Introduction

In this unit, students will be introduced to additional nonlinear functions such as radical and rational equations. Students learn how to simplify radical and rational expressions, and how to solve equations involving these expressions.

Students also explore triangles through the Pythagorean Theorem and trigonometric ratios.

Assessment Options

Unit 4 Test  Pages 779–780 of the Chapter 12 Resource Masters may be used as a test or review for Unit 4. This assessment contains both multiple-choice and short answer items.

ExamView® Pro

This CD-ROM can be used to create additional unit tests and review worksheets.

Yearly Progress

An online, research-based, instructional, assessment, and intervention tool that provides specific feedback on student mastery of state and national standards, instant remediation, and a data management system to track performance. For more information, contact mhdigitallearning.com.
Each year, amusement park owners compete to earn part of the billions of dollars Americans spend at amusement parks. Often the parks draw customers with new taller and faster roller coasters. In this project, you will explore how radical and rational functions are related to buying and building a new roller coaster.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 4.

### USA TODAY Snapshots

**A day in the park**

What the typical family of four pays to visit a park:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>'93</td>
<td>$180</td>
</tr>
<tr>
<td>'95</td>
<td>$120</td>
</tr>
<tr>
<td>'97</td>
<td>$60</td>
</tr>
<tr>
<td>'99</td>
<td>$0</td>
</tr>
<tr>
<td>'01</td>
<td>$163</td>
</tr>
<tr>
<td>'98</td>
<td>$116</td>
</tr>
</tbody>
</table>
| USA TODAY Snapshots® | Source: Amusement Business

### Teaching Suggestions

Have students study the USA TODAY Snapshot.

- Ask students how the data in the graph differs from that of a linear function. **The data in the graph does not follow a straight line, therefore it is not linear.**

- Why might math be important to a roller coaster designer? **Sample answer: A roller coaster designer might use math to calculate the speed of a roller coaster before it is built to make sure it is safe.**

### Additional USA TODAY Snapshots appearing in Unit 4:

- **Chapter 11** Global spending on construction (p. 615)
- **Chapter 12** Most Americans have one or two credit cards (p. 672)
- Cost of parenthood rising (p. 689)

### Problem-Based Learning

A WebQuest is an online project in which students do research on the Internet, gather data, and make presentations using word processing, graphing, page-making, or presentation software. In each chapter, students advance to the next step in their WebQuest. At the end of Chapter 12, the project culminates with a presentation of their findings.

Teaching notes and sample answers are available in the WebQuest and Project Resources.
Radical Expressions and Triangles

Chapter Overview and Pacing

Year-long and two-year pacing: pages T20–T21.

<table>
<thead>
<tr>
<th>LESSON OBJECTIVES</th>
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<th>Block</th>
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<tbody>
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<tr>
<td>Operations with Radical Expressions (pp. 593–597)</td>
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<td>1</td>
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<td>0.5</td>
</tr>
<tr>
<td>The Distance Formula (pp. 611–615)</td>
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</tr>
<tr>
<td>Similar Triangles (pp. 616–621)</td>
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<td>0.5</td>
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<tr>
<td>Trigonometric Ratios (pp. 622–630)</td>
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<td>1</td>
</tr>
<tr>
<td>Study Guide and Practice Test (pp. 632–637)</td>
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<td>0.5</td>
</tr>
<tr>
<td>Standardized Test Practice (pp. 638–639)</td>
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<tr>
<td>Chapter Assessment</td>
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</tr>
<tr>
<td>TOTAL</td>
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<td>7</td>
</tr>
</tbody>
</table>

An electronic version of this chapter is available on StudentWorks™. This backpack solution CD-ROM allows students instant access to the Student Edition, lesson worksheet pages, and web resources.
### Chapter Resource Manager

#### CHAPTER 11 RESOURCE MASTERS

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<thead>
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<th>Study Guide and Intervention</th>
<th>Practice (Skills and Average)</th>
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<th>Enrichment</th>
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<td>655–656</td>
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<td>660</td>
<td></td>
<td>GCS 43</td>
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<td>661–662</td>
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<td>SM 85–90</td>
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<td>675–676</td>
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<td>678</td>
<td>700</td>
<td>61–62</td>
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<tr>
<td>679–680</td>
<td>681–682</td>
<td>683</td>
<td>684</td>
<td>700</td>
<td></td>
<td>89</td>
<td>11-7</td>
<td>11-7</td>
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<td></td>
<td>685–698, 702–704</td>
<td>90</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

*Key to Abbreviations: GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual

ELL Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.
11-1 Simplifying Radical Expressions

A radical expression contains a square root. The expression is in simplest form if the expression inside the radical sign, or radicand, has only 1 as a perfect square factor. The Product Property of Square Roots states that the square root of a product equals the product of each square root. Prime factorizations combined with the Product Property of Square Roots can be used to simplify radical expressions. Principal square roots are never negative, so absolute value symbols must be used to signify that some results are not negative. The Quotient Property of Square Roots states that the square root of a quotient equals the quotient of each square root. The Quotient Property of Square Roots can be used to derive the Quadratic Formula.

A fraction does not have a radical in its denominator if it is in simplest form. Since squaring and taking a square root are inverse functions, you multiply the numerator and the denominator by the same number so that the denominator contains a perfect square. Remember that the numerator must be multiplied by the same amount so that the whole fraction is being multiplied by a value of 1. This process is called rationalizing the denominator. After rationalizing the denominator, check for coefficients of the radical in the numerator that will simplify with the denominator. If the denominator is \(\frac{a}{\sqrt{b}}\), multiply by \(\frac{\sqrt{b}}{\sqrt{b}}\). For example, if the denominator is \(a + \sqrt{b}\), multiply by \(a - \sqrt{b}\).

11-2 Operations with Radical Expressions

Use the process of combining like terms to simplify expressions in which radicals are added or subtracted. For terms to be combined, their radicands must be the same. As in combining monomials with variables, only the coefficients of the radicals are combined. Be sure to simplify all radicals first. When multiplying radical expressions, first multiply the coefficients, and then use the Product Property of Square Roots to multiply the radicals. Simplify each term and then combine like terms as necessary.

11-3 Radical Equations

Equations that contain variables in the radicand are called radical equations. To solve radical equations, the radical must first be isolated on one side. Then square each side. This will eliminate the radical. This process sometimes produces results that are not solutions of the original equation. These are called extraneous solutions. All solutions must be substituted back into the original equation to check their validity.
The Pythagorean Theorem

The two sides of a right triangle that form the right angle are the legs of the triangle. The third, and longest, side is the hypotenuse. The hypotenuse is the longest side because it is always across from the angle with the greatest measure. The Pythagorean Theorem states that the sum of the squares of the legs equals the square of the hypotenuse. The formula is \( c^2 = a^2 + b^2 \), where \( a \) and \( b \) are the measures of the legs and \( c \) is the measure of the hypotenuse. This formula can be used to find the length of any missing side of a right triangle if the lengths of the other two sides are known. Any three whole numbers that satisfy this equation are known as a Pythagorean Triple. These triples represent side lengths that always form right triangles. It follows that if three numbers do not satisfy the Pythagorean Theorem, then sides of their length will not form a right triangle.

The Distance Formula

If the Pythagorean Theorem is solved for \( c \), the result is the Distance Formula. The variable \( a \) is expressed as the difference of the \( x \)-coordinates of the endpoints of the hypotenuse and \( b \) is expressed as the difference of the \( y \)-coordinates. This formula is used to find the distance between any two points. You can also find one missing coordinate of an endpoint if you know the other coordinate, the coordinates of the other endpoint, and the distance between the two points.

Similar Triangles

Similar triangles have the same shape, but are not necessarily the same size. All of the corresponding angles will have equal measures, and the corresponding sides will all be proportional. If the sides have a 1 to 1 ratio of proportionality, then the similar triangles are the same size. When determining whether triangles are similar, all you need to check is if the corresponding angles have the same measure. If all the angle measures cannot be determined, then check the corresponding sides to see if they are proportional.

Proportions can be used to find the lengths of missing sides of similar triangles. You must know the lengths of at least one pair of corresponding sides and the length of the side that corresponds to the missing side’s length. Set up the proportion, and then cross multiply. Solve the resulting equation.

Trigonometric Ratios

Trigonometry is the mathematical study of angles and triangles. Ratios comparing the measures of two sides of a right triangle are called trigonometric ratios. The three most common trigonometric ratios are sine, cosine, and tangent. These ratios can be used to find the measures of missing sides or the measures of the acute angles. The relationship of the two sides necessary for the problem to a specific acute angle determines which ratio is used. If the measures of just two sides of a triangle or the measures of one side and one acute angle are known, then the measures of all of the rest of the sides and angles can be found. This is called solving the triangle. Trigonometric ratios are used to find distances in problems involving angles of elevation and angles of depression.

Quick Review

Math Handbook

Hot Words includes a glossary of terms while Hot Topics consists of explanations of key mathematical concepts with exercises to test comprehension. This valuable resource can be used as a reference in the classroom or for home study.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Hot Topics Section</th>
<th>Lesson</th>
<th>Hot Topics Section</th>
</tr>
</thead>
<tbody>
<tr>
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<td>3.2</td>
<td>11-6</td>
<td>8.6</td>
</tr>
<tr>
<td>11-2</td>
<td>3.2</td>
<td>11-7P</td>
<td>7.10</td>
</tr>
<tr>
<td>11-4</td>
<td>7.9</td>
<td>11-7</td>
<td>7.10</td>
</tr>
</tbody>
</table>

P = Preview

Additional mathematical information and teaching notes are available at www.algebra1.com/key_concepts.
### Key to Abbreviations:
TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

### Chapter 11: Radical Expressions and Triangles

<table>
<thead>
<tr>
<th>Type</th>
<th>Student Edition</th>
<th>Teacher Resources</th>
<th>Technology/Internet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed Review</td>
<td>pp. 592, 597, 603, 610, 615, 621, 630</td>
<td>Cumulative Review, CRM p. 702</td>
<td></td>
</tr>
<tr>
<td>Error Analysis</td>
<td>Find the Error, pp. 600, 618</td>
<td>Find the Error, TWE pp. 600, 618 Unlocking Misconceptions, TWE p. 612 Tips for New Teachers, TWE pp. 587, 624</td>
<td></td>
</tr>
</tbody>
</table>

**For more information on Yearly ProgressPro, see p. 582.**

### Yearly ProgressPro

<table>
<thead>
<tr>
<th>Algebra Lesson</th>
<th>Yearly ProgressPro Skill Lesson(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-1</td>
<td>Simplifying Radical Expressions Simplifying Rational Expressions with Radicals in the Denominator</td>
</tr>
<tr>
<td>11-2</td>
<td>Operations with Radical Expressions</td>
</tr>
<tr>
<td>11-3</td>
<td>Radical Equations</td>
</tr>
<tr>
<td>11-4</td>
<td>The Pythagorean Theorem</td>
</tr>
<tr>
<td>11-5</td>
<td>The Distance Formula</td>
</tr>
<tr>
<td>11-6</td>
<td>Similar Triangles</td>
</tr>
<tr>
<td>11-7</td>
<td>Trigonometric Ratios               Applying Trigonometric Ratios</td>
</tr>
</tbody>
</table>

**For more information on Intervention and Assessment, see pp. T8–T11.**
Reading and Writing in Mathematics

*Glencoe Algebra 1* provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

**Student Edition**
- Foldables Study Organizer, p. 585
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 589, 595, 600, 607, 612, 618, 627)
- Reading Mathematics, p. 631
- Writing in Math questions in every lesson, pp. 591, 597, 602, 610, 614, 620, 630
- Reading Study Tip, pp. 586, 611, 623
- WebQuest, p. 590

**Teacher Wraparound Edition**
- Foldables Study Organizer, pp. 585, 632
- Study Notebook suggestions, pp. 589, 595, 600, 608, 613, 618, 627, 631
- Modeling activities, pp. 603, 610, 621
- Speaking activities, pp. 597, 615
- Writing activities, pp. 592, 630
- Differentiated Instruction, (Verbal/Linguistic), p. 599
- ELL Resources, pp. 584, 591, 596, 599, 602, 609, 614, 620, 629, 631, 632

**Additional Resources**
- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 11 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 11 Resource Masters*, pp. 647, 653, 659, 665, 671, 677, 683)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*
- *Hot Words/Hot Topics* Sections 3.2, 7.9, 7.10, 8.6

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

---

**Encourage students to create frames to organize their notes.** The math frame below describes the Distance Formula, which students study in Lesson 11-5. Creating frames helps students understand topics by explaining them in their own words. Frames also allow students to quickly review information in the chapter.

<table>
<thead>
<tr>
<th>Term/Concept</th>
<th>Definition (in your own words)</th>
<th>Formula</th>
<th>Original Question with Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Distance Formula</td>
<td>A formula to find the distance between any two points on a coordinate plane.</td>
<td>$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</td>
<td>Find the distance between $(-7, 7)$ and $(2, -5)$.</td>
</tr>
</tbody>
</table>

$$d = \sqrt{(2 - (-7))^2 + (-5 - 7)^2}$$
$$= \sqrt{81 + 144}$$
$$= 15$$
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

**Key Vocabulary**
- radical expression  
- radical equation  
- Pythagorean Theorem  
- Distance Formula  
- trigonometric ratios  

**Physics problems are among the many applications of radical equations.** Formulas that contain the value for the acceleration due to gravity, such as free-fall times, escape velocities, and the speeds of roller coasters, can all be written as radical equations.  

You will learn how to calculate the time it takes for a skydiver to fall a given distance in Lesson 11-3.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>NCTM Standards</th>
<th>Local Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-1</td>
<td>1, 2, 6, 8, 9, 10</td>
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</tr>
<tr>
<td>11-2</td>
<td>1, 6, 8, 9, 10</td>
<td></td>
</tr>
<tr>
<td>11-3</td>
<td>2, 6, 8, 9, 10</td>
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</tr>
<tr>
<td>11-3  Follow-Up</td>
<td>2, 6, 8</td>
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</tr>
<tr>
<td>11-4</td>
<td>1, 2, 3, 6, 8, 9, 10</td>
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</tr>
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<td>11-5</td>
<td>1, 2, 3, 6, 8, 9, 10</td>
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<td>11-6</td>
<td>1, 2, 3, 6, 8, 9, 10</td>
<td></td>
</tr>
<tr>
<td>11-7  Preview</td>
<td>1, 3, 7</td>
<td></td>
</tr>
<tr>
<td>11-7</td>
<td>3, 6, 8, 9, 10</td>
<td></td>
</tr>
</tbody>
</table>

**Vocabulary Builder**

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 11 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 11 test.
This section provides a review of the basic concepts needed before beginning Chapter 11. Page references are included for additional student help. Additional review is provided in the Prerequisite Skills Workbook, pp. 37–38 and 61–62.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lessons 11-1 and 11-4

Find Square Roots

Find each square root. If necessary, round to the nearest hundredth.

(For review, see Lesson 2-7.)

1. \( \sqrt{25} \) 5  2. \( \sqrt{80} \) 8.94  3. \( \sqrt{56} \) 7.48  4. \( \sqrt{324} \) 18

For Lesson 11-2

Combine Like Terms

Simplify each expression.

(For review, see Lesson 1-6.)

5. \( 3a + 7b - 2a \)  \( a + 7b \)
6. \( 14x - 6y + 2y \)  \( 14x - 4y \)
7. \( (10c - 5d) + (6c + 5d) \) 16c  8. \( (21m + 15n) - (9n - 4m) \) 25m + 6n

For Lesson 11-3

Solve Quadratic Equations

Solve each equation.

(For review, see Lesson 9-3.)

