5)(2n - 7) \)

Who is correct? Explain your reasoning. **Craig;** see margin for explanation.
Solve each equation. Check your solutions.
10. $3x^2 + 11x + 6 = 0$
11. $10p^2 - 19p + 7 = 0$
12. $6n^2 + 7n = 20$

13. **GYMNASTICS** When a gymnast making a vault leaves the horse, her feet are 8 feet above the ground traveling with an initial upward velocity of 8 feet per second. Use the model for vertical motion to find the time $t$ in seconds it takes for the gymnast’s feet to reach the mat. (Hint: Let $h = 0$, the height of the mat.) 1 s

**Application**

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. 14–31. **See margin.**

14. $2x^2 + 7x + 5$
15. $3x^2 + 5x + 2$
16. $6p^2 + 5p - 6$

17. $5x^2 + 6x - 30$
18. $5x^2 - 19x + 9$
19. $9x^2 - 12x + 4$

20. $2x^2 - 9a - 18$
21. $2x^2 - 3x - 20$
22. $5c^2 - 17c + 14$

23. $3y^2 - 25y + 16$
24. $8y^2 - 6y - 9$
25. $10y^2 - 11y - 6$

26. $15x^2 + 17x - 12$
27. $14x^2 + 13x - 12$
28. $6x^2 - 14x - 12$

29. $30x^2 - 25x - 30$
30. $9x^2 + 30xy + 25y^2$
31. $36a^2 + 9ab - 10b^2$

**CRITICAL THINKING** Find all values of $k$ so that each trinomial can be factored as two binomials using integers.

32. $2x^2 + kx + 12$
33. $2x^2 + kx + 15$
34. $2x^2 + 12x + k$, $k > 0$

$\pm 25, \pm 14, \pm 11, \pm 10$
$\pm 31, \pm 17, \pm 13, \pm 11$

10, 16, 18

Solve each equation. Check your solutions. 35–48. **See p. 521A.**

35. $5x^2 + 7x + 10 = 0$
36. $3x^2 - 5x - 12 = 0$
37. $24x^2 - 11x - 3 = 3x$

38. $17x^2 - 11x + 2 = 2x$
39. $14x^2 = 25x + 25$
40. $12a^2 - 13a = 35$

41. $6x^2 - 14x = 12$
42. $21x^2 - 16 = 15x$
43. $24x^2 - 30x + 8 = -2x$

44. $24x^2 - 46x = 18$
45. $x^2 + \frac{2x}{3} - 4 = 0$
46. $i^2 - \frac{i}{6} = \frac{35}{6}$

47. $(3y + 2)(y + 3) = y + 14$
48. $(4n - 1)(n - 2) = 7a - 5$

**GEOMETRY** For Exercises 49 and 50, use the following information.
A rectangle with an area of 35 square inches is formed by cutting off strips of equal width from a rectangular piece of paper.

49. Find the width of each strip. 1 in.
50. Find the dimensions of the new rectangle. 5 in. by 7 in.

51. **CLIFF DIVING** Suppose a diver leaps from the edge of a cliff 80 feet above the ocean with an initial upward velocity of 8 feet per second. How long will it take the diver to enter the water below? 2.5 s

**Enrichment, p. 546**

**Lesson 9-4 Factoring Trinomials: $ax^2 + bx + c$**

**Study Guide and Intervention, p. 541 (shown) and p. 542**

**Skills Practice, p. 543 and Practice, p. 544 (shown)**

**Reading to Learn Mathematics, p. 545**

**Pre-Activity** How can algebra tiles be used to factor $x^2 + 3x + 2$?

Read the introduction to Lesson 9-4 at the top of page 499 in your textbook.

When you use the algebra tiles to form a rectangle, what is the first step?

Place the two $x^2$ tiles on the product mat and arrange the six $x$ tiles into a rectangular array.

What is the second step?

Arrange the seven $x$ tiles to complete the rectangle.

**Reading the Lesson**

1. Suppose you want to factor the trinomial $3a^2 + 4a - 4$.
   a. What is the first step?
    - Find integers with a product of 4 and a sum of 14. The integers are 2 and 12.
   b. What is the second step?
    - Rewrite the polynomials by breaking the middle term into two addends that use 2 and 12 as coefficients. You can use $12a^2 + 6a - 4a - 4$.
   c. Provide an explanation for the next two steps.
    - $(12a^2 + 6a) - (4a + 4)$
   d. Group terms with common factors.
    - $6a(2a + 1) - 4(2a + 1)$
   e. Factor the GCF from each grouping.
    - $2(2a + 1)(3a - 2)$
   f. Use the Distributive Property to rewrite the last expression in part e. You get $2(2a + 1)(3a - 2)$.

2. Repeat how you factored the trinomial $3a^2 + 4a - 4$ to form a prime polynomial.
   To factor $2x^2 + 7x + 4$, you would need two negative integers whose product is 4 and whose sum is 7. There are no such negative integers. Therefore, $2x^2 + 7x + 4$ is prime.

**Helping You Remember**

2. What is one step you should use to remember how to factor the trinomial of a quadratic written in the form of $ax^2 + bx + c$?

Sample answer: Look for two integers with a product equal to ac and a sum of b. Replace the middle term with a sum of a terms that have those two integers as coefficients. Then factor by grouping.

6. www.algebra1.com/self_check_quiz/sol
Open-Ended Assessment

Writing  Place students in pairs. Have each student pick a problem that involves solving an equation by factoring and intentionally make a mistake while solving the problem. Then have students exchange problems. Students should write a description of what is incorrect, and how to solve the problem correctly. Have students share their explanations and correct solutions with the class.

Getting Ready for Lesson 9-5

PREREQUISITE SKILL  Students will learn to factor binomials that are differences of squares in Lesson 9-5. Factoring differences of squares requires students to be able to quickly find the principal square roots of perfect squares. Use Exercises 63–70 to determine your students’ familiarity with finding square roots.

Assessment Options

Practice Quiz 2  The quiz provides students with a brief review of the concepts and skills in Lessons 9-3 and 9-4. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Answer

53. You can use algebra tiles to factor $2x^2 + 7x + 6$ by finding the dimensions of the rectangle that is formed by the tiles for $2x^2 + 7x + 6$. Answers should include the following.

- $2x + 3$ by $x + 2$
- With algebra tiles, you can try various ways to make a rectangle with the necessary tiles. Once you make the rectangle, however, the dimensions of the rectangle are the factors of the polynomial. In a way, you have to go through the guess-and-check process whether you are factoring algebraically or geometrically (using algebra tiles.)

Maintain Your Skills

Mixed Review

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. (Lesson 9-3)

56. $a^2 - 4a - 21$  57. $2t^2 + 2t + 2$  58. $a^2 + 15d + 44$

$(a + 3)(a - 7)$  prime  $(d + 4)(d + 11)$

Solve each equation. Check your solutions. (Lesson 9-2)

59. $(y - 4)(5y + 7) = 0$  60. $(2x + 9)(3x + 2) = 0$  61. $12u = u^2$

62. BUSINESS  Jake’s Garage charges $83 for a two-hour repair job and $185 for a five-hour repair job. Write a linear equation that Jake can use to bill customers for repair jobs of any length of time. (Lesson 5-3) $y = 34x + 15$

Getting Ready for the Next Lesson

PREPARATORY SKILL  Find the principal square root of each number. (To review square roots, see Lesson 2-7)

63. 16  4  64. 49  7  65. 36  6  66. 25  5

67. 100  10  68. 121  11  69. 169  13  70. 225  15

Practice Quiz 2  Lessons 9-3 and 9-4

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. (Lessons 9-3 and 9-4)

1. $x^2 - 14x - 72$  2. $8y^2 - 6y - 35$  3. $16a^2 - 24a + 5$

$(x + 4)(x - 18)$  $(2p - 5)(4p + 7)$  $(4a - 1)(4a - 5)$

4. $n^2 - 17n + 52$  5. $24c^2 + 62c + 18$  6. $3y^2 + 33y + 54$

$(n - 13)(n - 4)$  $(4c + 1)(4c + 9)$  $3(y + 2)(y + 9)$

Solve each equation. Check your solutions. (Lessons 9-3 and 9-4)

7. $b^2 + 14b - 32 = 0$  8. $x^2 + 45 = 18x$

$(-16, 2)$  $(3, 15)$

9. $12y^2 - 7y - 12 = 0$  10. $6u^2 - 25u - 14$

$\left\{-\frac{3}{4}, \frac{4}{3}\right\}$  $\left\{\frac{2}{3}, \frac{7}{2}\right\}$

Guess $(2x + 1)(x + 3)$ incorrect because $8 \times 1$ tiles are needed to complete the rectangle.
Factor binomials that are the differences of squares.

Solve equations involving the differences of squares.

A geometric model can be used to factor the difference of squares.

For example, consider the binomial $a^2 - b^2$. This can be factored using the difference of squares pattern.

**Algebra Activity**

**Difference of Squares**

**Step 1** Use a straightedge to draw two squares similar to those shown below. Choose any measures for $a$ and $b$.

```
   a-b   b-a
  +---+---+
  |   |   |
  +---+---+
  | a | b |
  +---+---+
```

Notice that the area of the large square is $a^2$, and the area of the small square is $b^2$.

**Step 2** Cut the small square from the large square.

```
  a-b
  +---+
  |   |
  +---+
  | b |
  +---+
  | a |
  +---+
```

The area of the remaining irregular region is $a^2 - b^2$.

**Step 3** Cut the irregular region into two congruent pieces as shown below.

```
  a-b
  +---+
  |   |
  +---+
  | b |
  +---+
  | a |
  +---+
```

**Step 4** Rearrange the two congruent regions to form a rectangle with length $a + b$ and width $a - b$.

```
  a-b
  +---+
  |   |
  +---+
  | b |
  +---+
  | a |
  +---+
```

**Make a Conjecture**

1. Write an expression representing the area of the rectangle. $(a + b)(a - b)$
2. Explain why $a^2 - b^2 = (a + b)(a - b)$. Since $a^2 - b^2$ and $(a + b)(a - b)$ describe the same area, $a^2 - b^2 = (a + b)(a - b)$.

**Lesson 9-5 Factoring Differences of Squares**

- **Virginia SOL** STANDARD A.12 The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.

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In-Class Examples

Teaching Tip Students may check their factoring by multiplying the factors using the FOIL method. The first-degree term will always drop out when the product is a difference of squares.

1. Factor each binomial.
   a. \( m^2 - 64 \) \((m + 8)(m - 8)\)
   b. \( 16y^2 - 81z^2 \) \((4y + 9z)(4y - 9z)\)

2. Factor \( 3b^3 - 27b \). \(3(b + 3)(b - 3)\)

Teaching Tip Students should notice that when the difference of squares factoring technique has been applied once, one of the factors should be prime.

3. Factor \( 4y^4 - 2500 \). \(4(y^2 + 25)(y + 5)(y - 5)\)

4. Factor \( 6x^3 + 30x^2 - 24x - 120 \). \(6(x + 2)(x - 2)(x + 5)\)

Study Tip

Common Misconception
Remember that the sum of two squares, like \( x^2 + 9 \), is not factorable using the difference of squares pattern. \( x^2 + 9 \) is a prime polynomial.

**Key Concept**

**Difference of Squares**

- **Symbols** \( a^2 - b^2 = (a + b)(a - b) \) or \( (a - b)(a + b) \)
- **Example** \( x^2 - 9 = (x + 3)(x - 3) \) or \( (x - 3)(x + 3) \)

We can use this pattern to factor binomials that can be written in the form \( a^2 - b^2 \).

**Example 1** Factor the Difference of Squares

Factor each binomial.

a. \( n^2 - 25 \)
   \[ n^2 - 25 = n^2 - 5^2 \]
   \[ = (n + 5)(n - 5) \]
   Factor the difference of squares.

b. \( 36x^2 - 49y^2 \)
   \[ 36x^2 - 49y^2 = (6x)^2 - (7y)^2 \]
   \[ = (6x + 7y)(6x - 7y) \]
   Factor the difference of squares.

If the terms of a binomial have a common factor, the GCF should be factored out first before trying to apply any other factoring technique.

**Example 2** Factor Out a Common Factor

Factor \( 48a^3 - 12a \).

\[ 48a^3 - 12a = 12a(4a^2 - 1) \]
\[ = 12a(2a)^2 - 1^2 \]
\[ = 12a(2a + 1)(2a - 1) \]
Factor the difference of squares.

Occasionally, the difference of squares pattern needs to be applied more than once to factor a polynomial completely.

**Example 3** Apply a Factoring Technique More Than Once

Factor \( 2x^4 - 162 \).

\[ 2x^4 - 162 = 2(x^4 - 81) \]
\[ = 2[(x^2)^2 - 9^2] \]
\[ = 2(x^2 + 9)(x^2 - 9) \]
\[ = 2(x^2 + 9)(x^2 - 3^2) \]
\[ = 2(x^2 + 9)(x + 3)(x - 3) \]
Factor the difference of squares.

The GCF of \( 2x^4 \) and \(-162 \) is 2. \( x^2 = x \cdot x \) and 81 = 9 \cdot 9

**Example 4** Apply Several Different Factoring Techniques

Factor \( 5x^3 + 15x^2 - 5x - 15 \).

\[ 5x^3 + 15x^2 - 5x - 15 \]
\[ = 5(x^3 + 3x^2 - x - 3) \]
\[ = 5[(x^3 - x) + (3x^2 - 3)] \]
\[ = 5[x(x^2 - 1) + 3(x^2 - 1)] \]
\[ = 5(x^2 - 1)(x + 3) \]
\[ = 5(x + 1)(x - 1)(x + 3) \]
Factor the difference of squares, \( x^2 - 1 \), into \( (x + 1)(x - 1) \).

Algebra Activity

**Materials:** straightedge, scissors

- Using graph paper, students are more likely to draw straight squares, which will make the final product appear more like a rectangle.
- Make sure students label their figures as shown. Explain that the sides of the original square have length of \( a \), and when the \( b \) square is cut out, the remaining sides have lengths of \( a - b \).
**SOLVE EQUATIONS BY FACTORING**  You can apply the Zero Product Property to an equation that is written as the product of any number of factors set equal to 0.

### Example 5  Solve Equations by Factoring

Solve each equation by factoring. Check your solutions.

**a.** \( p^2 - \frac{9}{16} = 0 \)

\[
\begin{align*}
p^2 - \frac{9}{16} &= 0 \\
p^2 - \left(\frac{3}{4}\right)^2 &= 0 \\
(p + \frac{3}{4})(p - \frac{3}{4}) &= 0
\end{align*}
\]

Original equation

Factor the difference of squares.

Zero Product Property

Solve each equation.

The solution set is \( \left\{-\frac{3}{4}, \frac{3}{4}\right\} \). Check each solution in the original equation.

**b.** \( 18x^3 = 50x \)

\[
\begin{align*}
18x^3 &= 50x \\
18x^3 - 50x &= 0 \\
2x(9x^2 - 25) &= 0 \\
2x(3x + 5)(3x - 5) &= 0
\end{align*}
\]

The GCF of \( 18x^3 \) and \(-50x \) is \( 2x \).

Applying the Zero Product Property, set each factor equal to 0 and solve the resulting three equations.

\[
\begin{align*}
2x &= 0 & \text{ or } & 3x + 5 &= 0 & \text{ or } & 3x - 5 &= 0 \\
x &= 0 & 3x &= -5 & 3x &= 5 \\
& & x &= -\frac{5}{3} & x &= \frac{5}{3}
\end{align*}
\]

The solution set is \( \left\{-\frac{5}{3}, 0, \frac{5}{3}\right\} \). Check each solution in the original equation.

### Example 6  Use Differences of Two Squares

**Extended-Response Test Item**

A corner is cut off a 2-inch by 2-inch square piece of paper. The cut is \( x \) inches from a corner as shown.

**a.** Write an equation in terms of \( x \) that represents the area \( A \) of the figure after the corner is removed. \( A = 64 - x^2 \)

**b.** What value of \( x \) will result in an area that is \( \frac{7}{4} \) the area of the original square piece of paper? Show how you arrived at your answer.

---

**Study Tip**

Alternative Method

The fraction could also be cleared from the equation in Example 5a by multiplying each side of the equation by 16.

\[
\begin{align*}
16p^2 - 9 &= 0 \\
(4p + 3)(4p - 3) &= 0
\end{align*}
\]

4p + 3 = 0 or 4p - 3 = 0

\[
\begin{align*}
p &= -\frac{3}{4} & p &= \frac{3}{4}
\end{align*}
\]

Read the Test Item

\( A \) is the area of the square minus the area of the triangular corner to be removed.

(continued on the next page)
Solve the Test Item

a. The area of the square is $2 \cdot 2$ or $4$ square inches, and the area of the triangle is $\frac{1}{2} \cdot 9 \cdot x$ or $\frac{9}{2}x^2$ square inches. Thus, $A = 4 - \frac{9}{2}x^2$.

b. Find $x$ so that $A = \frac{7}{9}$. The area of the original square piece of paper, $A_o$.

$$A = \frac{7}{9}A_o$$

Translate the verbal statement.

$$4 - \frac{1}{2}x^2 = \frac{7}{9}(4)$$

Simplify.

$$4 - \frac{1}{2}x^2 = \frac{28}{9}$$

Subtract $\frac{28}{9}$ from each side.

$$8 - \frac{1}{2}x^2 = 0$$

Simplify.

$$16 - 9x^2 = 0$$

Factor each side by 18 to remove fractions.

$$(4 + 3x)(4 - 3x) = 0$$

Zero Product Property

$$x = -\frac{4}{3}$$

Solve each equation.

Since length cannot be negative, the only reasonable solution is $\frac{4}{3}$. 

---

Check for Understanding

**Concept Check**

1. **Describe** a binomial that is the difference of two squares.

2. **OPEN ENDED** Write a binomial that is the difference of two squares. Then factor your binomial. **Sample answer:** $x^2 - 25 = (x + 5)(x - 5)$

3. **Determine** whether the difference of squares pattern can be used to factor $3n^2 - 48$. Explain your reasoning.

4. **FIND THE ERROR** Manuel and Jessica are factoring $64x^2 + 16y^2$.

Who is correct? Explain your reasoning. **Manuel:** $4x^2 + 4y^2$ is not the difference of squares.