9. \( x(x - 5) = 0 \) \{0, 5\}  10. \( x^2 + 10x + 24 = 0 \) \{-6, -4\}
11. \( x^2 - 6x - 27 = 0 \) \{-3, 9\}  12. \( 2x^2 + x + 1 = 2 \) \( \left\{ \frac{1}{2}, -1 \right\} \)

For Lesson 11-6

Proportions

Use cross products to determine whether each pair of ratios forms a proportion. Write yes or no.

(For review, see Lesson 3-6.)

13. \( \frac{2}{3} \) \( \frac{8}{12} \) yes  14. \( \frac{4}{5} \) \( \frac{16}{25} \) no  15. \( \frac{8}{10} \) \( \frac{12}{16} \) no  16. \( \frac{6}{30} \) \( \frac{3}{15} \) yes

For Prerequisite Lesson Skill

11-2 Multiplying Binomials (p. 592)
11-3 Finding Special Products (p. 597)
11-4 Evaluating Radical Expressions (p. 603)
11-5 Simplifying Radical Expressions (p. 610)
11-6 Solving Proportions (p. 615)
11-7 Evaluating Expressions (p. 621)

Fold in Half

Fold in half lengthwise.

Fold Again

Fold the top to the bottom.

Cut

Open. Cut along the second fold to the center to make two tabs.

Label

Label each tab as shown.

Radical Expressions and Equations

Make this Foldable to help you organize your notes. Begin with a sheet of plain \( \frac{8}{2} \) by 11" paper.

Reading and Writing

As you read and study the chapter, write notes and examples for each lesson under each tab.

Organization of Data: Annotating

As students read and work through the chapter, have them make annotations under the tabs of their Foldable. Explain to them that annotations are usually notes taken in the margins of books we own to organize the text for review or studying. Annotations often include questions that arise as we read the chapter, reader comments and reactions, sentence length summaries, steps or data numbered by the reader, and key points highlighted or underlined.
Focus

5-Minute Check Transparency 11-1 Use as a quiz or review of Chapter 10.

Mathematical Background notes are available for this lesson on p. 584C.

Building on Prior Knowledge

Students were introduced to the Quadratic Formula in Lesson 10-4 and they learned how to use it to solve quadratic equations. In this lesson, students learn how to derive the Quadratic Formula from the standard form of a quadratic equation using the Quotient Property of Square Roots.

How are radical expressions used in space exploration?

Ask students:

• In the formula for escape velocity, what does the radical sign mean? The radical sign means that you must find the square root of the value under the radical sign.

• Based on what you know about the order of operations, how do you think you should simplify the radical expression in the escape velocity formula? You should simplify the expression under the radical sign before finding the square root.

Vocabulary
• radical expression
• radicand
• rationalizing the denominator
• conjugate

5-Minute Check

Transparency 11-1

Product Property of Square Roots

For any numbers a and b, where a ≥ 0 and b ≥ 0, the square root of the product ab is equal to the product of each square root.

Example

Simplify Square Roots

a. \( \sqrt{12} \)

\[
\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} \quad \text{Prime factorization of 12}
\]

\[
= \sqrt{2^2 \cdot 3} \quad \text{Product Property of Square Roots}
\]

\[
= 2\sqrt{3} \quad \text{Simplify}
\]

b. \( \sqrt{90} \)

\[
\sqrt{90} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5} \quad \text{Prime factorization of 90}
\]

\[
= \sqrt{3^2 \cdot 2 \cdot 5} \quad \text{Product Property of Square Roots}
\]

\[
= 3\sqrt{10} \quad \text{Simplify}
\]

Resource Manager

Workbook and Reproducible Masters

Chapter 11 Resource Masters
- Study Guide and Intervention, pp. 643–644
- Skills Practice, p. 645
- Practice, p. 646
- Reading to Learn Mathematics, p. 647
- Enrichment, p. 648

Parent and Student Study Guide Workbook, p. 83
School-to-Career Masters, p. 21

Transparencies

5-Minute Check Transparency 11-1
Answer Key Transparencies

Technology

Interactive Chalkboard
The Product Property can also be used to multiply square roots.

**Example 2** Multiply Square Roots

Find \( \sqrt{3} \cdot \sqrt{15} \).

\[
\sqrt{3} \cdot \sqrt{15} = \sqrt{3 \cdot 15} = \sqrt{45} = 3 \sqrt{5}
\]

When finding the principal square root of an expression containing variables, be sure that the result is not negative. Consider the expression \( \sqrt{x^2} \). It may seem that \( \sqrt{x^2} = x \). Let’s look at \( x = -2 \).

\[
\sqrt{(-2)^2} = -2 \\
\sqrt{4} = 2
\]

For radical expressions where the exponent of the variable inside the radical is even and the resulting simplified exponent is odd, you must use absolute value to ensure nonnegative results.

\[
\sqrt{x^2} = |x| \\
\sqrt{x^3} = x\sqrt{x} \\
\sqrt{x^4} = x^2 \\
\sqrt{x^5} = x^2\sqrt{x} \\
\sqrt{x^6} = |x^3|
\]

**Example 3** Simplify a Square Root with Variables

Simplify \( \sqrt{40a^2b^2c^2} \).

\[
\sqrt{40a^2b^2c^2} = \sqrt{2^2 \cdot 5 \cdot a^2 \cdot b^2 \cdot c^2} = 2a \cdot b \cdot c \cdot \sqrt{5} = 2abc\sqrt{5}
\]

**Quotient Property of Square Roots** You can divide square roots and simplify radical expressions that involve division by using the Quotient Property of Square Roots.

**Key Concept**

**Quotient Property of Square Roots**

- **Words** For any numbers \( a \) and \( b \), where \( a \geq 0 \) and \( b > 0 \), the square root of the quotient \( \frac{a}{b} \) is equal to the quotient of each square root.
- **Symbols** \( \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \)
- **Example** \( \frac{49}{4} = \frac{\sqrt{49}}{\sqrt{4}} \)

You can use the Quotient Property of Square Roots to derive the Quadratic Formula by solving the quadratic equation \( ax^2 + bx + c = 0 \).

\[
ax^2 + bx + c = 0 \quad \text{Original equation} \\
x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{Divide each side by } a, \ a \neq 0.
\]

(continued on the next page)
In-Class Example

4 Simplify.

a. \( \sqrt{12} \div 5 \)

b. \( \sqrt{11y} \div \sqrt{27} \)

c. \( \sqrt{3} \div \sqrt{8} \)

Study Tip

Plus or Minus Symbol

The ± symbol is used with the radical expression since both square roots lead to solutions.

Thus, we have derived the Quadratic Formula.

A fraction containing radicals is in simplest form if no prime factors appear under the radical sign with an exponent greater than 1 and if no radicals are left in the denominator. Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction.

Example 4 Rationalizing the Denominator

Simplify.

a. \( \frac{\sqrt{10}}{\sqrt{3}} \)

b. \( \frac{\sqrt{7x}}{\sqrt{8}} \)

c. \( \frac{\sqrt{2}}{\sqrt{6}} \)

Logical Have students use the same method that is shown for the derivation of the Quadratic Formula to solve actual quadratic equations. Have students turn back to Lesson 10-4 and solve an example problem using this method.
Binomials of the form \( p\sqrt{q} + r\sqrt{s} \) and \( p\sqrt{q} - r\sqrt{s} \) are called **conjugates**. For example, \( 3 + \sqrt{2} \) and \( 3 - \sqrt{2} \) are conjugates. Conjugates are useful when simplifying radical expressions because if \( p, q, r, \) and \( s \) are rational numbers, their product is always a rational number with no radicals. Use the pattern \((a - b)(a + b) = a^2 - b^2\) to find their product.

\[
(3 + \sqrt{2})(3 - \sqrt{2}) = 3^2 - (\sqrt{2})^2 \quad a = 3, b = \sqrt{2} \\
= 9 - 2 \quad (\sqrt{2})^2 = \sqrt{2} \cdot \sqrt{2} \text{ or } 2
\]

**Example 5** Use Conjugates to Rationalize a Denominator

Simplify \( \frac{2}{6 - \sqrt{3}} \).

\[
\frac{2}{6 - \sqrt{3}} = \frac{2}{6 - \sqrt{3}} \cdot \frac{6 + \sqrt{3}}{6 + \sqrt{3}} = \frac{6 + \sqrt{3}}{6 - \sqrt{3}} \\
= \frac{2(6 + \sqrt{3})}{36 - 3} \quad (a - b)(a + b) = a^2 - b^2 \\
= \frac{12 + 2\sqrt{3}}{33} \quad \sqrt{3}^2 = 3
\]

When simplifying radical expressions, check the following conditions to determine if the expression is in simplest form.

**Concept Summary**  
**Simplest Radical Form**

A radical expression is in simplest form when the following three conditions have been met.

1. No radicands have perfect square factors other than 1.
2. No radicands contain fractions.

**Check for Understanding**

**Concept Check**

1. Explain why absolute value is not necessary for \( \sqrt{x^2} = x^2 \). **See margin.**
2. Show that \( \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a} \) for \( a > 0 \).
3. **OPEN ENDED** Give an example of a binomial in the form \( a\sqrt{b} + c\sqrt{d} \) and its conjugate. Then find their product. **Sample answer:** \( 2\sqrt{2} + 3\sqrt{3} \) and \( 2\sqrt{2} - 3\sqrt{3}; -19 \)

**Guided Practice**

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**In-Class Example**

5. Simplify \( \frac{3}{5 - \sqrt{2}} \).

**Practice/Apply**

**Study Notebook**

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.
- include examples of how to use the Product and Quotient Properties of Square Roots to simplify radical expressions.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

**Answer**

1. Both \( x^4 \) and \( x^2 \) are positive even if \( x \) is a negative number.
13. **GEOMETRY**  A square has sides each measuring $2\sqrt{7}$ feet. Determine the area of the square. $28 \text{ ft}^2$

14. **PHYSICS**  The period of a pendulum is the time required for it to make one complete swing back and forth. The formula of the period $P$ of a pendulum is $P = 2\pi \sqrt{\frac{\ell}{g}}$, where $\ell$ is the length of the pendulum in feet. If a pendulum in a clock tower is 8 feet long, find the period. Use 3.14 for $\pi$. $3.14 \text{ s}$

---

**Teacher to Teacher**

Judy Buchholtz  
Dublin Scioto H.S., Dublin, OH

“To make algebra more meaningful to my students, I bring in the school police officer to discuss the formula used in Exercises 45–47.”

---

590  Chapter 11  Radical Expressions and Triangles
45. Write a simplified expression for the speed if \( f = 0.6 \) for a wet asphalt road. \( 3\sqrt{2}d \)

46. What is a simplified expression for the speed if \( f = 0.8 \) for a dry asphalt road?

47. An officer measures skid marks that are 110 feet long. Determine the speed of the car for both wet road conditions and for dry road conditions. about 44.5 mph, about 51.4 mph

46. \( 2\sqrt{6}d \)

48. Find the value of \( s \) if the side lengths of a triangle are 13, 10, and 7 feet. 15

49. Determine the area of the triangle. \( 20\sqrt{3} \) or about 34.6 ft²

50. CRITICAL THINKING

Simplify \( \frac{a - 1 - \sqrt{a}}{a + 1 + \sqrt{a}} \).

51. WRITING IN MATH

Answer the question that was posed at the beginning of the lesson. See margin.

How are radical expressions used in space exploration?

Include the following in your answer:

- an explanation of how you could determine the escape velocity for a planet and why you would need this information before you landed on the planet, and
- a comparison of the escape velocity for two astronomical bodies with the same mass, but different radii.

52. If the cube has a surface area of \( 96a \),
what is its volume? 6

- A 32a³
- B 48a³
- C 6a³
- D 96a³

53. If \( x = 81b^2 \) and \( b > 0 \), then \( \sqrt[3]{x} = \)

- A \( -9b \)
- B 9b
- C \( 3b\sqrt{27} \)
- D \( 27b\sqrt{3} \)

WEATHER

For Exercises 54 and 55, use the following information.

The formula \( y = 91.4 - (91.4 - t)0.478 + 0.301(\sqrt{x} - 0.02) \) can be used to find the wind chill factor. In this formula, \( y \) represents the wind chill factor, \( t \) represents the air temperature in degrees Fahrenheit, and \( x \) represents the wind speed in miles per hour. Suppose the air temperature is 12°.

54. Use a graphing calculator to find the wind speed to the nearest mile per hour if it feels like \(-9°\) with the wind chill factor. 7 mph

55. What does it feel like to the nearest degree if the wind speed is 4 miles per hour? 8°F

Lesson 11-1 Simplifying Radical Expressions
Extending the Lesson

Radical expressions can be represented with fractional exponents. For example, \( x^{\frac{1}{2}} = \sqrt{x} \). Using the properties of exponents, simplify each expression.

56. \( x^{\frac{1}{2}} \cdot y^{\frac{1}{2}} \)
57. \( (x^{\frac{1}{2}})^4 \)
58. \( \frac{x^{\frac{1}{2}}}{x} \)

★ 59. Simplify the expression \( \frac{\sqrt{a}}{\sqrt{b}} \) or \( \frac{\sqrt{a}}{\sqrt{b}} \)

★ 60. Solve the equation \( |y^3| = \frac{1}{3} \) for \( y \). \( \pm \sqrt[3]{3} \)

★ 61. Write \( (s^{\frac{1}{2}})^8 \sqrt{s^{\frac{2}{3}}} \) in simplest form. \( s^{\frac{2}{3}} \)

Maintain Your Skills

Mixed Review

Find the next three terms in each geometric sequence. (Lesson 10-7)

62. 2, 6, 18, 54 \( 162, 486, 1458 \)
63. 1, –2, 4, –8 \( 16, –32, 64 \)
64. 384, 192, 96, 48 \( 24, 12, 6 \)
65. \( \frac{1}{2}, \frac{2}{3}, 4, 24 \) \( 144, 864, 5184 \)
66. \( 3, \frac{3}{4}, 16, \frac{3}{4} \) \( 256, 1024, 4096 \)
67. 50, 10, 2, 0.4 \( 0.08, 0.016, 0.0032 \)

68. BIOLOGY A certain type of bacteria, if left alone, doubles its number every 2 hours. If there are 1000 bacteria at a certain point in time, how many bacteria will there be 24 hours later? (Lesson 10-6) \( 4,996,000 \)

69. PHYSICS According to Newton’s Law of Cooling, the difference between the temperature of an object and its surroundings decreases in time exponentially. Suppose a cup of coffee is 95°C and it is in a room that is 20°C. The cooling of the coffee can be modeled by the equation \( y = 75(0.875)^t \), where \( y \) is the temperature difference and \( t \) is the time in minutes. Find the temperature of the coffee after 15 minutes. (Lesson 10-6) \( 84.9°C \)

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write prime. (Lesson 9-4)

70. \( 6x^2 + 7x - 5 \) \( (3x + 5)(2x - 1) \)
71. \( 35x^2 - 43x + 12 \) \( (5x - 4)(7x - 3) \)
72. \( 5x^2 + 3x + 31 \) \( \text{prime} \)
73. \( 3x^2 - 6x - 105 \) \( (3x - 7)(x + 5) \)
74. \( 4x^2 - 12x + 15 \) \( \text{prime} \)
75. \( 8x^2 - 10x + 3 \) \( (4x - 3)(2x - 1) \)

Find the solution set for each equation, given the replacement set. (Lesson 4-4)

76. \( y = 3x + 2; \) \( \{(1, 5), (2, 6), (-2, 2), (-4, -10)\} \) \( \{(1, 5), (-4, -10)\} \)
77. \( 5x + 2y = 10; \) \( \{(3, 5), (2, 0), (4, 2), (1, 2.5)\} \) \( \{(2, 0), (1, 2.5)\} \)
78. \( 3a + 2b = 11; \) \( \{(-3, 10), (4, 1), (2, 2.5), (3, -2)\} \) \( \{(-3, 10), (2, 2.5)\} \)
79. \( 5 - \frac{2}{3} = 2y; \) \( \{(0, 1), (8, 2), (4, -\frac{1}{2}), (2, 1)\} \) \( \{(4, -\frac{1}{2}), (2, 1)\} \)

Solve each equation. Then check your solution. (Lesson 3-3)

80. \( 40 = -5d \) \( d = -8 \)
81. \( 20.4 = 3.4y \) \( y = 6 \)
82. \( \frac{3}{5} = -25 \) \( d = 275 \)
83. \( -65 = \frac{r}{29} \) \( -1885 \)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each product. (To review multiplying binomials, see Lesson 8-7)

84. \( (x - 3)(x + 2) \) \( x^2 - x - 6 \)
85. \( (a + 2)(a + 5) \) \( a^2 + 7a + 10 \)
86. \( (2x + 1)(t - 6) \) \( 2t^2 - 11t - 6 \)
87. \( (4x - 3)(x + 1) \) \( 4x^2 + x - 3 \)
88. \( (5x + 3y)(3x - y) \) \( 15x^2 + 4xy - 3y^2 \)
89. \( (3a - 2b)(4a + 7b) \) \( 12a^2 + 13ab - 14b^2 \)
**Operations with Radical Expressions**

**What You’ll Learn**
- Add and subtract radical expressions.
- Multiply radical expressions.

**How can you use radical expressions to determine how far a person can see?**

![World's Tall Structures](image)

The formula \( d = \frac{2h}{\sqrt{2}} \) represents the distance \( d \) in miles that a person \( h \) feet high can see. To determine how much farther a person can see from atop the Sears Tower than from atop the Empire State Building, we can substitute the heights of both buildings into the equation.