---

**Guided Practice**

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

5. $n^2 - 81$ prime

6. $4 - 9x^2$ prime

7. $2x^2 - 98x^3$ $2x(x + 7)(x - 7)$

8. $32x^4 - 2y^4$ $16(2x^2 + y^2)(2x - y)$

9. $4x^2 - 27$ prime

10. $x^2 - 3x^2 - 9x + 27$ $(x + 3)(x - 3)(x - 3)$

Solve each equation by factoring. Check your solutions.

11. $4y^2 = 25$ $y = \pm\frac{5}{2}$

12. $17 - 68k^3 = 0$ $k = \frac{1}{2}$ or $k = \frac{1}{2}$

13. $x^2 - \frac{1}{3} = 0$ $x = \pm\frac{1}{\sqrt{3}}$

14. $121a = 49a^3$ $a = \frac{1}{7}$ or $a = \frac{1}{7}$

---

**About the Exercises...**

**Organization by Objective**

- Factor $a^2 - b^2$: 16–33
- Solve Equations by Factoring: 34–45

**Odd/Even Assignments**

Exercises 16–45 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**

**Basic:** 17–31 odd, 35–43 odd, 46, 47, 49, 51–70

**Average:** 17–43 odd, 46, 47, 49, 51–70

**Advanced:** 16–50 even, 51–64 (optional: 65–70)

---

**D A I L Y INTERVENTION**

**FIND THE ERROR**

Make sure students can explain what Jessica did wrong. Stress that Jessica’s error is a common one. Ask students what they can do to avoid making the same mistake themselves.

---

**D A I L Y INTERVENTION**

**Differentiated Instruction**

**Intrapersonal** Consider having students complete the Check for Understanding problems on one day, and then check their own work on the next day, examining their work and answers. Letting their own work sit for a day often allows students to see mistakes or problems in their work that they otherwise might not have noticed. It may also give students more of a chance to ask for help on difficult problems.
Factor each polynomial, if possible. If the polynomial cannot be factored, write prime. 19. \((5 + 2p)(5 - 2p)\) 20. \((7h + 4)/(7h - 4)\)

16. \(x^2 - 49 (x + 7)(x - 7)\) 17. \(n^2 - 36 (n + 6)(n - 6)\) 18. \(81 + 16k^2\) prime

19. \(25 - 4p^2\) 20. \(-16 + 49h^2\) 21. \(-9r^2 + 121\) prime

22. \(100c^2 - d^2\) 23. \(92 - 10x^2\) prime 24. \(144r^2 - 49b^2\) prime

25. \(169y^2 - 36z^2\) 26. \(8s^2 - 18\) 27. \(3x^2 - 75\)

28. \(8z^2 - 2\) 29. \(4y^2 - 50\) 30. \(18s^2 - 72c^2\)

31. \(20x^2 - 45y^2\) \(32. n^2 + 5n^2 - 4n - 20\) \(33. (a + b)^2 - c^2\)

5x(2x - 3y)(2x + 3y) \((n + 2)(n - 2)(n + 5)\) \((a + b)(c + a - b - c)\)

Solve each equation by factoring. Check your solutions. 34. \(25x^2 = 36\) \(35. 9y^2 = 64\) \(36. 12 - 27n^2 = 0\)

37. \(50 - 8a^2 = 0\) \(38. w^2 - \frac{4}{9} = 49\) \(39. 81 - 100 = p^2 = 0\)

40. \(36 - \frac{1}{9} = 2\) \(41. \frac{1}{4}x^2 - 25 = 0\)

42. \(12r^2 - 147d = 0\) \(43. 18m^3 - 50n = 0\) \(44. x^3 - 4x = 12 - 3x^2\)

45. \(36x - 16x^3 = 9x^2 - 4x^4\) \((-3, -2, 2)\)

46. CRITICAL THINKING Show that \(a^2 - b^2 = (a + b)(a - b)\) algebraically. (Hint: Rewrite \(a^2 - b^2\) as \(a^2 - ab + ab - b^2\).) See margin.

47. BOATING The United States Coast Guard’s License Exam includes questions dealing with the breaking strength of a line. The basic breaking strength \(b\) in pounds for a natural fiber line is determined by the formula \(900 + \text{length} = b\), where \(c\) is the circumference of the line in inches. What circumference of natural line would have 3600 pounds of breaking strength? 2 in.

48. AERODYNAMICS The formula for the pressure difference \(P\) above and below a wing is described by the formula \(P = \frac{1}{2} \rho v^2 \frac{d}{\rho} \frac{1}{2}\), where \(d\) is the density of the air, \(v_1\) is the velocity of the air passing above, and \(v_2\) is the velocity of the air passing below. Write this formula in factored form.

49. LAW ENFORCEMENT If a car skids on dry concrete, police can use the formula \(2d = \frac{1}{2} v^2\) to approximate the speed of a vehicle in miles per hour given the length \(d\) of the skid marks in feet. If the length of skid marks on dry concrete are 54 feet long, how fast was the car traveling when the brakes were applied?

36 mph

50. PACKAGING The width of a box is 9 inches more than its length. The height of the box is 1 inch less than its length. If the box has a volume of 72 cubic inches, what are the dimensions of the box?

3 in. by 12 in. by 2 in.
51. CRITICAL THINKING  The following statements appear to prove that 2 is equal to 1. Find the flaw in this “proof.”

Suppose $a$ and $b$ are real numbers such that $a = b, a \neq 0, b \neq 0$.

(1) $a = b$  Given.

(2) $a^2 = ab$  Multiply each side by $a$.

(3) $a^2 - b^2 = ab - b^2$  Subtract $b^2$ from each side.

(4) $(a - b)(a + b) = b(a - b)$  Factor.

(5) $a + b = b$  Divide each side by $a - b$.

(6) $a + a = a$  Substitution Property; $a - b$

(7) $2a = a$  Combine like terms.

(8) $2 = 1$  Divide each side by $a$.

52. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson. See margin.

How can you determine a basketball player’s hang time?

Include the following in your answer:

• a maximum height that is a perfect square and that would be considered a reasonable distance for a student athlete to jump, and

• a description of how to find the hang time for this maximum height.

53. What is the factored form of $25b^2 - 1$?  A

(A) $(b - 1)(5b + 1)$  (B) $(5b + 1)(5b - 1)$

(C) $(5b - 1)(5b - 1)$  (D) $(25b + 1)(b - 1)$

54. GRID IN  In the figure, the area between the two squares is 17 square inches. The sum of the perimeters of the two squares is 68 inches. How many inches long is a side of the larger square? 9 in.

---

51. The flaw is in line 5. Since $a = b$, $a - b = 0$. Therefore dividing by $a - b$ is dividing by zero, which is undefined.

52. Answers should include the following.

- 1 foot

- To find the hang time of a student athlete who attains a maximum height of 1 foot, solve the equation $4t^2 - 1 = 0$. You can factor the left side using the difference of squares pattern since $4t^2$ is the square of 2$t$ and 1 is the square of 1. Thus the equation becomes $(2t + 1)(2t - 1) = 0$. Using the Zero Product Property, each factor can be set equal to zero, resulting in two solutions, $t = -\frac{1}{2}$ and $t = \frac{1}{2}$. Since time cannot be negative, the hang time is $\frac{1}{2}$ second.
The Language of Mathematics

Mathematics is a language all its own. As with any language you learn, you must read slowly and carefully, translating small portions of it at a time. Then you must reread the entire passage to make complete sense of what you read.

In mathematics, concepts are often written in a compact form by using symbols. Break down the symbols and try to translate each piece before putting them back together. Read the following sentence:

\[ a^2 + 2ab + b^2 = (a + b)^2 \]

The trinomial \( a \) squared plus twice the product of \( a \) and \( b \) plus \( b \) squared equals the square of the binomial \( a + b \).

Below is a list of the concepts involved in that single sentence.

- The letters \( a \) and \( b \) are variables and can be replaced by monomials like 2 or 3, or by polynomials like \( x + 3 \).
- The square of the binomial \( a + b \) means \((a + b)(a + b)\). So, \( a^2 + 2ab + b^2 \) can be written as the product of two identical factors, \( a + b \) and \( a + b \).

Now put these concepts together. The algebraic statement \( a^2 + 2ab + b^2 = (a + b)^2 \) means that any trinomial that can be written in the form \( a^2 + 2ab + b^2 \) can be factored as the square of a binomial using the pattern \( (a + b)^2 \).

When reading a lesson in your book, use these steps.

- Read the “What You’ll Learn” statements to understand what concepts are being presented.
- Skim to get a general idea of the content.
- Take note of any new terms in the lesson by looking for highlighted words.
- Go back and reread in order to understand all of the ideas presented.
- Study all of the examples.
- Pay special attention to the explanations for each step in each example.
- Read any study tips presented in the margins of the lesson.

Reading to Learn

2. GCF, perfect square trinomial; \( x^2 + bx + c, ax^2 + bx + c \)

Turn to page 508 and skim Lesson 9-6.

1. List three main ideas from Lesson 9-6. Use phrases instead of whole sentences. See margin.

2. What factoring techniques should be tried when factoring a trinomial?

3. What should you always check for first when trying to factor any polynomial? a greatest common factor

4. Translate the symbolic representation of the Square Root Property presented on page 511 and explain why it can be applied to problems like \( (a + 4)^2 = 49 \) in Example 4a. See margin.