### ADD AND SUBTRACT RADICAL EXPRESSIONS
Radical expressions in which the radicands are alike can be added or subtracted in the same way that monomials are added or subtracted.

**Monomials**

- \( 2x + 7x = (2 + 7)x = 9x \)
- \( 15y - 3y = (15 - 3)y = 12y \)

**Radical Expressions**

- \( 2\sqrt{11} + 7\sqrt{11} = (2 + 7)\sqrt{11} = 9\sqrt{11} \)
- \( 15\sqrt{2} - 3\sqrt{2} = (15 - 3)\sqrt{2} = 12\sqrt{2} \)

Notice that the Distributive Property was used to simplify each radical expression.

**Example 1 Expressions with Like Radicands**

Simplify each expression.

**a.** \( 4\sqrt{3} + 6\sqrt{3} - 5\sqrt{3} \)

\[
4\sqrt{3} + 6\sqrt{3} - 5\sqrt{3} = (4 + 6 - 5)\sqrt{3} = 5\sqrt{3}
\]

**b.** \( 12\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} - 8\sqrt{5} \)

\[
12\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} - 8\sqrt{5} = (12 - 8)\sqrt{5} + (3 + 6)\sqrt{7} = 4\sqrt{5} + 9\sqrt{7}
\]

In Example 1b, \( 4\sqrt{5} + 9\sqrt{7} \) cannot be simplified further because the radicands are different. There are no common factors, and each radicand is in simplest form. If the radicals in a radical expression are not in simplest form, simplify them first.

**Mathematical Background**
- **Assuming that nothing blocks the view, how much farther can a person atop the Sears Tower see than a person atop the Empire State Building?** About 3.3 miles farther
- **Mountain Climbing** Mount Everest, the tallest mountain in the world, is about 12,000 feet above the plateau out of which it rises. How far would a climber be able to see from the peak of Mount Everest? About 134 miles

**Virginia SOL**
- **STANDARD A.3** The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects, pictorial representations, and the properties of real numbers, equality, and inequality.

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**Chapter 11 Resource Masters**
- Study Guide and Intervention, pp. 649–650
- Skills Practice, p. 651
- Practice, p. 652
- Reading to Learn Mathematics, p. 653
- Enrichment, p. 654
- Assessment, p. 699

**Parent and Student Study Guide Workbook,** p. 84
**Prerequisite Skills Workbook,** pp. 37–38

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**Technology**
- Interactive Chalkboard
ADD AND SUBTRACT RADICAL EXPRESSIONS

In-Class Examples

1. Simplify each expression.
   a. $6\sqrt{5} + 2\sqrt{5} - 5\sqrt{5} = 3\sqrt{5}$
   b. $7\sqrt{2} + 8\sqrt{11} - 4\sqrt{11} - 6\sqrt{2} \div 2 + 4\sqrt{11}$

2. $6\sqrt{27} + 8\sqrt{12} + 2\sqrt{75} = 44\sqrt{3}$

MULTIPLY RADICAL EXPRESSIONS

In-Class Example

3. Find the area of a rectangle with a width of $4\sqrt{6} - 2\sqrt{10}$ and a length of $5\sqrt{3} + 7\sqrt{5}$.
   $18\sqrt{30} - 10\sqrt{2}$

Example 2 Expressions with Unlike Radicands

Simplify $2\sqrt{20} + 3\sqrt{45} + \sqrt{180}$.

$2\sqrt{20} + 3\sqrt{45} + \sqrt{180} = 2\sqrt{2^2 \cdot 5} + 3\sqrt{3^2 \cdot 5} + \sqrt{6^2 \cdot 5}$

$= 2(2\sqrt{5}) + 3(3\sqrt{5}) + 6\sqrt{5}$

$= 4\sqrt{5} + 9\sqrt{5} + 6\sqrt{5}$

$= 19\sqrt{5}$

The simplified form is $19\sqrt{5}$.

You can use a calculator to verify that a simplified radical expression is equivalent to the original expression. Consider Example 2. First, find a decimal approximation for the original expression.

KEYSTROKES: 

Next, find a decimal approximation for the simplified expression.

KEYSTROKES: 

Since the approximations are equal, the expressions are equivalent.

Example 3 Multiply Radical Expressions

Find the area of the rectangle in simplest form.

To find the area of the rectangle multiply the measures of the length and width.

$\left(4\sqrt{5} - 2\sqrt{3}\right) \left(3\sqrt{6} - \sqrt{10}\right)$

First terms $\quad$ Outer terms $\quad$ Inner terms $\quad$ Last terms

$= \left(4\sqrt{5}\cdot3\sqrt{6}\right) + \left(4\sqrt{5}\cdot-\sqrt{10}\right) + \left(-2\sqrt{3}\cdot3\sqrt{6}\right) + \left(-2\sqrt{3}\cdot-\sqrt{10}\right)$

$= 12\sqrt{30} - 4\sqrt{50} - 6\sqrt{18} + 2\sqrt{30}$

Multiply.

$= 12\sqrt{30} - 4\sqrt{25 \cdot 2} - 6\sqrt{9 \cdot 2} + 2\sqrt{30}$

Prime factorization

$= 12\sqrt{30} - 20\sqrt{2} - 18\sqrt{2} + 2\sqrt{30}$

Simplify.

$= 14\sqrt{30} - 38\sqrt{2}$

Combine like terms.

The area of the rectangle is $14\sqrt{30} - 38\sqrt{2}$ square units.

Study Tip

Look Back

To review the FOIL method, see Lesson 8-7.

Differentiated Instruction

Visual/Spatial Have students write all the perfect squares from 1 to 100 on an index card using a colored pen or pencil. Then have them rework Example 3. After they multiply the binomials, have students use their colored pen or pencil to circle the terms inside radicals that have perfect square factors. Using the same color, have students write the factorization in the line below each radical expression that can be factored.
Check for Understanding

1. Explain why you should simplify each radical in a radical expression before adding or subtracting. **To determine if there are any like radicands**

2. Explain how you use the Distributive Property to simplify like radicands that are added or subtracted. **See margin.**

3. OPEN ENDED Choose values for $x$ and $y$. Then find $(\sqrt{x} + \sqrt{y})^2$. **Sample answer:** $(\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3$ or $5 + 2\sqrt{6}$

Guided Practice

Simplify each expression.

4. $\sqrt{3} + 7\sqrt{3} 11\sqrt{3}$
5. $2\sqrt{6} - 7\sqrt{6} - 5\sqrt{6}$
6. $\sqrt{5} - 3\sqrt{2} - \sqrt{5}$
7. $2\sqrt{3} + \sqrt{12} 4\sqrt{3}$
8. $\sqrt{5} + 5\sqrt{6} + 3\sqrt{20} 9\sqrt{5} + 5\sqrt{6}$
9. $\sqrt{8} + \sqrt{3} + \sqrt{9} 9\sqrt{3} + 3$

Find each product.

10. $\sqrt{2}\sqrt{8} + 4\sqrt{5} 4 + 4\sqrt{6}$
11. $\sqrt{4} + \sqrt{5}\sqrt{3} + \sqrt{5} 17 + 7\sqrt{5}$

Applications

12. **GEOMETRY** Find the perimeter and the area of a square whose sides measure 4 + 3\sqrt{6} feet. $P = 16 + 12\sqrt{6}$ ft; $A = 70 + 24\sqrt{6}$ ft$^2$

13. **ELECTRICITY** The voltage $V$ required for a circuit is given by $V = \sqrt{PR}$, where $P$ is the power in watts and $R$ is the resistance in ohms. How many more volts are needed to light a 100-watt bulb than a 75-watt bulb if the resistance for both is 110 ohms? $10\sqrt{110} - 5\sqrt{330} \approx 14.05$ volts

* indicates increased difficulty

Practice and Apply

Simplify each expression.

14. $8\sqrt{5} + 3\sqrt{5} 11\sqrt{5}$
15. $3\sqrt{5} + 10\sqrt{6} 13\sqrt{6}$
16. $2\sqrt{15} - 6\sqrt{15} - 3\sqrt{15} - 7\sqrt{15}$
17. $5\sqrt{19} + 6\sqrt{19} - 11\sqrt{19} 0$
18. $16\sqrt{x} + 2\sqrt{x} 18\sqrt{x}$
19. $3\sqrt{5b} - 4\sqrt{5b} + 11\sqrt{5b}$
20. $8\sqrt{3} - 2\sqrt{2} + 3\sqrt{2} + 5\sqrt{3}$
21. $4\sqrt{6} + \sqrt{17} - 6\sqrt{2} + 4\sqrt{17}$
22. $\sqrt{18} + \sqrt{12} + \sqrt{8} 5\sqrt{2} + 2\sqrt{3}$
23. $\sqrt{6} + 2\sqrt{3} + \sqrt{12} \sqrt{6} + 4\sqrt{3}$
24. $3\sqrt{7} - 2\sqrt{28} - \sqrt{7}$
25. $2\sqrt{50} - 3\sqrt{32} - 2\sqrt{2}$
26. $\sqrt{2} + \sqrt{\frac{1}{2}} 3\sqrt{2}$
27. $\sqrt{10} - \frac{2}{\sqrt{5}} 4\sqrt{10}$
28. $3\sqrt{3} - \sqrt{45} + 3\sqrt{\frac{1}{3}} 4\sqrt{3} - 3\sqrt{5}$
29. $6\sqrt{\frac{x}{4}} + 3\sqrt{28} - 10\sqrt{\frac{1}{7}} 5\sqrt{7}$

Find each product.

30. $\sqrt{6\sqrt{3} + 5\sqrt{2}} 3\sqrt{2} + 10\sqrt{3}$
31. $\sqrt{5\sqrt{10} + 3\sqrt{10} + 10\sqrt{2} + 3\sqrt{10}$
32. $(3 + 5\sqrt{3} - \sqrt{5}) 4$ 33. $(7 - \sqrt{10})^2 59 - 14\sqrt{10}$
34. $10\sqrt{3} + 16$ 35. $(\sqrt{5} - \sqrt{2})(\sqrt{14} + \sqrt{35}) 3\sqrt{7}$
36. $(2\sqrt{10} + 3\sqrt{15})(3\sqrt{3} - 2\sqrt{2})$ 37. $(5\sqrt{2} + 3\sqrt{5}/2\sqrt{10} - 3)|$ 19\sqrt{5}$
38. **GEOMETRY** Find the perimeter of a rectangle whose length is 8\sqrt{7} + 4\sqrt{5} inches and whose width is 5\sqrt{7} - 3\sqrt{5} inches. $26\sqrt{7} + 2\sqrt{5}$ in.

www.algebra1.com/extra_examples/sol

Answer

2. The Distributive Property allows you to add like terms. Radicals with like radicands can be added or subtracted.
Simplify each expression.

11. $\sqrt{3} + \sqrt{7}$

12. $\sqrt{5} - \sqrt{2}$

13. $\sqrt{10} + \sqrt{15}$

14. $\sqrt{8} - \sqrt{3}$

15. $\sqrt{20} \cdot \sqrt{5}$

16. $\sqrt{32} \cdot \sqrt{2}$

17. $\sqrt{48} \div \sqrt{3}$

18. $\sqrt{12} \div \sqrt{4}$

19. $\sqrt{18} \cdot \sqrt{2}$

20. $\sqrt{24} \cdot \sqrt{3}$

21. $\sqrt{50} \cdot \sqrt{2}$

22. $\sqrt{72} \cdot \sqrt{3}$

23. $\sqrt{64} \div \sqrt{8}$

24. $\sqrt{100} \div \sqrt{10}$

25. $\sqrt{16} \div \sqrt{4}$

26. $\sqrt{9} \div \sqrt{3}$

27. $\sqrt{25} \div \sqrt{5}$

28. $\sqrt{36} \div \sqrt{6}$

29. $\sqrt{49} \div \sqrt{7}$

30. $\sqrt{64} \div \sqrt{8}$

31. $\sqrt{100} \div \sqrt{10}$

32. $\sqrt{121} \div \sqrt{11}$

33. $\sqrt{144} \div \sqrt{12}$

34. $\sqrt{169} \div \sqrt{13}$

35. $\sqrt{225} \div \sqrt{15}$

36. $\sqrt{361} \div \sqrt{19}$

37. $\sqrt{400} \div \sqrt{20}$

38. $\sqrt{49} \div \sqrt{7}$

39. $\sqrt{64} \div \sqrt{8}$

40. $\sqrt{100} \div \sqrt{10}$

41. $\sqrt{121} \div \sqrt{11}$

42. $\sqrt{144} \div \sqrt{12}$

43. $\sqrt{169} \div \sqrt{13}$

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48. $\sqrt{64} \div \sqrt{8}$

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57. $\sqrt{64} \div \sqrt{8}$

58. $\sqrt{100} \div \sqrt{10}$

59. $\sqrt{121} \div \sqrt{11}$

60. $\sqrt{144} \div \sqrt{12}$

61. $\sqrt{169} \div \sqrt{13}$

62. $\sqrt{225} \div \sqrt{15}$

63. $\sqrt{361} \div \sqrt{19}$

64. $\sqrt{400} \div \sqrt{20}$

65. $\sqrt{49} \div \sqrt{7}$

66. $\sqrt{64} \div \sqrt{8}$

67. $\sqrt{100} \div \sqrt{10}$

68. $\sqrt{121} \div \sqrt{11}$

69. $\sqrt{144} \div \sqrt{12}$

70. $\sqrt{169} \div \sqrt{13}$

71. $\sqrt{225} \div \sqrt{15}$

72. $\sqrt{361} \div \sqrt{19}$

73. $\sqrt{400} \div \sqrt{20}$

74. $\sqrt{49} \div \sqrt{7}$

75. $\sqrt{64} \div \sqrt{8}$

76. $\sqrt{100} \div \sqrt{10}$

77. $\sqrt{121} \div \sqrt{11}$

78. $\sqrt{144} \div \sqrt{12}$

79. $\sqrt{169} \div \sqrt{13}$

80. $\sqrt{225} \div \sqrt{15}$

81. $\sqrt{361} \div \sqrt{19}$

82. $\sqrt{400} \div \sqrt{20}$

83. $\sqrt{49} \div \sqrt{7}$

84. $\sqrt{64} \div \sqrt{8}$

85. $\sqrt{100} \div \sqrt{10}$

86. $\sqrt{121} \div \sqrt{11}$

87. $\sqrt{144} \div \sqrt{12}$

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89. $\sqrt{225} \div \sqrt{15}$

90. $\sqrt{361} \div \sqrt{19}$

91. $\sqrt{400} \div \sqrt{20}$

92. $\sqrt{49} \div \sqrt{7}$

93. $\sqrt{64} \div \sqrt{8}$

94. $\sqrt{100} \div \sqrt{10}$

95. $\sqrt{121} \div \sqrt{11}$

96. $\sqrt{144} \div \sqrt{12}$

97. $\sqrt{169} \div \sqrt{13}$

98. $\sqrt{225} \div \sqrt{15}$

99. $\sqrt{361} \div \sqrt{19}$

100. $\sqrt{400} \div \sqrt{20}$

101. $\sqrt{49} \div \sqrt{7}$

102. $\sqrt{64} \div \sqrt{8}$

103. $\sqrt{100} \div \sqrt{10}$

104. $\sqrt{121} \div \sqrt{11}$

105. $\sqrt{144} \div \sqrt{12}$

106. $\sqrt{169} \div \sqrt{13}$

107. $\sqrt{225} \div \sqrt{15}$

108. $\sqrt{361} \div \sqrt{19}$

109. $\sqrt{400} \div \sqrt{20}$

110. $\sqrt{49} \div \sqrt{7}$

Use the figure above. Write each length on a radical expression in simplest form.