Answers

1. (1) explains how to factor a perfect square trinomial; (2) summarizes methods used to factor polynomials; (3) explains how to solve equations involving perfect squares using the Square Root Property

4. For any number \( n \), where \( n \) is positive, the square of \( x \) equals \( n \), then \( x \) equals plus or minus the square root of \( n \). This property can be applied to the equation \( (a + 4)^2 = 49 \) since the variable \( x = a + 4 \) and \( n = 49 \) in the equation \( x^2 = n \).
The expression numbers like 144, 16, and 49.

What does the expression 144 represent?

What feature of the pavilion tells you that 144 must be a perfect square? The building is square, so the area must be the square of the side length. Since the area is 144, 8 + 2x = 12, and x = 2.

Study Tip

Look Back
To review the square of a sum or difference, see Lesson 8-8.

FACTOR PERFECT SQUARE TRINOMIALS

The senior class has decided to build an outdoor pavilion. It will have an 8-foot by 8-foot portrayal of the school’s mascot in the center. The class is selling bricks with students’ names on them to finance the project. If they sell enough bricks to cover 80 square feet and want to arrange the bricks around the art, how wide should the border of bricks be? To solve this problem, you would need to solve the equation (8 + 2x)² = 144.

For a trinomial to be factorable as a perfect square, three conditions must be satisfied as illustrated in the example below.

- The first term must be a perfect square.
- The middle term must be twice the product of the square roots of the first and last terms.
- The last term must be a perfect square.

Numbers like 144, 16, and 49 are perfect squares, since each can be expressed as the square of an integer.

144 = 12 · 12 or 12² 16 = 4 · 4 or 4² 49 = 7 · 7 or 7²

Products of the form (a + b)² and (a - b)², such as (8 + 2x)², are also perfect squares. Recall that these are special products that follow specific patterns.

\[(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2\]
\[(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2\]

These patterns can help you factor perfect square trinomials, trinomials that are the square of a binomial.

<table>
<thead>
<tr>
<th>Squaring a Binomial</th>
<th>Factoring a Perfect Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + 7)² = x² + 2(x)(7) + 7²</td>
<td>x² + 14x + 49 = x² + 2(x)(7) + 7²</td>
</tr>
<tr>
<td>(3x - 4)² = (3x)² - 2(3x)(4) + 4²</td>
<td>9x² - 24x + 16 = (3x)² - 2(3x)(4) + 4²</td>
</tr>
</tbody>
</table>

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- AlgePASS: Tutorial Plus, Lesson 27
- Interactive Chalkboard
- Multimedia Applications
### Key Concept

**Factoring Perfect Square Trinomials**

- **Words**: If a trinomial can be written in the form \(a^2 + 2ab + b^2\) or \(a^2 - 2ab + b^2\), then it can be factored as \((a + b)^2\) or \((a - b)^2\), respectively.

- **Symbols**: \(a^2 + 2ab + b^2\) and \(a^2 - 2ab + b^2\)

- **Example**: \(4x^2 - 20x + 25 = (2x)^2 - 2(2x)(5) + (5)^2\) or \((2x - 5)^2\)

### Example 1

**Factor Perfect Square Trinomials**

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

**a. 16x^2 + 32x + 64**

- Is the first term a perfect square? Yes, \(16x^2 = (4x)^2\).
- Is the last term a perfect square? Yes, \(64 = 8^2\).
- Is the middle term equal to \(2(4x)(8)\)? No, \(32x \neq 2(4x)(8)\).

16x^2 + 32x + 64 is not a perfect square trinomial.

**b. 9y^2 - 12y + 4**

- Is the first term a perfect square? Yes, \(9y^2 = (3y)^2\).
- Is the last term a perfect square? Yes, \(4 = 2^2\).
- Is the middle term equal to \(2(3y)(2)\)? Yes, \(12y = 2(3y)(2)\).

9y^2 - 12y + 4 is a perfect square trinomial.

\[9y^2 - 12y + 4 = (3y)^2 - 2(3y)(2) + 2^2 = (3y - 2)^2\]

Write as \(a^2 - 2ab + b^2\). Factor using the pattern.

In this chapter, you have learned to factor different types of polynomials. The Concept Summary lists these methods and can help you decide when to use a specific method.

### Concept Summary

<table>
<thead>
<tr>
<th>Number of Terms</th>
<th>Factoring Technique</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 or more</td>
<td>greatest common factor</td>
<td>3x^3 + 6x^2 - 15x = 3x(x^2 + 2x - 5)</td>
</tr>
<tr>
<td>2</td>
<td>difference of squares</td>
<td>a^2 - b^2 = (a + b)(a - b)</td>
</tr>
<tr>
<td>3</td>
<td>perfect square trinomial</td>
<td>a^2 + 2ab + b^2 = (a + b)^2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a^2 - 2ab + b^2 = (a - b)^2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>x^2 + bx + c = (x + m)(x + n), when m + n = b and mn = c.</td>
</tr>
<tr>
<td>4 or more</td>
<td>factoring by grouping</td>
<td>ax^2 + bx + c = ax^2 + mx + nx + c, when m + n = b and mn = ac.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)</td>
</tr>
</tbody>
</table>

### Building on Prior Knowledge

In Chapter 8, students learned how to square a binomial such as \((a + b)^2\) or \((a - b)^2\). In this lesson, students will learn how to undo this process to factor perfect square trinomials.

### FACTOR PERFECT SQUARE TRINOMIALS

In-Class Example

**Teaching Tip** Remind students to look closely at the operation sign in front of the second term of the trinomial. This sign signifies whether the factors are in the form \((a + b)^2\) or \((a - b)^2\).

1 Determine whether each trinomial is a perfect square trinomial. If so, factor it.

**a. 25x^2 - 30x + 9**  yes; \((5x - 3)^2\)

**b. 49y^2 + 42y + 36**  not a perfect square trinomial

---

**Tips for New Teachers**

**Assessment** During the last lesson of a chapter, it is often good to review some of the major concepts of the chapter to assess whether students have mastered the concepts. The concept summary table on Factoring Polynomials provides a perfect opportunity for review. Briefly review each factoring technique with students. After your review, you might consider giving students a quiz on the different techniques to assess student mastery.
When there is a GCF other than 1, it is usually easier to factor it out first. Then, check the appropriate factoring methods in the order shown in the table. Continue factoring until you have written the polynomial as the product of a monomial and/or prime polynomial factors.

**Example 2** Factor Completely

Factor each polynomial.

a. $4x^2 - 36$

First check for a GCF. Then, since the polynomial has two terms, check for the difference of squares.

$$4x^2 - 36 = 4(x^2 - 9)$$

$4$ is the GCF.

$$= 4(x^2 - 3^2)$$

$x^2 = x \cdot x$ and $9 = 3 \cdot 3$

$$= 4(x + 3)(x - 3)$$

Factor the difference of squares.

b. $25x^2 + 5x - 6$

This polynomial has three terms that have a GCF of 1. While the first term is a perfect square, $25x^2 = (5x)^2$, the last term is not. Therefore, this is not a perfect square trinomial.

This trinomial is of the form $ax^2 + bx + c$. Are there two numbers $m$ and $n$ whose product is $25 \cdot -6 = 150$ and whose sum is 5? Yes, the product of 15 and $-10$ is $150$ and their sum is 5.

$$25x^2 + 5x - 6$$

$$= 25x^2 + mx + nx - 6$$

Write the pattern.

$$= 25x^2 + 15x - 10x - 6$$

$m = 15$ and $n = -10$

Group terms with common factors.

$$= (25x^2 + 15x) + (-10x - 6)$$

Factor out the GCF from each grouping.

$$= 5x(5x + 3) - 2(5x + 3)$$

$5x + 3$ is the common factor.

**Solve Equations with Repeated Factors**

When solving equations involving repeated factors, it is only necessary to set one of the repeated factors equal to zero.

**Example 3** Solve Equations with Repeated Factors

Solve $x^2 - x + \frac{1}{4} = 0$.

$x^2 - x + \frac{1}{4} = 0$ Original equation

$x^2 - 2\left(x\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 0$ Recognize $x^2 - x + \frac{1}{4}$ as a perfect square trinomial.

$\left(x - \frac{1}{2}\right)^2 = 0$ Factor the perfect square trinomial.

$x - \frac{1}{2} = 0$ Set repeated factor equal to zero.

$x = \frac{1}{2}$ Solve for $x$.

Thus, the solution set is $\left\{\frac{1}{2}\right\}$. Check this solution in the original equation.
You have solved equations like \( x^2 - 36 = 0 \) by using factoring. You can also use the definition of square root to solve this equation.

\[
\begin{align*}
  x^2 - 36 &= 0 & \text{Original equation} \\
  x^2 &= 36 & \text{Add 36 to each side.} \\
  x &= \pm \sqrt{36} & \text{Take the square root of each side.}
\end{align*}
\]

Remember that there are two square roots of 36, namely 6 and \(-6\). Therefore, the solution set is \{-6, 6\}. This is sometimes expressed more compactly as \( \{\pm 6\} \). This and other examples suggest the following property.

**Key Concept**

**Square Root Property**

- **Symbols**
  
  For any number \( n > 0 \), if \( x^2 = n \), then \( x = \pm \sqrt{n} \).

- **Example**
  
  \[
  \begin{align*}
  x^2 &= 9 \\
  x &= \pm \sqrt{9} \text{ or } \pm 3
  \end{align*}
  \]

**Example 4** Use the Square Root Property to Solve Equations

Solve each equation. Check your solutions.

a. \((a + 4)^2 = 49\)

\[
\begin{align*}
  (a + 4)^2 &= 49 & \text{Original equation} \\
  a + 4 &= \pm \sqrt{49} & \text{Square Root Property} \\
  a + 4 &= \pm 7 & 49 = 7 \cdot 7 \\
  a &= -4 \pm 7 & \text{Subtract 4 from each side.} \\
  a &= -4 + 7 \text{ or } a &= -4 - 7 & \text{Separate into two equations.} \\
  &= 3 & = -11 & \text{Simplify.}
\end{align*}
\]

The solution set is \{-11, 3\}. Check each solution in the original equation.

b. \(y^2 - 4y + 4 = 25\)

\[
\begin{align*}
  y^2 - 4y + 4 &= 25 & \text{Original equation} \\
  (y - 2)^2 &= 25 & \text{Recognize perfect square trinomial.} \\
  y - 2 &= \pm \sqrt{25} & \text{Square Root Property} \\
  y &= 2 \pm 5 & 25 = 5 \cdot 5 \\
  y &= 2 \pm 5 & \text{Add 2 to each side.} \\
  y &= 2 + 5 \text{ or } y &= 2 - 5 & \text{Separate into two equations.} \\
  &= 7 & = -3 & \text{Simplify.}
\end{align*}
\]

The solution set is \{-3, 7\}. Check each solution in the original equation.

c. \((x - 3)^2 = 5\)

\[
\begin{align*}
  (x - 3)^2 &= 5 & \text{Original equation} \\
  x - 3 &= \pm \sqrt{5} & \text{Square Root Property} \\
  x &= 3 \pm \sqrt{5} & \text{Add 3 to each side.}
\end{align*}
\]

Since 5 is not a perfect square, the solution set is \(3 \pm \sqrt{5}\). Using a calculator, the approximate solutions are \(3 + \sqrt{5}\) or about 5.24 and \(3 - \sqrt{5}\) or about 0.76.
Check for Understanding

1. **Concept Check**
   - Explain how to determine whether a trinomial is a perfect square trinomial.
   - Determine whether the following statement is sometimes, always, or never true. Explain your reasoning. **never**; \( (a - b)^2 = a^2 - 2ab + b^2 \)

2. **Advanced:** Solve Equations With Perfect Square Trinomials: 60–80
   - Determine whether each trinomial is a perfect square trinomial. If so, factor it.
   - Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

3. **Odd/Even Assignments**
   - Odd/Even Assignments: 17–53 are structured to help students practice the same concepts whether they are assigned odd or even problems.

4. **Guided Practice**
   - Determine whether each trinomial is a perfect square trinomial. If so, factor it.
   - Solve each equation. Check your solutions.

5. **Application**
   - **HISTORY** Galileo demonstrated that objects of different weights fall at the same velocity by dropping two objects of different weights from the top of the Leaning Tower of Pisa. A model for the height \( h \) in feet of an object dropped from an initial height \( h_0 \) in feet is \( h = -16t^2 + h_0 \) where \( t \) is the time in seconds after the object is dropped. Use this model to determine approximately how long it took for the objects to hit the ground if Galileo dropped them from a height of 180 feet. **about 3.35 s**

6. **Answers**
   - Determine if the first term is a perfect square. Then determine if the last term is a perfect square. Finally, check to see if the middle term is equal to twice the product of the square roots of the first and last terms.

   - **1.** Yes; \((y + 4)^2\)
   - **2.** Yes; \((y + 3)(y + 6)\)
   - **3.** No
   - **4.** Yes; \((y + 4)^2\)
   - **5.** No
   - **6.** Yes; \((2x + 9)^2\)
   - **7.** **Never**
   - **8.** Yes; \((5x - 2)^2\)
   - **9.** Yes; \((3x + 2)^2\)
   - **10.** Yes; \((x - 2)^2\)
   - **11.** Yes; \((x - 3)^2\)
   - **12.** Yes; \((x - 3)^2\)
   - **13.** Yes; \((x - 3)^2\)
   - **14.** Yes; \((x - 3)^2\)
   - **15.** Yes; \((x - 3)^2\)
   - **16.** Yes; \((x - 3)^2\)
   - **17.** No
   - **18.** Yes; \((a - 12)^2\)
   - **19.** Yes; \((2y - 11)^2\)
   - **20.** No
   - **21.** Yes; \((3n + 7)^2\)
   - **22.** Yes; \((5a - 12b)^2\)

   - **17–22.** See margin.

   - **23.** **GEOMETRY** The area of a circle is \((16x^2 + 80x + 100)\pi\) square inches. What is the diameter of the circle? **8x + 20**

   - **24.** **GEOMETRY** The area of a square is \(81 - 90x + 25x^2\) square meters. If \(x\) is a positive integer, what is the least possible perimeter measure for the square? **16 m**
Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

25. $4k^2 - 100$  
26. $9x^2 - 3x - 20$  
27. $x^2 + 6x - 9$  
28. $50x^2 + 40x + 8$  
29. $9f^3 + 66f^2 - 48f$  
30. $4a^2 - 36ab + 9b^2$  
31. $20m^2 + 34n + 6$  
32. $5y^2 - 90$  
33. $24x^3 - 78x^2 + 45x$  
34. $18y^3 - 48y + 32$  
35. $90x^2 - 27x^2 - 75 - 3(3g - 5)^2$  
36. $45c^3 - 32cd(c + 32d - 4c)$  
37. $(a^2 + 2)(4a + 3b^2)$

**Answer:** prime, $(3x + 4)(3x - 5)$, prime, $(5a + 3b)(7a + 6b - 4b)$, prime, $(4m^2 + 6mn - 16n^2 + 24n^2m^2)(x^2 + 3)(x - 1)$

**41. GEOMETRY** The volume of a rectangular prism is $x^2y - 63x^2 + 73x - 9xy^2$ cubic meters. Find the dimensions of the prism if it can be represented by binomials with integral coefficients. $x - 3y$, $x + 3y$, $xy + 7m$

**42. GEOMETRY** If the area of the square shown below is $16x^2 - 56x + 49$ square inches, what is the area of the rectangle in terms of $x$?

**Solve each equation. Check your solutions.**

43. $3x^2 + 24x + 48 = 0$  
44. $7x^2 = 70x - 175$  
45. $49x^2 + 16 = 56a$  
46. $18y^2 + 24y + 8 = 0$  
47. $y^2 - \frac{2}{3}y + \frac{1}{9} = 0$  
48. $a^2 + \frac{4}{5}a + \frac{2}{5} = 0$  
49. $z^2 + 2z + 1 = 16$  
50. $x^2 + 10x + 25 = 81$  
51. $(y - 8)^2 = 7$  
52. $(w + 3)^2 = 2$  
53. $p^2 + 2p + 1 = 6$  
54. $x^2 - 12x + 36 = 11$  

**FORESTRY** For Exercises 55 and 56, use the following information.

Lumber companies need to be able to estimate the number of board feet that a given log will yield. One of the most commonly used formulas for estimating board feet is $L = \frac{D^2 - 8D + 16}{16}$, where $D$ is the diameter in inches, and $L$ is the length of the log in feet.

55. Write this formula in factored form. $B = \frac{L}{16}(D - 4)^2$

56. For logs that are 16 feet long, what diameter will yield approximately 256 board feet? **20 in.**

**FREE-FALL RIDE** For Exercises 57 and 58, use the following information.

The height $h$ in feet of a car above the exit ramp of an amusement park’s free-fall ride can be modeled by $h = -16t^2 + s$, where $t$ is the time in seconds after the car drops and $s$ is the starting height of the car.

57. How high above the car’s exit ramp should the ride’s designer start the drop in order for riders to experience free fall for at least 3 seconds? **144 ft**

58. Approximately how long will riders be in free fall if their starting height is 160 feet above the exit ramp? **3.16 s**

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**Study Guide and Intervention, p. 555 (shown) and p. 554**

**Reading to Learn Mathematics, p. 557**

**Read the Lesson**

3. Three conditions must be met if a trinomial can be factored in a perfect square trinomial. Complete the following sentences.

- The first term of the trinomial $b^2 - 2ac + c^2$ is (certain or not) a perfect square.
- The last term of the trinomial $b^2 - 2ac + c^2$ is (certain or not) a perfect square.
- The middle term of the trinomial $b^2 - 2ac + c^2$ is (certain or not) a perfect square trinomial.

**Reading the Lesson**

3. Three conditions must be met if a trinomial can be factored in a perfect square trinomial. Complete the following sentences.

- The first term of the trinomial $b^2 - 2ac + c^2$ is (certain or not) a perfect square.
- The last term of the trinomial $b^2 - 2ac + c^2$ is (certain or not) a perfect square.
- The middle term of the trinomial $b^2 - 2ac + c^2$ is (certain or not) a perfect square trinomial.