1. hypotenuse $= \sqrt{2}$
2. leg $= \sqrt{3}$
3. leg $= \sqrt{5}$
4. leg $= \sqrt{7}$

**Answer**

42. Approximately 1000 feet; solve $\sqrt{\frac{3(1250)}{2}} - \sqrt{\frac{37}{2}} = 4.57$; may use guess and test, graphical, or analytical methods.
49. **CRITICAL THINKING** Find a counterexample to disprove the following statement.  
   **Sample answer:**  \( a = 4, b = 9: \sqrt{4 + 9} \neq \sqrt{4} + \sqrt{9} \).  
   For any numbers \( a \) and \( b \), where \( a > 0 \) and \( b > 0 \), \( \sqrt{a + b} = \sqrt{a} + \sqrt{b} \).  

50. **CRITICAL THINKING** Under what conditions is \( (\sqrt{a + b})^2 = (\sqrt{a})^2 + (\sqrt{b})^2 \) true? \( a = 0 \) or \( b = 0 \) or both.

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See margin.
   How can you use radical expressions to determine how far a person can see? Include the following in your answer:
   - an explanation of how this information could help determine how far apart lifeguard towers should be on a beach, and
   - an example of a real-life situation where a lookout position is placed at a high point above the ground.

52. Find the difference of \( 9\sqrt{7} \) and \( 2\sqrt{28} \).  
   **C** 
   \( \text{A} \sqrt{7} \quad \text{B} 4\sqrt{7} \quad \text{C} 5\sqrt{7} \quad \text{D} 7\sqrt{7} \)

53. Simplify \( \sqrt{3^4 + 12^2} \).  
   **D** 
   \( \text{A} 4\sqrt{3} + 6 \quad \text{B} 28\sqrt{3} \quad \text{C} 28 + 16\sqrt{3} \quad \text{D} 48 + 28\sqrt{3} \)

---

**Maintain Your Skills**

**Mixed Review**  
54. \( \sqrt{40} \quad 2\sqrt{10} \)

55. \( \sqrt{128} \quad 8\sqrt{2} \)

56. \(-\sqrt{196x^2y^2} \)

57. \( \frac{\sqrt{50}}{\sqrt{8}} \frac{5}{2} \)

58. \( \sqrt{\frac{225a^4b^4}{18a^2b^2}} \times \frac{\sqrt{2a}}{2} \)

59. \( \sqrt{\frac{64a^6b^8}{128a^4b^2}} \frac{3\sqrt{14}}{16} \sqrt{ab} \)

Find the \( n \)th term of each geometric sequence.  
60. \( a_1 = 4, n = 6, r = 4 \quad \text{61.} \quad a_1 = -\frac{7}{11}, n = 4, r = 9 \quad \text{62.} \quad a_1 = 2, n = 8, r = -0.8 \)

63. \( 81 = 49y^2 \{ \pm \frac{9}{7} \} \)

64. \( q^2 - \frac{36}{121} = 0 \{ \pm \frac{6}{11} \} \)

65. \( 48n^3 - 75n = 0 \{ -\frac{5}{4}, 0, \frac{5}{4} \} \)

66. \( 5x^3 - 80x = 240 - 15x^2 \{ -4, -3, 4 \} \)

Solve each inequality. Then check your solution.  
67. \( 8n \geq 5 \quad n \geq \frac{5}{8} \)

68. \( \frac{11}{9} < 14 \quad w < 126 \)

69. \( \frac{7k}{2} > \frac{21}{10} \quad k > \frac{3}{5} \)

70. **PROBABILITY** A student rolls a die three times. What is the probability that each roll is a 1?  
   \( \text{Lesson 2-6} \quad \frac{1}{216} \)

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**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find each product.  
(To review special products, see Lesson 8-8.)  
71. \( (x - 2)^2 \quad x^2 - 4x + 4 \)

72. \( (x + 5)^2 \quad x^2 + 10x + 25 \)

73. \( (x + 6)^2 \quad x^2 + 12x + 36 \)

74. \( (3x - 1)^2 \quad 9x^2 - 6x + 1 \)

75. \( (2x - 3)^2 \quad 4x^2 - 12x + 9 \)

76. \( (4x + 7)^2 \quad 16x^2 + 56x + 49 \)

---

**Open-Ended Assessment**

**Speaking** Ask students to compare and contrast adding, subtracting, and multiplying radical expressions and variable expressions. What are the similarities? What are the differences? Allow students to give examples on the chalkboard if they wish.

**Getting Ready for Lesson 11-3**

**PREREQUISITE SKILL** Students will learn how to solve radical equations in Lesson 11-3. In order to solve radical equations, students will need to be able to recognize and find special products. Use Exercises 71–76 to determine your students’ familiarity with finding special products.

**Assessment Options**

**Quiz (Lessons 11-1 and 11-2)** is available on p. 699 of the Chapter 11 Resource Masters.

**Answer**

51. The distance a person can see is related to the height of the person using \( d = \sqrt{\frac{3h}{2}} \). Answers should include the following.
   - You can find how far each lifeguard can see from the height of the lifeguard tower. Each tower should have some overlap to cover the entire beach area.
   - On early ships, a lookout position (Crow’s nest) was situated high on the foremast. Sailors could see farther from this position than from the ship’s deck.
5-Minute Check

Transparency 11-3 Use as a quiz or review of Lesson 11-2.

Mathematical Background notes are available for this lesson on p. 584D.

How are radical equations used to find free-fall times?

Ask students:

• Is \( \sqrt{h} \) in simplest form? Explain. Yes, since there is no radical in the denominator, the expression is in simplest form.

• How would you solve \( t = \frac{x}{4} \) for \( x \)? Multiply each side by 4.

• What do you think might be a way to remove the radical sign from \( 4t = \sqrt{h} \)? Square each side of the equation.

Vocabulary

- radical equation
- extraneous solution

\[ t = \frac{\sqrt{h}}{4} \]

Radical Equation with a Variable

FREE-FALL HEIGHT Two objects are dropped simultaneously. The first object reaches the ground in 2.5 seconds, and the second object reaches the ground 1.5 seconds later. From what heights were the two objects dropped?

Find the height of the first object. Replace \( t \) with 2.5 seconds.

\[ t = \frac{\sqrt{h}}{4} \]

Original equation

2.5 = \( \frac{\sqrt{h}}{4} \) Replace \( t \) with 2.5.

10 = \( \sqrt{h} \) Multiply each side by 4.

\( 10^2 = (\sqrt{h})^2 \) Square each side.

100 = \( h \) Simplify.

CHECK \( t = \frac{\sqrt{h}}{4} \)

Original equation

\[ t \geq \frac{\sqrt{100}}{4} \]

\( h = 100 \)

\[ t \geq \frac{10}{4} \]

\( \sqrt{100} = 10 \)

\( t = 2.5 \) Simplify.

The first object was dropped from 100 feet.
Example 2 Radical Equation with an Expression

Solve $\sqrt{x + 1} + 7 = 10$.

1. Original equation:
   $\sqrt{x + 1} + 7 = 10$

2. Subtract 7 from each side:
   $\sqrt{x + 1} = 3$

3. Square each side:
   $\sqrt{x + 1}^2 = 3^2$
   $x + 1 = 9$

4. Subtract 1 from each side:
   $x = 8$

The solution is 8. Check this result.

EXTRANEOUS SOLUTIONS Squaring each side of an equation sometimes produces extraneous solutions. An extraneous solution is a solution derived from an equation that is not a solution of the original equation. Therefore, you must check all solutions in the original equation when you solve radical equations.

Example 3 Variable on Each Side

Solve $\sqrt{x + 2} = x - 4$.

1. Original equation:
   $\sqrt{x + 2} = x - 4$

2. Square each side:
   $\sqrt{x + 2}^2 = (x - 4)^2$
   $x + 2 = x^2 - 8x + 16$

3. Subtract $x$ and 2 from each side:
   $0 = x^2 - 9x + 14$

4. Factor:
   $0 = (x - 7)(x - 2)$

5. Solve:
   $x = 7$ or $x = 2$

CHECK $\sqrt{7 + 2} = 7 - 4$ $\sqrt{9} = 3$

Since 2 does not satisfy the original equation, 7 is the only solution.

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Lesson 11-3 Radical Equations 599

Differentiated Instruction

Verbal/Linguistic Have students write a short paragraph in their own words explaining why checking solutions is important when solving radical equations. Students should include an example of a radical equation that has an extraneous solution.
### Concept Check

1. **Describe** the steps needed to solve a radical equation.
2. **Explain** why it is necessary to check for extraneous solutions in radical equations. The solution may not satisfy the original equation.
3. **OPEN ENDED** Give an example of a radical equation. Then solve the equation for the variable. **Sample answer:** \( \sqrt{x + 1} = 8, 63 \)
4. **FIND THE ERROR** Alex and Victor are solving \( \sqrt{x - 5} = -2 \).

#### Alex
- \( \sqrt{x - 5} = -2 \)
- \( (-\sqrt{x - 5})^2 = (-2)^2 \)
- \( x - 5 = 4 \)
- \( x = 9 \)

#### Victor
- \( -\sqrt{x - 5} = -2 \)
- \( (-\sqrt{x - 5})^2 = (-2)^2 \)
- \( x - 5 = 4 \)
- \( x = 9 \)

Who is correct? Explain your reasoning.

#### Guided Practice

Solve each equation. Check your solution.

5. \( \sqrt{x} = 5 \) \( 25 \)
6. \( \sqrt{2b} = -8 \) no solution
7. \( \sqrt{7x} = 7 \) \( 7 \)
8. \( \sqrt{-3a} = 6 \) \( -12 \)
9. \( \sqrt{8s + 1} = 5 \) \( 2 \)
10. \( \sqrt{7x + 18} = 9 \) \( 9 \)
11. \( \sqrt{5x + 1} + 2 = 6 \) \( 3 \)
12. \( \sqrt{6x - 8} = x - 4 \) \( 6 \)
13. \( 4 + \sqrt{x - 2} = x \) \( 6 \)

#### Application

**OCEANS** For Exercises 14–16, use the following information.

Tsunamis, or large tidal waves, are generated by undersea earthquakes in the Pacific Ocean. The speed of the tsunami in meters per second is \( s = 3.1 \sqrt{d} \), where \( d \) is the depth of the ocean in meters. 

15. Find the speed of the tsunami if the depth of the water is 10 meters. **about 9.8 m/s**
16. Find the depth of the water if a tsunami’s speed is 240 meters per second. 
17. A tsunami may begin as a 2-foot high wave traveling 450–500 miles per hour. It can approach a coastline as a 50-foot wave. How much speed does the wave lose if it travels from a depth of 10,000 meters to a depth of 20 meters? **approximately 296 m/s**

### Practice/Apply

#### Study Notebook

- **Include examples of how to solve radical equations.**
- **Include any other item(s) that they find helpful in mastering the skills in this lesson.**

#### DAILY INTERVENTION

**FIND THE ERROR** This problem may require close inspection for students to find the error. Tell students to focus on checking the given solutions. One solution produces an impossible radicand. Also point out that Alex and Victor could have begun by multiplying each side by \(-1\) to eliminate the negative signs.

#### Answer

1. 

2. \((10, 5)\)
3. \(x = 10\); the solution is the same as the solution from the graph. However, when solving algebraically, you have to check that \(x = 3\) is an extraneous solution.
4. Alex; the square of \(-\sqrt{x - 5}\) is \(x - 5\).
Solve each equation. Check your solution.

17. \( \sqrt{a} = 10 \) 100  
19. \( 5\sqrt{2} = \sqrt{x} \) 50  
21. \( 3\sqrt{4\pi} - 2 = 10 \) 4  
23. \( \sqrt{x + 3} = -5 \) no solution  
25. \( \sqrt{3x + 12} = 3\sqrt{3} \) 5  
27. \( \sqrt{4b} + 1 = 3 \) 2  
29. \( \sqrt[4]{x^2} - 9 = 3 \) 180  
31. \( \sqrt{x^2 + 9x + 14} = x + 4 \) 2

33. The square root of the sum of a number and 7 is 8. Find the number. 57

34. The square root of the quotient of a number and 6 is 9. Find the number. 486

Solve each equation. Check your solution.

35. \( x = \sqrt{6 - x} \) 2  
37. \( \sqrt{5x - 6} = x \) 2, 3  
39. \( \sqrt{x + 1} = x - 1 \) 3  
41. \( 4 + \sqrt{m} - 2 = m \) 6  
43. \( x + \sqrt{6 - x} = 4 \) 2  
45. \( \sqrt{2x^2 - 121} = r \) 11

47. State whether the following equation is sometimes, always, or never true. \( \sqrt{(x - 5)^2} = x - 5 \) sometimes

AVIATION For Exercises 48 and 49, use the following information.

The formula \( L = \sqrt{kP} \) represents the relationship between a plane’s length \( L \) in feet and the pounds \( P \) its wings can lift, where \( k \) is a constant of proportionality calculated for a plane.

48. The length of the Douglas D-558-II, called the Skyrocket, was approximately 42 feet, and its constant of proportionality was \( k = 0.1669 \). Calculate the maximum takeoff weight of the Skyrocket. 10,569 lb

49. A Boeing 747 is 232 feet long and has a takeoff weight of 870,000 pounds. Determine the value of \( k \) for this plane. about 0.0619

GEOMETRY For Exercises 50–53, use the figure below. The area \( A \) of a circle is equal to \( \pi r^2 \) where \( r \) is the radius of the circle.

50. Write an equation for \( r \) in terms of \( A \). \( r = \sqrt{\frac{A}{\pi}} \)

51. The area of the larger circle is 96\( \pi \) square meters. Find the radius. 4\( \sqrt{6} \) or about 9.8 m

52. The area of the smaller circle is 48\( \pi \) square meters. Find the radius. 4\( \sqrt{3} \) or about 6.9 m

53. If the area of a circle is doubled, what is the change in the radius? It increases by a factor of \( \sqrt{2} \).
Radical Equations

To solve a radical equation, one first isolates the radical on one side of the equation. Why you then square each side of the equation.