**Helping You Remember**

3. Sometimes it is necessary to remember a set of instructions if you can store them in a chart or reference frame. Demonstrate the compact method that must be met if a trinomial can be factored into a perfect square trinomial. **Sample Answer:** The first and last terms are perfect squares, and the middle term is twice the product of the square roots of the first and last terms.
59. **HUMAN CANNONBALL** A circus acrobat is shot out of a cannon with an initial upward velocity of 64 feet per second. If the acrobat leaves the cannon 6 feet above the ground, will he reach a height of 70 feet? If so, how long will it take him to reach that height? Use the model for vertical motion.

**Yes:** 2 s

**CRITICAL THINKING** Determine all values of $k$ that make each of the following a perfect square trinomial. 62. 70, $-70$  
60. $x^2 + 16$  
61. $4x^2 + kx + 4$  
62. $25x^2 + kx + 49$

63. $x^2 + 8x + 16$  
64. $x^2 - 18x + k$  
65. $x^2 + 20x + k$  

66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See margin.

How can factoring be used to design a pavilion?

Include the following in your answer:

• an explanation of how the equation $(8 + 2x)^2 = 144$ models the given situation, and

• an explanation of how to solve this equation, listing any properties used, and an interpretation of its solutions.

67. During an experiment, a ball is dropped off a bridge from a height of 205 feet. The formula $205 = 16t^2$ can be used to approximate the amount of time, in seconds, it takes for the ball to reach the surface of the water of the river below the bridge. Find the time it takes the ball to reach the water to the nearest tenth of a second. **C**

(A) 2.3 s  
(B) 3.4 s  
(C) 3.6 s  
(D) 12.8 s

68. If $\sqrt{a^2 - 2ab + b^2} = a - b$, then which of the following statements best describes the relationship between $a$ and $b$? **D**

(A) $a < b$  
(B) $a = b$  
(C) $a > b$  
(D) $a \geq b$

## Maintain Your Skills

### Mixed Review

Solve each equation. Check your solutions. (Lessons 9-4 and 9-5)

69. $s^2 = 25$  
70. $9x^2 - 16 = 0$  
71. $49m^2 = 81$

72. $8k^2 + 22k - 6 = 0$  
73. $12w^2 + 23w = -5$  
74. $6z^2 + 7 = 17z$

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation. (Lesson 5-6)

75. $(1, 4), y = 2x - 1$  
76. $(-4, 7), y = -\frac{1}{2}x + \frac{9}{2}$

77. **NATIONAL LANDMARKS** At the Royal Gorge in Colorado, an inclined railway takes visitors down to the Arkansas River. Suppose the slope is 50% or $\frac{1}{2}$ and the vertical drop is 1015 feet. What is the horizontal change of the railway? (Lesson 5-1)

78. Find the next three terms of each arithmetic sequence. (Lesson 4-7)

(A) 17, 13, 9, …  
(B) $-5, -4.5, -4, -3.5, …$  
(C) 45, 54, 63, 72, …

80. $1, -3, -7$, $-3, -2.5, -2$, $81, 90, 99$

- $(8 + 2x)^2 = 144$  
  **Original equation**  
  \[
  8 + 2x = \pm 12 \quad \text{Square Root Property}
  \]

- $8 + 2x = 12$ or $8 + 2x = -12$  
  **Separate into two equations**

- $2x = 4$  
  $2x = -20$  
  **Solve each equation**

- $x = 2$  
  $x = -10$

Since length cannot be negative, the border should be 2 feet wide.
Vocabulary and Concept Check

State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. The number 27 is an example of a prime number. false, composite
2. \(2x\) is the greatest common factor (GCF) of \(12x^2\) and \(14xy\). true
3. \(66\) is an example of a perfect square. false, sample answer: \(64\)
4. \(61\) is a factor of \(183\). true
5. The prime factorization for \(48\) is \(3 \cdot 4^2\). false, \(2^4 \cdot 3\)
6. \(x^2 - 25\) is an example of a perfect square trinomial. false, difference of squares
7. The number 35 is an example of a composite number. true
8. \(x^2 - 3x - 70\) is an example of a prime polynomial. false, sample answer: \(x^2 + 2x + 2\)
9. Expressions with four or more unlike terms can sometimes be factored by grouping. true
10. \((b - 7)(b + 7)\) is the factorization of a difference of squares. true

Lesson-by-Lesson Review

9-1 Factors and Greatest Common Factors

Concept Summary

• The greatest common factor (GCF) of two or more monomials is the product of their common prime factors.

Example

Find the GCF of \(15x^2y\) and \(45xy^2\).

\[
15x^2y = 3 \cdot 5 \cdot x \cdot x \cdot y \\
45xy^2 = 3 \cdot 3 \cdot 5 \cdot x \cdot y \cdot y
\]

Factor each number.
Circle the common prime factors.
The GCF is \(3 \cdot 5 \cdot y \cdot x = 15xy\).

Exercises

Find the prime factorization of each integer.
See Examples 2 and 3 on page 1475.

\[
11. \ 28 \quad 2^2 \cdot 7 \\
12. \ 33 \quad 3 \cdot 11 \\
13. \ 150 \quad 2 \cdot 3 \cdot 5^2 \\
14. \ 301 \quad 7 \cdot 43 \\
15. \ -83 \quad -1 \cdot 83 \\
16. \ -378 \quad -1 \cdot 2 \cdot 3^3 \cdot 7
\]

Find the GCF of each set of monomials.
See Example 5 on page 476.

\[
17. \ 35, \ 30 \quad 5 \\
18. \ 12, \ 18, \ 40 \quad 2 \\
19. \ 12ab, \ 4a^2b^2 \quad 4ab \\
20. \ 16mrt, \ 30mr^2 \quad 2mr \\
21. \ 20u^2, \ 25uy^5 \quad 5u \\
22. \ 60x^2y^2, \ 35xz^3 \quad 5x
\]
9-2 Factoring Using the Distributive Property

Concept Summary
- Find the greatest common factor and then use the Distributive Property.
- With four or more terms, try factoring by grouping.
- Factoring by Grouping: \(ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)\)
- Factoring can be used to solve some equations.
- Zero Product Property: For any real numbers \(a\) and \(b\), if \(ab = 0\), then either \(a = 0\), \(b = 0\), or both \(a\) and \(b\) equal zero.

Example

Factor \(2x^2 - 3xz - 2xy + 3yz\).

\[
2x^2 - 3xz - 2xy + 3yz = (2x^2 - 3xz) + (-2xy + 3yz) = x(2x - 3z) - y(2x - 3z) = (x - y)(2x - 3z).
\]

Exercises

25. \(2a(13b + 9c + 16a)\) See Examples 1 and 2 on pages 481 and 482.

23. \(13x + 26y\)
24. \(24a^2b^2 - 18ab\)
25. \(26ab + 18ac + 32a^2\)
26. \(a^2 - 4ac + ab - 4bc\)
27. \(4rs + 12ps + 2mr + 6mp\)
28. \(24am - 9an + 40nm - 15bn\)
29. \(2(r + 3p)(2s + m)\)

30. Solve each equation. Check your solutions. See Examples 2 and 5 on pages 482 and 483.
30. \(x(2x - 5) = 0\) \(\{0, \frac{5}{2}\}\)
31. \((3u + 8)(2u - 6) = 0\) \(-\frac{8}{3}, 3\)
32. \(x^2 - 3x - 4 = 0\) \(-1, 4\)

9-3 Factoring Trinomials: \(x^2 + bx + c\)

Concept Summary
- Factoring \(x^2 + bx + c\): Find \(m\) and \(n\) whose sum is \(b\) and whose product is \(c\). Then write \(x^2 + bx + c\) as \((x + m)(x + n)\).

Example

Solve \(a^2 - 3a - 4 = 0\). Then check the solutions.

\[
a^2 - 3a - 4 = 0 \quad \text{Original equation}
\]
\[
(a + 1)(a - 4) = 0 \quad \text{Factor.}
\]
\[
a + 1 = 0 \quad \text{or} \quad a - 4 = 0 \quad \text{Zero Product Property}
\]
\[
a = -1 \quad a = 4 \quad \text{Solve each equation.}
\]

The solution set is \(-1, 4\).

Exercises

32. \((y + 3)(y + 4)\)
33. \((x + 12)(x + 3)\)

32. \(y^2 + 7y + 12\)
33. \(x^2 - 9x - 36\)
34. \(b^2 + 5b - 6\) \((b + 6)(b - 1)\)
35. \(18 - 9r + r^2\)
36. \(a^2 + 6ax - 40x^2\)
37. \(m^2 - 4mn - 32n^2\)
38. \((r - 3)(r - 6)\) \((a + 10x)(a - 4x)\)
39. \((m + 4n)(m - 8n)\)

30. Solve each equation. Check your solutions. See Example 5 on page 491.
30. \(y^2 + 13y + 40 = 0\) \(-5, -8\)
31. \(x^2 - 5x - 66 = 0\) \(-6, 11\)
32. \(m^2 - m - 12 = 0\) \(-3, 4\)
Factoring Trinomials: $ax^2 + bx + c$

Concept Summary
- Factoring $ax^2 + bx + c$: Find $m$ and $n$ whose product is $ac$ and whose sum is $b$. Then, write as $ax^2 + mx + nx + c$ and use factoring by grouping.

Example
Factor $12x^2 + 22x - 14$.

First, factor out the GCF, 2: $12x^2 + 22x - 14 = 2(6x^2 + 11x - 7)$. In the new trinomial, $a = 6$, $b = 11$ and $c = -7$. Since $b$ is positive, $m + n$ is positive. Since $c$ is negative, $mn$ is negative. So either $m$ or $n$ is negative, but not both. Therefore, make a list of the factors of $6(-7)$ or $-42$, where one factor in each pair is negative. Look for a pair of factors whose sum is 11.

<table>
<thead>
<tr>
<th>Factors of $-42$</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1, 42$</td>
<td>41</td>
</tr>
<tr>
<td>$1, -42$</td>
<td>$-41$</td>
</tr>
<tr>
<td>$-2, 21$</td>
<td>19</td>
</tr>
<tr>
<td>$2, -21$</td>
<td>$-19$</td>
</tr>
<tr>
<td>$-3, 14$</td>
<td>11</td>
</tr>
</tbody>
</table>

The correct factors are $-3$ and $14$.

$6x^2 + 11x - 7 = 6x^2 + mx + nx - 7$ Write the pattern.

$= 6x^2 - 3x + 14x - 7$ $m = -3$ and $n = 14$

$= (6x^2 - 3x) + (14x - 7)$ Group terms with common factors.

$= 3x(2x - 1) + 7(2x - 1)$ Factor the GCF from each grouping.

$= (2x - 1)(3x + 7)$ $2x - 1$ is the common factor.

Thus, the complete factorization of $12x^2 + 22x - 14$ is $2(2x - 1)(3x + 7)$.

42. $(2m - 3)(m + 8)$ 43. $(5r + 2)(5r + 2)$

Exercises Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime.

41. $2a^2 - 9a + 3$ prime 42. $2m^2 + 13m - 24$ 43. $25r^2 + 20r + 4$
44. $6z^2 + 7z + 3$ prime 45. $12b^2 + 17b + 6$ 46. $3n^2 - 6n - 15$

$4b + 3$ $(3b + 2)$ $3(n - 5)(n + 3)$

Solve each equation. Check your solutions.

47. $2m^2 - 3m - 20 = 0$ 48. $3a^2 - 13a + 14 = 0$ 49. $40x^2 + 2x = 24$

$\left\{\frac{3}{4}, -\frac{4}{5}\right\}$ $\left\{\frac{2}{3}, \frac{7}{3}\right\}$

Factoring Differences of Squares

Concept Summary
- Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$
- Sometimes it may be necessary to use more than one factoring technique or to apply a factoring technique more than once.

Example
Factor $3x^3 - 75x$.

$3x^3 - 75x = 3x(x^2 - 25)$ The GCF of $3x^3$ and $75x$ is $3x$.

$= 3x(x + 5)(x - 5)$ Factor the difference of squares.
Perfect Squares and Factoring

**Concept Summary**

- If a trinomial can be written in the form \(a^2 + 2ab + b^2\) or \(a^2 - 2ab + b^2\), then it can be factored as \((a + b)^2\) or \((a - b)^2\), respectively.
- For a trinomial to be factorable as a perfect square, the first term must be a perfect square, the middle term must be twice the product of the square roots of the first and last terms, and the last term must be a perfect square.
- **Square Root Property**: For any number \(n \geq 0\), if \(x^2 = n\), then \(x = \pm \sqrt{n}\).

**Examples**

1. Determine whether \(9x^2 + 24xy + 16y^2\) is a perfect square trinomial. If so, factor it.

   - Is the first term a perfect square? Yes, \(9x^2 = (3x)^2\).
   - Is the last term a perfect square? Yes, \(16y^2 = (4y)^2\).
   - Is the middle term equal to \(2(3x)(4y)\)? Yes, \(24xy = 2(3x)(4y)\).

   \[9x^2 + 24xy + 16y^2 = (3x)^2 + 2(3x)(4y) + (4y)^2\]

   **Write as** \(a^2 + 2ab + b^2\).

2. Solve \((x - 4)^2 = 121\).

   \[(x - 4)^2 = 121\]

   Original equation

   \[x - 4 = \pm \sqrt{121}\]

   Square Root Property

   \[x - 4 = 11\]

   \[x - 4 = -11\]

   \[121 = 11 \cdot 11\]

   \[x = 4 \pm 11\]

   Add 4 to each side.

   \[x = 4 + 11\] or \[x = 4 - 11\] Separate into two equations.

   \[x = 15\] or \[x = -7\]

   The solution set is \{-7, 15\}.

**Exercises**

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime. See Example 2 on page 510.

50. \(2y^3 - 288y\)
51. \(9b^2 - 20\) prime
52. \(\frac{1}{4}n^2 - \frac{9}{16}r^2\)

Solve each equation. Check your solutions. See Examples 3 and 4 on pages 510 and 511.

60. \(6b^3 - 24b^2 + 24b = 0\) \((-2, 2)\)
61. \(49m^2 - 126m + 81 = 0\) \(\left\{\frac{9}{7}\right\}\)
62. \((c - 9)^2 = 144\) \((-3, 21)\)
63. \(144b^2 = 36\) \(\left\{\pm \frac{1}{2}\right\}\)
### Vocabulary and Concepts

1. Give an example of a prime number and explain why it is prime. **Sample answer:** 7; Its only factors are 1 and itself.
2. Write a polynomial that is the difference of two squares. Then factor your polynomial. **Sample answer:** \( n^2 - 100; (n + 10)(n - 10) \)
3. Describe the first step in factoring any polynomial. **Check for a GCF other than 1 and factor it out.**

### Skills and Applications

**Find the prime factorization of each integer.**

4. \( 63 \)  \( 3^2 \cdot 7 \)
5. \( 81 \)  \( 3^4 \)
6. \( -210 \)  \( -1 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \)

**Find the GCF of the given monomials.**

7. \( 48, 64 \)  \( 16 \)
8. \( 28, 75 \)  \( 1, \) relatively prime
9. \( 18a^2b^2, 28a^2b^2 \)  \( 2a^2b^2 \)

**Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write prime.**

10. \( 25y^2 - 49w^2 \)
11. \( t^2 - 16t + 64 \)  \( (t - 8)^2 \)
12. \( x^2 + 14x + 24 \)  \( (x + 12)(x + 2) \)
13. \( 28m^2 + 18m \)  \( 2m(14m + 9) \)
14. \( a^2 - 11ab + 18b^2 \)
15. \( 12x^2 + 23x - 24 \)  \( (3x + 8)(4x - 3) \)
16. \( 2b^2 - 3b - 18 \)  prime
17. \( 6x^3 + 15x^2 - 9x \)
18. \( 64p^2 - 63p + 16 \)  prime
19. \( 2d^2 + d - 1 \)  \( (2d - 1)(d + 1) \)
20. \( 36a^2b^3 - 45ab^4 \)  \( 9ab^2(4a - 5b) \)
21. \( 36m^2 + 60mn + 25n^2 \)  \( (6m + 5n)^2 \)
22. \( a^2 - 4 \)  \( 22 - 27. \) See margin.
23. \( 4n^2 - 20m + 3n^3 - 15p \)
24. \( 15a^2b + 5a^2 - 10a \)
25. \( 6y^2 - 5y - 6 \)
26. \( 4x^2 - 100 \)
27. \( x^3 - 4x^2 - 9x + 36 \)

**Write an expression in factored form for the area of each shaded region.**

28. 8(x + y + 6)
29. 4r^2(4 - \pi)

**Solve each equation. Check your solutions.**

30. \( (4x - 3)(3x + 2) = 0 \)  \( \{ \frac{3}{4}, \frac{2}{3} \} \)
31. \( 18x^2 + 72x = 0 \)  \( \{0, -4\} \)
32. \( 4x^2 = 36 \)  \( \{-3, 3\} \)
33. \( t^2 + 25 = 10t \)  \( \{5\} \)
34. \( a^2 - 9a - 52 = 0 \)  \( \{-4, 13\} \)
35. \( x^3 - 5x^2 - 66x = 0 \)  \( \{-6, 0, 11\} \)
36. \( 2x^2 = 9x + 5 \)  \( \{-2, 5\} \)
37. \( 3b^2 + 6 = 11b \)  \( \{\frac{2}{3}, 3\} \)

38. **GEOMETRY** A rectangle is 4 inches wide by 7 inches long. When the length and width are increased by the same amount, the area is increased by 26 square inches. What are the dimensions of the new rectangle? 6 in. by 9 in.

39. **CONSTRUCTION** A rectangular lawn is 24 feet wide by 32 feet long. A sidewalk will be built along the inside edges of all four sides. The remaining lawn will have an area of 425 square feet. How wide will the walk be? 3.5 ft

40. **STANDARDIZED TEST PRACTICE** The area of the shaded part of the square shown at the right is 98 square meters. Find the dimensions of the square. 14 m by 14 m

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**Portfolio Suggestion**

**Introduction** Have you ever noticed that when you are learning the concepts in a chapter, such as how to factor polynomials in this chapter, that there is often more than one way to solve a problem?