Example

\[ \sqrt{5x - 3} = 1 \]

1. \[ \sqrt{5x - 3} = 1 \]
   \[ \text{Original equation} \]
2. \[ 5x - 3 = 1 \]
   \[ \text{Square each side} \]
3. \[ 5x = 4 \]
   \[ \text{Simplify} \]
4. \[ x = \frac{4}{5} \]
   \[ \text{Solution} \]

HELPING YOU REMEMBER

1. How do you determine whether an equation has extraneous solutions?

   Substitute the solution(s) into the original equation. If a solution does not satisfy the original equation, then it is an extraneous solution.

   It is necessary to check all solutions to eliminate extraneous solutions. Remember, you square each side of a radical equation, and squaring each side can sometimes produce an extraneous solution, you need to check all solutions. The only way to be sure that a solution is not extraneous is to check it in the original equation.

     \[ \sqrt{5x - 3} = 1 \]
   
   \[ x = \frac{4}{5} \]

     \[ \text{Solution} \]

     \[ \text{Check} \]

2. How do you determine whether an equation has extraneous solutions?

   Substitute the solution(s) into the original equation. If a solution does not satisfy the original equation, then it is an extraneous solution.

   It is necessary to check all solutions to eliminate extraneous solutions. Remember, you square each side of a radical equation, and squaring each side can sometimes produce an extraneous solution, you need to check all solutions. The only way to be sure that a solution is not extraneous is to check it in the original equation.

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   \[ \sqrt{5x - 3} = 1 \]

   \[ x = \frac{4}{5} \]

   \[ \text{Solution} \]

   \[ \text{Check} \]

2. How do you determine whether an equation has extraneous solutions?

   Substitute the solution(s) into the original equation. If a solution does not satisfy the original equation, then it is an extraneous solution.

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   \[ \sqrt{5x - 3} = 1 \]

   \[ x = \frac{4}{5} \]

   \[ \text{Solution} \]

   \[ \text{Check} \]

2. How do you determine whether an equation has extraneous solutions?

   Substitute the solution(s) into the original equation. If a solution does not satisfy the original equation, then it is an extraneous solution.

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   \[ \sqrt{5x - 3} = 1 \]

   \[ x = \frac{4}{5} \]

   \[ \text{Solution} \]

   \[ \text{Check} \]
Graphing Calculator

**RADICAL EQUATIONS** Use a graphing calculator to solve each radical equation. Round to the nearest hundredth.

64. \( 3 + \sqrt{2x} = 7 \) 8  
65. \( \sqrt{3x - 8} = 5 \) 11  
66. \( \sqrt{x + 6} - 4 = x \) -2  
67. \( \sqrt{4x + 5} = x - 7 \) 15.08  
68. \( x + \sqrt{7 - x} = 4 \) 1.70  
69. \( \sqrt{3x - 9} = 2x + 6 \) no solution

**Maintain Your Skills**

**Mixed Review** (Lesson 11-1)

70. \( 5\sqrt[5]{6} + 12\sqrt{6} \) 17\( \sqrt[6]{6} \) 71. \( \sqrt{12} + 6\sqrt{27} \) 20\( \sqrt{3} \) 72. \( \sqrt{18} + 5\sqrt{2} - 3\sqrt{32} \) -4\( \sqrt{2} \)

73. \( \sqrt{192} \) 8\( \sqrt{3} \) 74. \( \sqrt{6} \cdot \sqrt{10} \) 2\( \sqrt{15} \) 75. \( \frac{21}{\sqrt{10} + \sqrt{3}} \)

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

(Lesson 9-6)

76. \( a^2 + 50a + 225 \) no 77. \( 4n^2 - 28n + 49 \) yes; \( (2n - 7)^2 \) 78. \( 16b^2 - 56bc + 49c^2 \) yes; \( (4b - 7c)^2 \)

(Lesson 8-7)

79. \( (r + 3)(r - 4) \) 80. \( (3z + 7)(2z + 10) \) 81. \( (2p + 5)(3p^2 - 4p + 9) \)

\( r^2 - r - 12 \) 82. \( 6z^2 + 44z + 70 \) \( 6p^3 + 7p^2 - 2p + 45 \)

82. PHYSICAL SCIENCE A European-made hot tub is advertised to have a temperature of 35°C to 40°C, inclusive. What is the temperature range for the hot tub in degrees Fahrenheit? Use \( F = \frac{9}{5}C + 32 \). (Lesson 6-4) \( 95^\circ \leq F \leq 104^\circ \)

Write each equation in standard form. (Lesson 5-5)

83. \( y = 2x + \frac{3}{14x - 7} = -3 \) 84. \( y - 3 = -2(x - 6) \) 85. \( y + 2 = 7.5(x - 3) \)

86. \( 2x + y = 15 \) 10x - 2y = 49

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Evaluate \( \sqrt{a^2 + b^2} \) for each value of \( a \) and \( b \).

(To review evaluating expressions, see Lesson 1-2.)

86. \( a = 3, b = 4 \) 87. \( a = 24, b = 7 \) 88. \( a = 1, b = 1 \) 89. \( a = 8, b = 12 \) 4\( \sqrt{13} \)

**Practice Quiz 1**

**Lessons 11-1 through 11-3**

Simplify. (Lesson 11-1)

1. \( \sqrt{48} \) 4\( \sqrt{3} \) 2. \( \sqrt{3} \cdot \sqrt{6} \) 3\( \sqrt[2]{2} \) 3. \( \frac{3}{2 + \sqrt{10}} \) -2 + \( \sqrt{10} \) 2

Simplify. (Lesson 11-2)

4. \( 6\sqrt[5]{5} + 3\sqrt[2]{5} + 5\sqrt[5]{5} \) 5. \( 2\sqrt[5]{3} + 9\sqrt[12]{12} \) 20\( \sqrt[3]{3} \) 6. \( 3 - \sqrt{6}^2 \) 15 - 6\( \sqrt{6} \)

7. GEOMETRY Find the area of a square whose side measure is \( 2 + \sqrt{7} \) centimeters. (Lesson 11-2) \( 11 + 4\sqrt{7} \) or 21.6 cm²

Solve each equation. Check your solution. (Lesson 11-3)

8. \( \sqrt{15} - x = 4 \) -1 9. \( \sqrt{3x^2 - 32} = x \) 4 10. \( \sqrt{2x - 1} = 2x - 7 \) 5

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Lesson 11-3 Radical Equations 603

**Open-Ended Assessment**

**Modeling** Write radical equations on the overhead projector, using a cut-out overlay for the radical. Have students explain what must be done to the equation in order to “remove” the radical. When students explain the correct procedure, remove the overlay and solve the equation.

**Getting Ready for Lesson 11-4**

**PREREQUISITE SKILL** Students will learn about the Pythagorean Theorem in Lesson 11-4. In order to use the Pythagorean Theorem, students must be able to evaluate radical expressions for given values. Use Exercises 86–89 to determine your students’ familiarity with evaluating radical expressions.

**Assessment Options**

**Practice Quiz 1** The quiz provides students with a brief review of the concepts and skills in Lessons 11-1 through 11-3. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

**Answer**

61. You can determine the time it takes an object to fall from a given height using a radical equation. Answers should include the following.

- It would take a skydiver approximately 42 seconds to fall 10,000 feet. Using the equation, it would take 25 seconds. The time is different in the two calculations because air resistance slows the skydiver.

- A skydiver can increase the speed of his fall by lowering air resistance. This can be done by pulling his arms and legs close to his body. A skydiver can decrease his speed by holding his arms and legs out, which increases the air resistance.
Graphs of Radical Equations

In order for a square root to be a real number, the radicand cannot be negative. When graphing a radical equation, determine when the radicand would be negative and exclude those values from the domain.

Example 1
Graph \( y = \sqrt{x} \). State the domain of the graph.
Enter the equation in the \( Y= \) list.

**KEYSTROKES:**
\[
\text{\texttt{Y= 2nd [\sqrt] X,T,\theta,n 41 GRAPH}}
\]

From the graph, you can see that the domain of \( x \) is \( \{x \mid x \geq 0\} \).

Example 2
Graph \( y = \sqrt{x + 4} \). State the domain of the graph.
Enter the equation in the \( Y= \) list.

**KEYSTROKES:**
\[
\text{\texttt{Y= 2nd [\sqrt] X,T,\theta,n + 4 41 GRAPH}}
\]

The value of the radicand will be positive when \( x + 4 \geq 0 \), or when \( x \geq -4 \). So the domain of \( x \) is \( \{x \mid x \geq -4\} \).

This graph looks like the graph of \( y = \sqrt{x} \) shifted left 4 units.

**Exercises** 1–9. See pp. 639A–639B.
Graph each equation and sketch the graph on your paper. State the domain of the graph. Then describe how the graph differs from the parent function \( y = \sqrt{x} \).

1. \( y = \sqrt{x + 1} \)  
2. \( y = \sqrt{x - 3} \)  
3. \( y = \sqrt{x + 2} \)
4. \( y = \sqrt{x - 5} \)  
5. \( y = \sqrt{-x} \)  
6. \( y = \sqrt{3x} \)
7. \( y = -\sqrt{x} \)  
8. \( y = \sqrt{1 - x + 6} \)  
9. \( y = \sqrt{2x + 5 - 4} \)

10. Is the graph of \( y = x^2 \) a function? Explain your reasoning. 10–11. See margin.

11. Does the equation \( x^2 + y^2 = 1 \) determine \( y \) as a function of \( x \)? Explain.

12. Graph \( y = \frac{|x|}{\sqrt{1 - x^2}} \) in the window defined by \([-2, 2] \text{ scl: 1 by } [-2, 2] \text{ scl: 1}. Describe the graph. heart

**Answers**

10. No; you must consider the graph of \( y = \sqrt{x} \) and the graph of \( y = -\sqrt{x} \). This graph fails the vertical line test. For every value of \( x > 0 \), there are two values for \( y \).

11. No; the equation \( y = \frac{\pm \sqrt{1 - x^2}}{x} \) is not a function since there are both positive and negative values for \( y \) for each value of \( x \).
Vocabulary
- hypotenuse
- legs
- Pythagorean triple
- corollary

Study Tip
Triangles
Sides of a triangle are represented by lowercase letters \( a, b, \) and \( c \).

Mathematical Background
notes are available for this lesson on p. 584D.

is the Pythagorean Theorem used in roller coaster design?
The roller coaster Superman: Ride of Steel in Agawam, Massachusetts, is one of the world’s tallest roller coasters at 208 feet. It also boasts one of the world’s steepest drops, measured at 78 degrees, and it reaches a maximum speed of 77 miles per hour. You can use the Pythagorean Theorem to estimate the length of the first hill.

The Pythagorean Theorem
In a right triangle, the side opposite the right angle is called the hypotenuse. This side is always the longest side of a right triangle. The other two sides are called the legs of the triangle.

To find the length of any side of a right triangle when the lengths of the other two are known, you can use a formula developed by the Greek mathematician Pythagoras.

**Example 1** Find the Length of the Hypotenuse
Find the length of the hypotenuse of a right triangle if \( a = 8 \) and \( b = 15 \).

\[
\begin{align*}
\text{Pythagorean Theorem} & \quad c^2 = a^2 + b^2 \\
\text{Given:} & \quad a = 8, b = 15 \\
\text{Simplify:} & \quad c^2 = 8^2 + 15^2 \\
& \quad c^2 = 64 + 225 \\
& \quad c^2 = 289 \\
& \quad c = \pm \sqrt{289} \\
& \quad c = \pm 17
\end{align*}
\]

The length of the hypotenuse is 17 units.
In-Class Examples

1. Find the length of the hypotenuse of a right triangle if \( a = 18 \) and \( b = 24 \). The length of the hypotenuse is 30 units.

2. Find the length of the missing side.

3. What is the area of triangle \( XYZ \)?

A. 94 units\(^2\)  
B. 128 units\(^2\)  
C. 294 units\(^2\)  
D. 588 units\(^2\)

**Concept Check**

Pythagorean Theorem Ask students to rewrite the formula for the Pythagorean Theorem in terms of \( c \). \( c = \sqrt{a^2 + b^2} \)

**Example 2 Find the Length of a Side**

Find the length of the missing side.

In the triangle, \( c = 25 \) and \( b = 10 \) units.

\[
\begin{align*}
\frac{c^2}{a^2} &= a^2 + b^2 & \text{Pythagorean Theorem} \\
25^2 &= a^2 + 10^2 & b = 10 \text{ and } c = 25 \\
625 &= a^2 + 100 & \text{Evaluate squares.} \\
525 &= a^2 & \text{Subtract 100 from each side.} \\
\pm \sqrt{525} &= a & \text{Use a calculator to evaluate } \sqrt{525}. \\
22.91 &= a & \text{Use the positive value.}
\end{align*}
\]

To the nearest hundredth, the length of the leg is 22.91 units.

**Example 3 Pythagorean Triples**

Multiple-Choice Test Item

What is the area of triangle \( ABC \)?

A. 96 units\(^2\)  
B. 120 units\(^2\)  
C. 160 units\(^2\)  
D. 196 units\(^2\)

Read the Test Item

The area of a triangle is \( A = \frac{1}{2}bh \). In a right triangle, the legs form the base and height of the triangle. Use the measures of the hypotenuse and the base to find the height of the triangle.

Solve the Test Item

**Step 1** Check to see if the measurements of this triangle are a multiple of a common Pythagorean triple. The hypotenuse is 4 \( \times \) 5 units, and the leg is 4 \( \times \) 3 units. This triangle is a multiple of a (3, 4, 5) triangle.

\[
\begin{align*}
4 \cdot 3 &= 12 \\
4 \cdot 4 &= 16 \\
4 \cdot 5 &= 20
\end{align*}
\]

The height of the triangle is 16 units.

**Step 2** Find the area of the triangle.

\[
\begin{align*}
A &= \frac{1}{2}bh & \text{Area of a triangle} \\
A &= \frac{1}{2} \cdot 12 \cdot 16 & b = 12 \text{ and } h = 16 \\
A &= 96 & \text{Simplify}
\end{align*}
\]

The area of the triangle is 96 square units. Choice A is correct.

**Standardized Test Practice**

Example 3 Caution students that this is a two-step problem. First they must find the missing side length, then they must calculate the area of the triangle. Also stress to students that it is important to read each answer choice carefully before selecting the correct answer. In Example 3, choices B and C look very similar, and one could mistakenly choose D, 196, when the correct answer is A, 96.
Lesson 11-4  The Pythagorean Theorem

**In-Class Example**

**Teaching Tip** Point out to students that with Pythagorean triples, the greatest value is always the measure of the hypotenuse, and the two lesser values are the measures of the two legs.

4. Determine whether the following side measures form right triangles.
   
   a. 7, 12, 15  **not a right triangle**
   
   b. 27, 36, 45  **right triangle**

**Answers**

1. 

   ![Diagram of a right triangle with hypotenuse and legs]

   2. Compare the lengths of the sides. The hypotenuse is the longest side, which is always the side opposite the right angle.

**Check for Understanding**

1. **OPEN ENDED** Draw a right triangle and label each side and angle. Be sure to indicate the right angle.  **See margin.**

2. **Explain** how you can determine which angle is the right angle of a right triangle if you are given the lengths of the three sides.  **See margin.**

3. **Write** an equation you could use to find the length of the diagonal \( d \) of a square with side length \( s \).  \[ d = \sqrt{2s^2} \text{ or } d = s\sqrt{2} \]

**Guided Practice**

Find the length of each missing side. If necessary, round to the nearest hundredth.

4.  

   ![Diagram with sides 12 and 14]

   18.44

5.  