**Ask Students** Pick a trinomial that can be factored by more than one method that you learned in this chapter, and explain how to factor it using these methods. Make sure you include a worked-out example with your descriptions in your portfolio.
SOL/EOC Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which equation best describes the function graphed below? (Lesson 5-3) A
   - A: $y = \frac{3}{5}x - 3$
   - B: $y = \frac{5}{3}x - 3$
   - C: $y = \frac{5}{3}x + 3$
   - D: $y = \frac{3}{5}x + 3$

2. The school band sold tickets to their spring concert every day at lunch for one week. Before they sold any tickets, they had $80 in their account. At the end of each day, they recorded the total number of tickets sold and the total amount of money in the band's account.
   - Day | Total Number of Tickets Sold $t$ | Total Amount in Account $a$
   - Monday | 12 | $176$
   - Tuesday | 18 | $224$
   - Wednesday | 24 | $272$
   - Thursday | 30 | $320$
   - Friday | 36 | $368$

Which equation describes the relationship between the total number of tickets sold $t$ and the amount of money in the band's account $a$? (Lesson 5-4) D
   - A: $a = \frac{1}{8}t + 80$
   - B: $a = \frac{1}{8}t - 80$
   - C: $a = 6t + 8$
   - D: $a = 8t + 8$

3. Which inequality represents the shaded portion of the graph? (Lesson 6-6) A
   - A: $y \leq \frac{1}{3}x - 1$
   - B: $y \geq \frac{1}{3}x - 1$
   - C: $y \leq 3x + 1$
   - D: $y \geq 3x - 1$

4. Today, the refreshment stand at the high school football game sold twice as many bags of popcorn as were sold last Friday. The total sold both days was 258 bags. Which system of equations will determine the number of bags sold today $n$ and the number of bags sold last Friday $f$? (Lesson 7-2) D
   - A: $n = f - 258$
   - B: $n = f - 258$
   - C: $n + f = 258$
   - D: $n + f = 258$

5. Express $5.387 \times 10^{-3}$ in standard notation. (Lesson 8-3) B
   - A: $0.0005387$
   - B: $0.005387$
   - C: $53.87$
   - D: $5387$

6. The quotient $\frac{16x^8}{x^3} \neq 0$, is
   - A: $2x^2$
   - B: $8x^2$
   - C: $2x^4$
   - D: $8x^4$

7. What are the solutions of the equation $3x^2 - 48 = 0$? (Lesson 9-1) A
   - A: $4, -4$
   - B: $4, \frac{1}{3}$
   - C: $16, -16$
   - D: $16, \frac{1}{3}$

8. What are the solutions of the equation $x^2 - 3x + 8 = 6x - 6$? (Lesson 9-4) D
   - A: $2, -7$
   - B: $-2, -4$
   - C: $2, 4$
   - D: $2, 7$

9. The area of a rectangle is $12x^2 - 21x - 6$. The width is $3x - 6$. What is the length? (Lesson 9-5) B
   - A: $4x - 1$
   - B: $4x + 1$
   - C: $9x + 1$
   - D: $12x - 18$

Test-Taking Tip

Questions 7 and 9
When answering a multiple-choice question, first find an answer on your own. Then, compare your answer to the answer choices given in the item. If your answer does not match any of the answer choices, check your calculations.

Additional Practice

See pp. 577–578 in the Chapter 9 Resource Masters for additional standardized test practice.

ExamView® Pro

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.
10. Write an equation of a line that has a $y$-intercept of $-1$ and is perpendicular to the graph of $2 - 2y = -5x$. (Lesson 5-6) $y = \frac{2}{5}x - 1$

11. Find all values of $x$ that make the equation $\left| 6x - 2 \right| = 18$ true. (Lesson 6-5) $5$ and $-1$

12. Graph the inequality $x + y \leq 3$. (Lesson 6-6) See margin.

13. A movie theater charges $7.50 for each adult ticket and $4 for each child ticket. If the theater sold a total of 145 tickets for a total of $790, how many adult tickets were sold? (Lesson 7-2) 60

14. Solve the following system of equations. $3x + y = 8$ $4x - 2y = 14$ (Lesson 7-3) $\{3, -1\}$

15. Write an expression to represent the volume of the rectangular prism. (Lesson 8-7) $8x^3 + 4x^2 - 60x$

16. Jon is cutting a 64-inch-long board and a 48-inch-long board to make shelves. He wants the shelves to be the same length without wasting any wood. What is the longest possible length of the shelves? (Lesson 9-1) 16 in.

17. Write $(x + 1)(x + 1)$ as the product of two factors. (Lesson 9-3) $(x + 1)(x + 1)$

18. The product of two consecutive odd integers is 195. Find the integers. (Lesson 9-4) 13 and 15 or $-13$ and $-15$

19. Solve $2x^2 + 5x - 12 = 0$ by factoring. (Lesson 9-5) $\frac{3}{2}$ or $-4$

20. Factor $2x^2 + 7x + 3$. (Lesson 9-5) $(2x + 1)(x + 3)$

21. The length and width of an advertisement in the local newspaper had to be increased by the same amount in order to double its area. The original advertisement had a length of 6 centimeters and a width of 4 centimeters. (Lesson 9-3)
   a. Find an equation that represents the area of the enlarged advertisement. $(x + 6)(x + 4) = 2(6)(4)$
   b. What are the new dimensions of the advertisement? Round to the nearest tenth. length $= 8$ cm, width $= 6$ cm
   c. What is the new area of the advertisement? $48$ cm$^2$
   d. If the entire page has 200 square centimeters of space, about what fraction does the advertisement take up? $\frac{6}{25}$

22. Suppose the area of a rectangular plot of land is $(6c^2 + 7c - 3)$ square miles. (Lesson 9-4)
   a. Find algebraic expressions for the length and the width. length $= (3c - 1)$ mi, width $= (2c + 3)$ mi
   b. If the area is 21 square miles, find the value of $c$. $1.5$
   c. What are the length and the width? $3.5$ mi, $6$ mi

23. Madison is building a fenced, rectangular dog pen. The width of the pen will be 3 yards less than the length. The total area enclosed is 28 square yards. (Lesson 9-4)
   a. Using $L$ to represent the length of the pen, write an equation showing the area of the pen in terms of its length.
   b. What is the length of the pen?
   c. How many yards of fencing will Madison need to enclose the pen completely? $a-c$. See margin.

23a. a polynomial equation equivalent to $28 = L(L - 3)$
23b. The length is 7 yards.
23c. 22 yd

Calculate $W$. Calculate the perimeter.
$W = L - 3$
$W = 7 - 3 or 4$
$P = 2L + 2W$
$P = 2(7) + 2(4)$
$P = 14 + 8 or 22$
Pages 477–479, Lesson 9-1

69. Scientists listening to radio signals would suspect that a modulated signal beginning with prime numbers would indicate a message from an extraterrestrial. Answers should include the following.
• Sample answer: It is unlikely that any natural phenomenon would produce such an artificial and specifically mathematical pattern.

Page 480, Preview of Lesson 9-2

5. \[
\begin{array}{c|c|c|c|c}
  x & x & -1 & -1 & -1 \\
  \hline
  2x - 5 & x & x & -1 & -1 & -1 & -1
\end{array}
\]

6. \[
\begin{array}{c|c|c|c|c|c|c}
  x & x & -1 & -1 & -1 & -1 & -1 \\
  \hline
  x & x & -1 & -1 & -1 & -1 & -1
\end{array}
\]

7. \[
\begin{array}{c|c|c|c|c|c}
  x & x & x & x & x & x \\
  \hline
  x^2 & x & x
\end{array}
\]

8. \[
\begin{array}{c|c|c|c}
  x^2 & x^2 & 1 & 1 & 1 \\
  \hline
\end{array}
\]

Pages 484–486, Lesson 9-2

16. $5(x + 6y)$
18. $a(a^4b - 1)$
20. $3d(7c - 1)$
22. $15ay(a - 2)$
24. $4xy^2z(3x + 10yz)$
26. $a(1 + ab^2 + a^2b^3)$
28. $4x(3ax^2 + 5bx + 8c)$
30. $(x + 3)(x + 2)$
32. $(2x + 3)(2x + 7)$
34. $(3a - 4)(2a - 5)$
36. $(a + b)(4x + 3y)$
38. $(2x - 3)(4a - 3)$
40. $(2a - 3)(5x - 7y)$

Pages 498–500, Lesson 9-4

35. $\left[-5, \frac{-2}{5}\right]$
36. $\left[\frac{-4}{3}, 3\right]$  
37. $\left[-\frac{1}{6}, \frac{3}{4}\right]$  
38. $\left[\frac{1}{3}, \frac{2}{5}\right]$  
39. $\left[-\frac{5}{7}, \frac{5}{2}\right]$  
40. $\left[\frac{5}{4}, \frac{7}{3}\right]$  
41. $\left[-\frac{2}{3}, 3\right]$  
42. $\left[-\frac{2}{7}, 1\right]$  
43. $\left[\frac{1}{2}, \frac{2}{3}\right]$  
44. $\left[-\frac{1}{3}, \frac{9}{4}\right]$  
45. $(-4, 12)$  
46. $\left[\frac{7}{3}, \frac{5}{2}\right]$  
47. $\left[-4, \frac{2}{3}\right]$  
48. $\left[\frac{1}{2}, \frac{7}{2}\right]$