   ![Diagram with sides 40 and 9]

   41

---

**Differentiated Instruction**

**Kinesthetic** Give students blocks, algebra tiles, or some other square or cube-shaped manipulatives. Have them use the manipulatives and what they know about the Pythagorean Theorem and Pythagorean triples to construct a right triangle, placing the manipulatives along the outside of the triangle. Have students construct the triangle on a piece of paper, then trace inside the blocks to transfer the triangle to the paper. Then ask them to use a protractor to confirm that the right angle is 90°.

---

**Right Triangles**

A statement that can be easily proved using a theorem is often called a **corollary**. The following corollary, based on the Pythagorean Theorem, can be used to determine whether a triangle is a right triangle.

**Key Concept**

**Corollary to the Pythagorean Theorem**

If \( a \) and \( b \) are measures of the shorter sides of a triangle, \( c \) is the measure of the longest side, and \( c^2 = a^2 + b^2 \), then the triangle is a right triangle.

If \( c^2 \neq a^2 + b^2 \), then the triangle is not a right triangle.

**Example 4 Check for Right Triangles**

Determine whether the following side measures form right triangles.

a. 20, 21, 29

Since the measure of the longest side is 29, let \( c = 29 \), \( a = 20 \), and \( b = 21 \).

Then determine whether \( c^2 = a^2 + b^2 \).

\[
\begin{align*}
29^2 & = 20^2 + 21^2 \\
841 & = 400 + 441 \\
841 & = 841 \quad \text{Add.}
\end{align*}
\]

Since \( c^2 = a^2 + b^2 \), the triangle is a right triangle.

b. 8, 10, 12

Since the measure of the longest side is 12, let \( c = 12 \), \( a = 8 \), and \( b = 10 \).

Then determine whether \( c^2 = a^2 + b^2 \).

\[
\begin{align*}
12^2 & = 8^2 + 10^2 \\
144 & = 64 + 100 \\
144 & \neq 164 \quad \text{Add.}
\end{align*}
\]

Since \( c^2 \neq a^2 + b^2 \), the triangle is not a right triangle.
If $c$ is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

6. $a = 10, b = 24, c = ?$  
7. $a = 11, c = 61, b = ?$  
8. $b = 13, c = \sqrt{233}, a = ?$  
9. $a = 7, b = 4, c = ? \sqrt{65} \approx 8.06$

Determine whether the following side measures form right triangles. Justify your answer.

10. $4, 6, 9$ No; $4^2 + 6^2 \neq 9^2$.
11. $16, 30, 34$ Yes; $16^2 + 30^2 = 34^2$.

12. In right triangle $XYZ$, the length of $YZ$ is 6, and the length of the hypotenuse is 8. Find the area of the triangle. $A$

| A | 6$\sqrt{7}$ units$^2$ | B | 30 units$^2$ | C | 40 units$^2$ | D | 48 units$^2$ |

Find the length of each missing side. If necessary, round to the nearest hundredth.

13. 14. 15. 28 53

14. 17

15. 20

16. 19. $a = 16, b = 63, c = ?$  
20. $a = 16, c = 34, b = ?$  
21. $b = 3, a = \sqrt{112}, c = ?$  
22. $a = \sqrt{15}, b = \sqrt{10}, c = ?$  
23. $c = 14, a = 9, b = ? \sqrt{115} \approx 10.72$  
24. $a = 6, b = 3, c = ? \sqrt{45} \approx 6.71$  
25. $b = \sqrt{77}, c = 12, a = ? \sqrt{67} \approx 8.19$  
26. $a = 4, b = \sqrt{11}, c = ? \sqrt{27} \approx 5.20$  
27. $a = \sqrt{225}, b = \sqrt{28}, c = ?$  
28. $a = \sqrt{31}, c = \sqrt{155}, b = ?$  
29. $a = 8x, b = 15x, c = ? 17x$  
30. $b = 3x, c = 7x, a = ? \sqrt{40}x \approx 6.32x$

Determine whether the following side measures form right triangles. Justify your answer.

31. 30, 40, 50 Yes; $30^2 + 40^2 = 50^2$.
32. 6, 12, 18 No; $6^2 + 12^2 \neq 18^2$.
33. 24, 30, 36 No; $24^2 + 30^2 \neq 36^2$.
34. 45, 60, 75 Yes; $45^2 + 60^2 = 75^2$.
35. 15, $\sqrt{31}, 16$ Yes; $15^2 + (\sqrt{31})^2 = 16^2$.

Use an equation to solve each problem. If necessary, round to the nearest hundredth.

37. Find the length of a diagonal of a square if its area is 162 square feet. $18$ ft

Answer

48. The area of the largest semicircle is $\frac{\pi c^2}{4} = \frac{\pi}{4}c^2$.

The sum of the other two areas is $\frac{\pi}{4}(a^2 + b^2)$. Using the Pythagorean Theorem, $c^2 = a^2 + b^2$, we can show that the sum of the two small areas is equal to the area of the largest semicircle.
38. A right triangle has one leg that is 5 centimeters longer than the other leg. The hypotenuse is 25 centimeters long. Find the length of each leg of the triangle. 15 cm, 20 cm

39. Find the length of the diagonal of the cube if each side of the cube is 4 inches long. $4\sqrt{3}$ in. or about 6.93 in.

40. The ratio of the length of the hypotenuse to the length of the shorter leg in a right triangle is 8:3. The hypotenuse measures 144 meters. Find the length of the longer leg, about 112.41 m

45. SAILING A sailboat’s mast and boom form a right angle. The sail itself, called a mainsail, is in the shape of a right triangle. If the edge of the mainsail that is attached to the mast is 100 feet long and the edge of the mainsail that is attached to the boom is 60 feet long, what is the length of the longest edge of the mainsail? about 112.6 ft

46. Determine the missing length shown in the rafter. 13 ft

47. If the roof is 30 feet long and it hangs an additional 2 feet over the garage walls, how many square feet of shingles are needed for the entire garage roof? 900 ft²

48. CRITICAL THINKING Compare the area of the largest semicircle to the areas of the two smaller semicircles. Justify your reasoning. See margin.

**Roller Coasters** For Exercises 41–43, use the following information and the figure.

Suppose a roller coaster climbs 208 feet higher than its starting point making a horizontal advance of 360 feet. When it comes down, it makes a horizontal advance of 44 feet.

41. How far will it travel to get to the top of the ride? about 415.8 ft

42. How far will it travel on the downhill track? about 212.6 ft

43. Compare the total horizontal advance, vertical height, and total track length. The roller coaster makes a total horizontal advance of 404 feet, reaches a vertical height of 208 feet, and travels a total track length of about 628.4 feet.

**RESEARCH** Use the Internet or other reference to find the measurements of your favorite roller coaster or a roller coaster that is at an amusement park close to you. Draw a model of the first drop. Include the height of the hill, length of the vertical drop, and steepness of the hill. See students’ work.

**Study Guide and Intervention, p. 651 and p. 662**

**Skills Practice, p. 663 and Practice, p. 664 (shown)**

Find the length of each missing side. If necessary, round to the nearest hundredth.

1. $a \approx 15.65$ ft

2. $c \approx 18.71$ ft

**Reading to Learn Mathematics, p. 665**

Pre-Activity How is the Pythagorean Theorem used in roller coaster design?

Read the introduction to Lesson 11-4 at the top of page 651 in your text.

The diagram in the introduction shows a right triangle and part of the roller coaster. Which side of the right triangle has a length approximately equal to the length of the first hill of the roller coaster?

The longest side, which is the side opposite the right angle

**Reading the Lesson**

Complete each sentence.

1. The words leg and hypotenuse refer to the sides of a **right triangle**.

2. In a right triangle, each of the two sides that form the right angle is a **leg** of the right triangle.

3. The longest side of a right triangle is called the **hypotenuse** of the right triangle.

Write an equation that you could solve to find the missing side length of each right triangle.

4. $a^2 + b^2 = c^2$

5. $a = \sqrt{c^2 - b^2}$

Sample answer: Put the numbers in order from least to greatest. Square each number. Use the integers 3, 4, and 5 to satisfy the Pythagorean Theorem. If the hypotenuse is $c$, then $c = \sqrt{3^2 + 4^2}$.

Helping You Remember

K. Think of a cereal box that you can associate with the Pythagorean Theorem to help you remember the equation $a^2 + b^2 = c^2$.

Sample answer: Think of the word “eat.” The letters of the word are the same as those in the equation $a^2 + b^2 = c^2$.
4. **Assess**

**Open-Ended Assessment**

**Modeling** Have students use pencils, uncooked spaghetti, or twigs to model right triangles. Ask students to explain the Pythagorean Theorem and relate it to their triangle.

**Getting Ready for Lesson 11-5**

**PREREQUISITE SKILL** Students will learn about the Distance Formula in Lesson 11-5. In order to use the Distance Formula, students must be able to simplify radical expressions. Use Exercises 63–68 to determine your students' familiarity with simplifying radical expressions.

**Assessment Options**

**Quiz (Lessons 11-3 and 11-4)** is available on p. 699 of the Chapter 11 Resource Masters.

**Mid-Chapter Test (Lessons 11-1 through 11-4)** is available on p. 701 of the Chapter 11 Resource Masters.

**Answer**

50. Engineers can use the Pythagorean Theorem to find the total length of the track, which determines how much material and land area they need to build the attraction. Answers should include the following.

- A tall hill requires more track length both going uphill and downhill, which will add to the total length of the tracks. Tall, steep hills will increase the speed of the roller coaster. So a coaster with a tall, steep first hill will have more speed and a longer track length.
- The steepness of the hill and speed are limited for safety and to keep the cars on the track.

**Maintain Your Skills**

**Mixed Review**

- **Solve each equation. Check your solution.** (Lesson 11-3)
  - 53. \( \sqrt{y} = 12 \)  
  - 54. \( 3 \sqrt{s} = 126 \)  
  - 55. \( 4 \sqrt{2v} + 1 = 3 = 17 \)

- **Simplify each expression.** (Lesson 11-2)
  - 56. \( \sqrt{72} \)  
  - 57. \( 7 \sqrt{z} - 10 \sqrt{z} \)  
  - 58. \( \sqrt{\frac{3}{7}} + \sqrt{21} \)

- **Simplify. Assume that no denominator is equal to zero.** (Lesson 8-2)
  - 59. \( \frac{5^7}{5^5} \)  
  - 60. \( d^{-7} \)  
  - 61. \( \frac{-26a^4b^7c^5}{-13a^2b^6c^3} \)

- **AVIATION** Flying with the wind, a plane travels 300 miles in 40 minutes. Flying against the wind, it travels 300 miles in 45 minutes. Find the air speed of the plane. (Lesson 7-4) \( 425 \) mph

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Simplify each expression.

- **To review simplifying radical expressions, see Lesson 11-1.**
  - 63. \( \sqrt{(6 - 3)^2 + (8 - 4)^2} \)  
  - 64. \( \sqrt{(10 - 4)^2 + (13 - 5)^2} \)
  - 65. \( \sqrt{(5 - 3)^2 + (2 - 9)^2} \)  
  - 66. \( \sqrt{(-9 - 5)^2 + (7 - 3)^2} \)
  - 67. \( \sqrt{(-4 - 5)^2 + (-4 - 3)^2} \)  
  - 68. \( \sqrt{(20 - 5)^2 + (-2 - 6)^2} \)
What You’ll Learn

• Find the distance between two points on the coordinate plane.
• Find a point that is a given distance from a second point in a plane.

How can the distance between two points be determined?

Consider two points A and B in the coordinate plane. Notice that a right triangle can be formed by drawing lines parallel to the axes through the points at A and B. These lines intersect at C forming a right angle. The hypotenuse of this triangle is the distance between A and B. You can determine the length of the legs of this triangle and use the Pythagorean Theorem to find the distance between the two points. Notice that AC is the difference of the y-coordinates, and BC is the difference of the x-coordinates.

\[\text{So, } AB = \sqrt{(AC)^2 + (BC)^2}.\]

THE DISTANCE FORMULA  You can find the distance between any two points in the coordinate plane using a similar process. The result is called the Distance Formula.

**Key Concept**

**The Distance Formula**

- **Words**  The distance \(d\) between any two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is given by \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).

- **Model**

![Diagram](Image)

**Example 1 Distance Between Two Points**

Find the distance between the points at \((2, 3)\) and \((-4, 6)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
= \sqrt{(-4 - 2)^2 + (6 - 3)^2} \\
= \sqrt{(-6)^2 + 3^2} \\
= \sqrt{36 + 9} \\
= \sqrt{45} \\
= 3\sqrt{5} \text{ or about 6.71 units}
\]

**Virginia SOL**

STANDARD A.1 The student will solve multistep linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable, and apply these skills to solve practical problems. Graphing calculators will be used to confirm algebraic solutions.

**5-Minute Check Transparency 11-5** Use as a quiz or review of Lesson 11-4.

**Mathematical Background** notes are available for this lesson on p. 584D.

**How can the distance between two points be determined?**

Ask students:

- What must you know in order to use the Pythagorean Theorem to determine the distance between the two points? You must know the lengths of the two legs in order to find the distance between the two points, which is the hypotenuse.

- How do you find the length of the vertical leg? Find the difference in the \(y\)-coordinates of \(A\) and \(B\).

- How do you find the length of the horizontal leg? Find the difference in the \(x\)-coordinates of \(A\) and \(B\).

**Resource Manager**

- **Workbook and Reproducible Masters**
  - Chapter 11 Resource Masters
    - Study Guide and Intervention, pp. 667–668
    - Skills Practice, p. 669
    - Practice, p. 670
    - Reading to Learn Mathematics, p. 671
    - Enrichment, p. 672
  - Workbook, p. 87
  - Science and Mathematics Lab Manual, pp. 85–90

- **Parent and Student Study Guide Transparencies**

- **Technology**
  - Interactive Chalkboard
**Teach**

**THE DISTANCE FORMULA**

1. Find the distance between the points at (1, 2) and (−3, 0). The distance is $2\sqrt{5}$, or about 4.47 units.

2. **BIATHLON** Julianne is sighting in her rifle for an upcoming biathlon competition. Her first shot is 2 inches to the right and 7 inches below the bull’s-eye. What is the distance between the bull’s-eye and where her first shot hit the target? The distance is $\sqrt{53}$ or about 7.28 inches.

**Find Coordinates**

3. Find the value of $a$ if the distance between the points at (2, −1) and ($a$, −4) is 5 units. $−2$ or $6$

**Answers**

1. The values that are subtracted are squared before being added and the square of a negative number is always positive. The sum of two positive numbers is positive, so the distance will never be negative.

2. See students’ graph; the distance from $A$ to $B$ equals the distance from $B$ to $A$. Using the Distance Formula, the solution is the same no matter which ordered pair is used first.

3. See students’ diagrams; there are exactly two points that lie on the line $y = −3$ that are 10 units from the point (7, 5).

---

**Example** 2 **Use the Distance Formula**

**GOLF** Tracy hits a golf ball that lands 20 feet short and 8 feet to the right of the cup. On her first putt, the ball lands 2 feet to the left and 3 feet beyond the cup. Assuming that the ball traveled in a straight line, how far did the ball travel on her first putt?

Draw a model of the situation on a coordinate grid. If the cup is at (0, 0), then the location of the ball after the first hit is $(8, −20)$. The location of the ball after the first putt is $(−2, 3)$. Use the Distance Formula.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Distance Formula

\[ = \sqrt{(-2 - 8)^2 + (3 - (-20))^2} \]

\[ = \sqrt{(-10)^2 + 23^2} \]

\[ = \sqrt{629} \text{ or about 25 feet} \]

**Check for Understanding**

1. **Concept Check** 1–3. See margin.

2. **Open Ended** Plot two ordered pairs and find the distance between their graphs. Does it matter which ordered pair is first when using the Distance Formula? Explain.

3. **Explain** why there are two values for $a$ in Example 3. Draw a diagram to support your answer.

---

**Daily Intervention**

Unlocking Misconceptions

Not only does it not matter which point is designated $(x_1, y_1)$ and $(x_2, y_2)$ when using the Distance Formula, it also does not matter in which order the $x$- and $y$-coordinates are subtracted. For example,

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

will produce the same results as

\[ \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

or

\[ \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2} \].

Have students test each method with two points.
Guided Practice

Find the distance between each pair of points whose coordinates are given. Express in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

**GUIDED PRACTICE KEY**

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4. (5, –1), (11, 7)  
5. (3, 7), (–2, –5)  
6. (2, 2), (5, –1)  
7. (–3, –5), (–6, –4)  

3√2 ≈ 4.24  
√10 ≈ 3.16

Find the possible values of a if the points with the given coordinates are the indicated distance apart.

8. (3, –1), (a, 7); d = 10  
9. (10, a), (1, –6); d = 145  
9 or –3  
2 or –14

Applications

10. GEOMETRY An isosceles triangle has two sides of equal length. Determine whether triangle ABC with vertices A(–3, 4), B(5, 2), and C(–1, –5) is an isosceles triangle. yes; AB = √68, BC = √85, AC = √85

FOOTBALL For Exercises 11 and 12, use the information at the right.

11. A quarterback can throw the football to one of the two receivers. Find the distance from the quarterback to each receiver.

12. What is the distance between the two receivers? about 20.6 yd

Practice and Apply

Find the distance between each pair of points whose coordinates are given. Express in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

13. (12, 3), (–8, 3)  
14. (0, 0), (5, 12)  
15. (6, 8), (3, 4)  
16. (–4, 2), (4, 17)  
17. (–3, 8), (5, 4)  
18. (9, –2), (3, –6)  
19. (–8, –4), (–3, –8)  
20. (2, 7), (10, –4)  
21. (4, 2), (6, –2)  
22. (5, 1), (3, 4)  
23. (4√5, 7), (6√5, 5)  
24. (3, 3), (3, 2)  
25. (4, 7), (a, 3); d = 5  
26. (5, 2), (7, 2, 10)  
27. (4, 7), (a, 3); d = 5  
28. (–4, a), (4, 2); d = 17  
29. (5, a), (6, 1); d = √10  
30. (a, 5), (–7, 3); d = √29  
31. (6, –3), (–3, a); d = √130  
32. (20, 5), (a, 9); d = √340  

1 or 7  
17 or –13  
–2 or 4  
2 or 12  
10 or 4  
2 or 38

Find the possible values of a if the points with the given coordinates are the indicated distance apart.

33. Triangle ABC has vertices at A(7, –4), B(–1, 2), and C(5, –6). Determine whether the triangle has three, two, or no sides that are equal in length. two; AB = BC = 10

34. √157 ≠ √101; The trapezoid is not isosceles.

If the diagonals of a trapezoid have the same length, then the trapezoid is isosceles. Find the length of the diagonals of trapezoid ABCD with vertices A(–2, 2), B(10, 6), C(9, 8), and D(0, 5) to determine if it is isosceles.

www.algebra1.com/extra_examples/sol

Lesson 11-5 The Distance Formula 613

Differentiated Instruction

Interpersonal Place students in pairs. Have each person in a pair drop a penny on a coordinate grid, and record the coordinates of the point closest to where the center of the penny landed. Then have the students work together to find the distance between the two points, using the Distance Formula.
What is the distance between Players B and C?

21. 15.

Three players are warming up for a baseball game. Player B (2, 13, 10) and Player C (3, 15, 12) are both warming up for a baseball game. Player A (0, 0) is warming up for a baseball game.

Find the distance between each pair of points whose coordinates are given.

1. (1, -1, 3) and (3, -5, 1)
2. (2, 3, -1) and (5, -2, -3)
3. (x, y, z) and (0, 0, 0)
4. (1, 2, 3) and (4, 5, 6)
5. (2, 3, 4) and (5, 6, 7)
6. (x, y, z) and (a, b, c)

Helping You Remember

Reading the Lesson

How can the distance between two points be determined?

Distance Formula

The distance between two points with coordinates (x, y, z) and (a, b, c) is given by:

\[ d = \sqrt{(a-x)^2 + (b-y)^2 + (c-z)^2} \]

Example

Maria and Jackson live in adjacent neighborhoods. If they superimpose a coordinate grid and the distance between their homes is about 110 ft, estimate the coordinates of each home.

Finding the distance between B and C:

Given the coordinates of B (11, 4) and C (9, 8), we can find the distance between them using the Distance Formula:

\[ d = \sqrt{(9-11)^2 + (8-4)^2} = \sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.47 

So, the distance between B and C is approximately 4.47 units.

Exercise 41: Minneapolis-St. Cloud, 53 mi; St. Paul-Rochester, 64 mi; Minneapolis-Eau Claire, 79 mi; Duluth-St. Cloud, 118 mi

41. Minneapolis-St. Cloud, 53 mi; St. Paul-Rochester, 64 mi; Minneapolis-Eau Claire, 79 mi; Duluth-St. Cloud, 118 mi

CRITICAL THINKING

Plot A(−4, 4), B(−7, −3), and C(4, 0), and connect them to form triangle ABC. Demonstrate two different ways to show that ABC is a right triangle.

43. WRITING IN MATH

Answer the question that was posed at the beginning of the lesson. See margin.

How can the distance between two points be determined?

Include the following in your answer:

- an explanation how the Distance Formula is derived from the Pythagorean Theorem, and
- an explanation why the Distance Formula is not needed to find the distance between points P(−24, 18) and Q(−24, 10).

614 Chapter 11 Radical Expressions and Triangles
45. Find the distance between points at (6, 11) and (−2, −4). B
   - A 16 units
   - B 17 units
   - C 18 units
   - D 19 units

46. Find the perimeter of a square ABCD if two of the vertices are A(3, 7) and B(−3, 4). B
   - A 12 units
   - B $12\sqrt{2}$ units
   - C $9\sqrt{5}$ units
   - D 45 units

**Maintain Your Skills**

**Mixed Review**

If $c$ is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth. (Lesson 11-4)

47. $a = 7, b = 24, c = ?$ 25
48. $b = 30, c = 34, a = ?$ 16
49. $a = \sqrt{7}, c = \sqrt{16}, b = ?$ 3
50. $a = \sqrt{13}, b = \sqrt{50}, c = ?$ $3\sqrt{7} = 7.94$

Solve each equation. Check your solution. (Lesson 11-3)

51. $\sqrt{p} + 2 = p$ 11
52. $\sqrt{r} + 5 = r - 1$ 4
53. $\sqrt{5t^2 + 29} = 2t + 3 (2, 10)$

**COST OF DEVELOPMENT** For Exercises 54–56, use the graph that shows the amount of money being spent on worldwide construction. (Lesson 8-3)

54. Write the value shown for each continent or region listed in standard notation.
55. Write the value shown for each continent or region in scientific notation.
56. How much more money is being spent in Asia than in Latin America? $5.72 \times 10^{11}$ or $872$ billion

Solve each inequality. Then check your solution and graph it on a number line. (Lesson 6-1) 57–62. See pp. 639A–639B for graphs.

57. $8 \leq m - 1 \quad |m| 
58. $3 > 10 + k \quad |k| 
59. $3x \leq 2x - 3 \quad |x| 
60. v - (-4) > 6 \quad |v| 
61. $r - 5.2 \geq 3.9 \quad |r| 
62. s + \frac{1}{6} \geq \frac{2}{3} \quad |s| 

**PREREQUISITE SKILL** Solve each proportion. (To review proportions, see Lesson 3-6.)

63. $\frac{x}{4} = \frac{3}{2}$ 6
65. $\frac{6}{9} = \frac{8}{x}$ 12
67. $\frac{x + 2}{7} = \frac{3}{7}$ 1

64. $\frac{20}{x} = \frac{-5}{2}$ -8
66. $\frac{10}{12} = \frac{x}{18}$ 15
68. $\frac{2}{3} = \frac{6}{x + 4}$ 5

**Getting Ready for the Next Lesson**

43. Compare the slopes of the two potential legs to determine whether the slopes are negative reciprocals of each other. You can also compute the lengths of the three sides and determine whether the square of the longest side length is equal to the sum of the squares of the other two side lengths. Neither test holds true in this case because the triangle is not a right triangle.

**Online Lesson Plans**

USA TODAY Education’s Online site offers resources and interactive features connected to each day’s newspaper. *Experience TODAY*, USA TODAY’s daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

**Open-Ended Assessment**

**Speaking** Have students describe a real-world situation in which the Distance Formula could be used to find the distance between two points. If students have trouble thinking of situations, suggest that they think of anything that involves measurement such as construction, art, sports, and so on. Have them explain how the Distance Formula is used in the situation.
**Focus**

5-Minute Check Transparency 11-6 Use as a quiz or review of Lesson 11-5.

Mathematical Background notes are available for this lesson on p. 584D.

**Building on Prior Knowledge**

In Lesson 3-6, students learned how to solve proportions. In this lesson, students will use their knowledge of proportions to determine whether two triangles are similar.

**How** are similar triangles related to photography?

Ask students:

- If you photograph two people from the same distance, one of whom is twice the height of the other, what are the heights of their images in the photograph? One image will be twice the height of the other.
- How would you describe this relationship using ratios? The ratio of the heights of the people is the same as the ratio of the heights of the images.
- What do you call an equation that states two ratios are equal? A proportion

**What You’ll Learn**

- Determine whether two triangles are similar.
- Find the unknown measures of sides of two similar triangles.

**Similiar Triangles**

Similar triangles have the same shape, but not necessarily the same size. There are two main tests for similarity.

- If the angles of one triangle and the corresponding angles of a second triangle have equal measures, then the triangles are similar.
- If the measures of the sides of two triangles form equal ratios, or are proportional, then the triangles are similar.

The triangles below are similar. This is written as \( \triangle ABC \sim \triangle DEF \). The vertices of similar triangles are written in order to show the corresponding parts.

<table>
<thead>
<tr>
<th>Corresponding angles</th>
<th>Corresponding sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle A ) and ( \angle D )</td>
<td>( \frac{AB}{DE} = \frac{2}{4} = \frac{1}{2} )</td>
</tr>
<tr>
<td>( \angle B ) and ( \angle E )</td>
<td>( \frac{BC}{EF} = \frac{2.5}{5} = \frac{1}{2} )</td>
</tr>
<tr>
<td>( \angle C ) and ( \angle F )</td>
<td>( \frac{AC}{DF} = \frac{3}{6} = \frac{1}{2} )</td>
</tr>
</tbody>
</table>

**Key Concept**

- **Words** If two triangles are similar, then the measures of their corresponding sides are proportional, and the measures of their corresponding angles are equal.
- **Symbols** If \( \triangle ABC \sim \triangle DEF \), then \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \).
- **Model**

**TEACHING TIP**

Arrows are used to show angles that have equal measures.

**Resource Manager**

**Workbook and Reproducible Masters**

- Chapter 11 Resource Masters
  - Study Guide and Intervention, pp. 673–674
  - Skills Practice, p. 675
  - Practice, p. 676
  - Reading to Learn Mathematics, p. 677
  - Enrichment, p. 678
  - Assessment, p. 700

- Parent and Student Study Guide Workbook, p. 88
- Prerequisite Skills Workbook, pp. 61–62

**Transparencies**

- 5-Minute Check Transparency 11-6
- Answer Key Transparencies

**Technology**

- Interactive Chalkboard
Example 1 Determine Whether Two Triangles Are Similar

Determine whether the pair of triangles is similar. Justify your answer.

Remember that the sum of the measures of the angles in a triangle is 180°.

The measure of $\angle P$ is $180° - (51° + 51°)$ or $78°$.

In $\triangle MNO$, $\angle N$ and $\angle O$ have the same measure.

Let $x =$ the measure of $\angle N$ and $\angle O$.

\[ x + x + 78° = 180° \]
\[ 2x = 102° \]
\[ x = 51° \]

So $\angle N = 51°$ and $\angle O = 51°$. Since the corresponding angles have equal measures, $\triangle MNO \sim \triangle PQR$.

FIND UNKNOWN MEASURES Proportions can be used to find the measures of the sides of similar triangles when some of the measurements are known.

Example 2 Find Missing Measures

Find the missing measures if each pair of triangles below is similar.

a. Since the corresponding angles have equal measures, $\triangle TUV \sim \triangle WXY$.

The lengths of the corresponding sides are proportional.

\[
\frac{WX}{TU} = \frac{XY}{UV}
\]

Corresponding sides of similar triangles are proportional.

\[
\frac{a}{3} = \frac{16}{4}
\]

Find the cross products.

\[
a = 12
\]

Divide each side by 4.

\[
\frac{WY}{TV} = \frac{XY}{UV}
\]

Corresponding sides of similar triangles are proportional.

\[
\frac{b}{6} = \frac{16}{4}
\]

Find the cross products.

\[
b = 24
\]

Divide each side by 4.

The missing measures are 12 and 24.

b. $\triangle ABE \sim \triangle ACD$

\[
\frac{BE}{CD} = \frac{AE}{AD}
\]

Corresponding sides of similar triangles are proportional.

\[
\frac{10}{x} = \frac{6}{9}
\]

Find the cross products.

\[
x = 15
\]

The missing measure is 15.
3 SHADOWS  Richard is standing next to the General Sherman Giant Sequoia tree in Sequoia National Park. The shadow of the tree is 22.5 meters, and Richard’s shadow is 5.6 centimeters. If Richard’s height is 2 meters, how tall is the tree? The tree is about 84 m tall.

3 Practice/Apply

Study Notebook

Have students—
* add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.
* include examples of how to determine whether two triangles are similar, and how to find unknown measures using triangles.
* include any other item(s) that they find helpful in mastering the skills in this lesson.

Guided Practice

<table>
<thead>
<tr>
<th>GUIDED PRACTICE KEY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercises</td>
</tr>
<tr>
<td>4, 5</td>
</tr>
<tr>
<td>6–9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

Check for Understanding

1. Explain how to determine whether two triangles are similar.

2. OPEN ENDED Draw a pair of similar triangles. List the corresponding angles and the corresponding sides. See pp. 639A–639B.

3. FIND THE ERROR Russell and Consuela are comparing the similar triangles below to determine their corresponding parts. See margin.

Who is correct? Explain your reasoning.

Determine whether each pair of triangles is similar. Justify your answer.

4. No; the angle measures are not equal.

5. Yes; the angle measures are equal.

For each set of measures given, find the measures of the missing sides if \( \triangle ABC \sim \triangle DEF \).

6. \( a = 15, d = 7, e = 9, f = 5 \) \( a = 21, b = 27 \)

7. \( a = 18, c = 9, e = 10, f = 6 \) \( b = 15, d = 12 \)

8. \( a = 5, d = 7, f = 6, e = 5 \) \( b = 5x, c = 30 \)

9. \( a = 17, b = 15, c = 10, f = 6 \) \( d = 10.2, e = 9 \)

DAILY INTERVENTION FIND THE ERROR

Remind students to pay close attention to the arcs that denote which angles are congruent.

Answers

3. Consuela; the arcs indicate which angles correspond. The vertices of the triangles are written in order to show the corresponding parts.

Naturalist Have students use the method in Example 3 to find the heights of trees that are native to your area. Make sure students record the location and type of tree along with the height. Students will need tape measures and will need to take measurements on a sunny day.

Example 3 Use Similar Triangles to Solve a Problem

SHADOWS Jenelle is standing near the Washington Monument in Washington, D.C. The shadow of the monument is 302.5 feet, and Jenelle’s shadow is 3 feet. If Jenelle is 5.5 feet tall, how tall is the monument?

The shadows form similar triangles. Write a proportion that compares the heights of the objects and the lengths of their shadows.

Let \( x \) = the height of the monument.

\[
\frac{Jenelle's\ shadow}{monument's\ shadow} = \frac{Jenelle's\ height}{monument's\ height}\]

\[
\frac{3}{302.5} = \frac{5.5}{x}\]

Cross products

\( 3x = 1663.75 \)

Divide each side by 3.

\( x = 554.6 \) feet

The height of the monument is about 554.6 feet.
**Application**

10. **SHADOWS**  If a 25-foot flagpole casts a shadow that is 10 feet long and the nearby school building casts a shadow that is 26 feet long, how high is the building?  **65 ft**

★ indicates increased difficulty

---

**Practice and Apply**

**Exercises Examples Extra Practice**

indicates increased difficulty

11–16 1

See page 845.

See page 845.

For See

10.

**SHADOWS**

Lenno is playing billiards on a table like the one shown at the right. He wants to strike the cue ball at D, bank it at C, and hit another ball at the mouth of pocket A. Use similar triangles to find where Lenno’s cue ball should strike the rail.  **24 in. from pocket B**
Determine whether each pair of triangles is similar. Justify your answer.

1. Yes; corresponding angles have equal measures.
2. No; corresponding angles do not have equal measures.
3. Yes; corresponding angles have equal measures.
4. Yes; corresponding angles have equal measures.
5. No; corresponding angles do not have equal measures.
6. Yes; corresponding angles have equal measures.
7. No; corresponding angles do not have equal measures.
8. Yes; corresponding angles have equal measures.
9. No; corresponding angles do not have equal measures.
10. Yes; corresponding angles have equal measures.

Skills Practice, p. 675 and Practice, p. 676 (shown)

Determine whether each pair of triangles is similar. Justify your answer.

1. Yes; Q = (70°, 30°), P = (70°, 30°) since the corresponding angles have equal measures, \( \triangle ABC \sim \triangle DEF \).
2. No; Q = 180° - \( 30° \) = 150°, P = 180° - \( 30° \) = 150° but the corresponding angles do not have equal measures, \( \triangle ABC \not\sim \triangle DEF \).

For each set of measures given, find the measures of the missing sides if \( \triangle ABC \sim \triangle DEF \):

1. \( a = 6, b = 8 \)
2. \( a = 10, d = 12, e = 8, f = 10 \)
3. \( a = 6, c = 8, d = 12, f = 10 \)
4. \( a = 3, b = 5, c = 4, d = 3, e = 4 \)
5. \( a = 3, b = 5, c = 4, d = 3, e = 4 \)
6. \( a = 3, b = 5, c = 4, d = 3, e = 4 \)
7. \( a = 3, b = 5, c = 4, d = 3, e = 4 \)
8. \( a = 3, b = 5, c = 4, d = 3, e = 4 \)

11. SHADOWS Suppose you are standing near a building and you want to know its height. The building casts a 66-foot shadow. You cast a 3-foot shadow. If you are 5 feet 6 inches tall, how tall is the building? (Round to the nearest foot.) Answer: 53 ft

12. MODELS Test how big can triangles be in their parent house. Molly made a model of a frame bridge on the scale of 1 inch = 0 feet. If the height of the bridge was 4 feet, what is the height of the bridge in the actual bridge? (Round to the nearest inch.) Answer: 24 ft

For Exercises 29 and 30, use the following information.

Melinda is working on a quilt pattern containing isosceles right triangles whose sides measure 2 inches, 2 inches, and about 2.8 inches.

29. She has several square pieces of material that measure 4 inches on each side. From each square piece, how many triangles with the required dimensions can she cut? 8

30. She wants to enlarge the pattern to make similar triangles for the center of the quilt. What is the largest similar triangle she can cut from the square material? 4 by 4 by about 5.6 in.

For Exercises 31 and 32, use the diagram and the following information.

Vinho wanted to measure the height of a nearby building. He placed a mirror on the pavement at point P, 80 feet from the base of the building. He then backed away until he saw an image of the top of the building in the mirror.

31. If Vinho is 6 feet tall and he is standing 9 feet from the mirror, how tall is the building? about 53 ft

32. What assumptions did you make in solving the problem?

CRITICAL THINKING For Exercises 33–35, use the following information.

The radius of one circle is twice the radius of another. 34–35. See margin.

33. Are the circles similar? Explain your reasoning.

34. What is the ratio of their circumferences? Explain your reasoning.

35. What is the ratio of their areas? Explain your reasoning.

36. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How are similar triangles related to photography?

Include the following in your answer:

• an explanation of the effect of moving a camera with a zoom lens closer to the object being photographed, and

• a description of what you could do to fit the entire image of a large object on the picture.

For Exercises 37 and 38, use the figure at the right.

37. Which statement must be true? D
   \( \triangle ABC \sim \triangle ADC \)

38. Which statement is always true? A
   \( \triangle ABC \sim \triangle CAD \)
   \( \triangle ABC \sim \triangle CAD \)

620 Chapter 11 Radical Expressions and Triangles
Maintain Your Skills

### Mixed Review
Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary. (Lesson 11-5)

39. (1, 8), (–2, 4) 40. (6, –3), (12, 5) 41. (4, 7), (3, 12) \( \sqrt{26} \approx 5.1 \) 42. \( (1, 5\sqrt{6}), (6, 7\sqrt{6}) \) \( 7 \)

Determine whether the following side measures form right triangles. Justify your answer. (Lesson 11-4)

43. 25, 60, 65 Yes; \( 25^2 + 60^2 = 65^2 \).
44. 20, 25, 35 No; \( 20^2 + 25^2 \neq 35^2 \).
45. 49, 168, 175 Yes; \( 49^2 + 168^2 = 175^2 \).
46. 7, 9, 12 No; \( 7^2 + 9^2 \neq 12^2 \).

Arrange the terms of each polynomial so that the powers of the variable are in descending order. (Lesson 8-4)

47. \( 1 + 3x^2 - 7x \) \( 3x^2 - 7x + 1 \)
48. \( 7 - 4x - 2x^2 + 5x^3 \)
49. \( 6x + 3 - 3x^2 - 6x + 3 \) \( -x^2 + 6x + 3 \)
50. \( ax^2 + bxy + cy^2 + 3x - x^2 + abx^2 - bcy + 34 \)

Use elimination to solve each system of equations. (Lesson 7-3)

51. \( 2x + y = 4 \) \( (3, -2) \)
\( x - y = 5 \)
52. \( 3x + 2y = -13 \) \( (-5, -1) \)
\( 2x - 5y = -5 \)
53. \( 0.6m - 0.2n = 0.9 \) \( (1.5, 0) \)
\( 0.3m + 0.45 - 0.1n = 0 \)
54. \( \frac{1}{3}x + \frac{1}{2}y = 8 \) \( (6, 12) \)
\( \frac{1}{2}x - \frac{1}{3} y = 0 \)

55. **AVIATION** An airplane passing over Sacramento at an elevation of 37,000 feet begins its descent to land at Reno, 140 miles away. If the elevation of Reno is 4500 feet, what should be the approximate slope of descent? (Hint: 1 mi = 5280 ft) (Lesson 5-4) about \(-0.044 \)

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Evaluate if \( a = 6, b = -5, \) and \( c = -1.5 \).

56. \( \frac{a}{c} \)
57. \( \frac{b}{a} \) \( \frac{5}{6} \) or \(-0.83 \)
58. \( \frac{a + b}{c} \) \( \frac{2}{3} \) or \(-0.6 \)
59. \( \frac{ac}{b} \)
60. \( \frac{b}{a + c} \) \( \frac{-10}{9} \) or \(-1.1 \)
61. \( \frac{c}{a + c} \) \( \frac{1}{3} \) or \(-0.3 \)

### Practice Quiz 2

**Lessons 11-4 through 11-6**

If \( c \) is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth. (Lesson 11-4)

1. \( a = 14, b = 48, c = ? \) 250
2. \( a = 40, c = 41, b = ? \) 9
3. \( b = 8, c = \sqrt{84}, a = ? \) \( 2\sqrt{5} \approx 4.47 \)
4. \( a = \sqrt{5}, b = \sqrt{8}, c = ? \) \( \sqrt{13} \approx 3.61 \)

Find the distance between each pair of points whose coordinates are given. (Lesson 11-5)

5. \( (6, -12), (-3, 3) \) \( \sqrt{306} \approx 17.49 \)
6. \( (1, 3), (-5, 11) \) \( 10 \)
7. \( (2, 4), (4, 7) \) \( 2\sqrt{6} \approx 2.83 \)
8. \( (-2, -9), (-5, 4) \) \( \sqrt{178} \approx 13.34 \)

Find the measures of the missing sides if \( \triangle BCA \sim \triangle EFD \). (Lesson 11-6)

9. \( b = 10, d = 2, e = 1, f = 1.5 \) \( a = 20, c = 15 \)
10. \( a = 12, c = 9, d = 8, e = 12 \) \( b = 18, f = 6 \)

### Answers

34. 2:1; Let the first circle have radius \( r \) and the larger have radius \( 2r \). The circumference of the first is \( 2\pi r \) and the other has circumference \( 2\pi (2r) = 4\pi r \).

35. 4:1; The area of the first is \( \pi r^2 \) and the area of the other is \( \pi (2r)^2 = 4\pi r^2 \).

36. The size of an object on the film of a camera can be related to its actual size using similar triangles. Answers should include the following.

- Moving the lens closer to the object (and farther from the film) makes the object appear larger.
- Taking a picture of a building; you would need to be a great distance away to fit the entire building in the picture.

Open-Ended Assessment

**Modeling** Construct a triangle using unsharpened pencils, uncooked spaghetti or other items that have a uniform length. Have students construct a triangle similar to yours and explain how they would show that the two triangles are similar.

### Assessment Options

**Practice Quiz 2** The quiz provides students with a brief review of the concepts and skills in Lessons 11-4 through 11-6. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

**Quiz (Lessons 11-5 and 11-6)** is available on p. 700 of the Chapter 11 Resource Masters.

[www.algebra1.com/self_check_quiz/sol](http://www.algebra1.com/self_check_quiz/sol)
Algebra Activity

A Preview of Lesson 11-7

Getting Started

**Objective** Construct right triangles with legs whose lengths are equal to a specific ratio, in order to investigate the relationship between the sides and the angles as a preview of trigonometric ratios.

**Materials**
- ruler
- grid paper
- protractor

**Teach**

- Remind students that the triangles they are drawing are similar because the ratio of the corresponding sides is the same for each triangle.
- Point out to students that because they drew the triangles by hand, the measures of the angles may not be exact, according to the protractor. However, the measures should be close to 35°, 55°, and 90°.

**Assess**

Ask students to conjecture why knowing the relationship between the ratios of the sides of a triangle to each other and the angles might be useful. **Sample answer:** Knowing these relationships might allow you to calculate a missing side length or angle measure.

**Collect the Data**

**Step 1** Use a ruler and grid paper to draw several right triangles whose legs are in a 7:10 ratio. Include a right triangle with legs 3.5 units and 5 units, a right triangle with legs 7 units and 10 units, another with legs 14 units and 20 units, and several more right triangles similar to these three. Label the vertices of each triangle as A, B, and C, where C is at the right angle, B is opposite the longest leg, and A is opposite the shortest leg.

**Step 2** Copy the table below. Complete the first three columns by measuring the hypotenuse (side AB) in each right triangle you created and recording its length.

**Step 3** Calculate and record the ratios in the middle two columns. Round to the nearest tenth, if necessary.

**Step 4** Use a protractor to carefully measure angles A and B in each right triangle. Record the angle measures in the table.

**Analyze the Data**

1. Examine the measures and ratios in the table. What do you notice? Write a sentence or two to describe any patterns you see. **All ratios and angle measures are the same for any 7:10 right triangle.**

**Make a Conjecture**

2. For any right triangle similar to the ones you have drawn here, what will be the value of the ratio of the length of the shortest leg to the length of the longest leg? **7:10**

3. If you draw a right triangle and calculate the ratio of the length of the shortest leg to the length of the hypotenuse to be approximately 0.574, what will be the measure of the larger acute angle in the right triangle? **55°**

**Resource Manager**

- **Teaching Algebra with Manipulatives**
  - p. 1 (master for grid paper)
  - p. 23 (master for protractors)
  - p. 24 (master for rulers)
  - p. 191 (student recording sheet)

- **Glencoe Mathematics Classroom Manipulative Kit**
  - coordinate grid stamp
  - protractors
  - rulers
11-7 Trigonometric Ratios

**What You’ll Learn**
- Define the sine, cosine, and tangent ratios.
- Use trigonometric ratios to solve right triangles.

**Vocabulary**
- trigonometric ratios
- sine
- cosine
- tangent
- solve a triangle
- angle of elevation
- angle of depression

**How are trigonometric ratios used in surveying?**

Surveyors use triangle ratios called trigonometric ratios to determine distances that cannot be measured directly.
- In 1852, British surveyors measured the altitude of the peak of Mt. Everest at 29,002 feet using these trigonometric ratios.
- In 1954, the official height became 29,028 feet, which was also calculated using surveying techniques.
- On November 11, 1999, a team using advanced technology and the Global Positioning System (GPS) satellite measured the mountain at 29,035 feet.

**TRIGONOMETRIC RATIOS**

Trigonometry is an area of mathematics that involves angles and triangles. If enough information is known about a right triangle, certain ratios can be used to find the measures of the remaining parts of the triangle.

**Key Concept**

- **Words**
  - sine of $\angle A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of hypotenuse}}$
  - cosine of $\angle A = \frac{\text{measure of leg adjacent to } \angle A}{\text{measure of hypotenuse}}$
  - tangent of $\angle A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of leg adjacent to } \angle A}$

- **Symbols**
  - $\sin \angle A = \frac{BC}{AB}$
  - $\cos \angle A = \frac{AC}{AB}$
  - $\tan \angle A = \frac{BC}{AC}$

- **Model**

**Reading Math**

Notice that sine, cosine, and tangent are abbreviated sin, cos, and tan respectively.

**Study Tip**

**Reading Math**

Notice that sine, cosine, and tangent are abbreviated sin, cos, and tan respectively.

**Lesson Notes**

**Focus**

5-Minute Check Transparency 11-7 Use as a quiz or review of Lesson 11-6.

Mathematical Background notes are available for this lesson on p. 584D.

**How are trigonometric ratios used in surveying?**

Ask students:
- In the previous lesson, you calculated the height of the Washington Monument using ratios. What is required in order to make this calculation? Two similar triangles formed by the shadow of the monument and the shadow of a person standing next to it. You know the length of both shadows and the height of the person. You can use ratios to calculate the height of the monument.
- Based on the Algebra Activity on the previous page, how do you suppose a surveyor could calculate the height of a mountain? Sample answer: A surveyor could use the angle between him or herself and the top of the mountain, and the distance to the mountain to construct a similar right triangle. Then based on the side measures of that similar triangle, the height of the mountain could be calculated using ratios.

**Resource Manager**

**Workbook and Reproducible Masters**

- Chapter 11 Resource Masters
  - Study Guide and Intervention, pp. 679–680
  - Skills Practice, p. 681
  - Practice, p. 682
  - Reading to Learn Mathematics, p. 683
  - Enrichment, p. 684
  - Assessment, p. 700

- Parent and Student Study Guide Workbook, p. 89

- Teaching Algebra With Manipulatives Masters, pp. 23, 24, 192

**Technology**

- Interactive Chalkboard

- Transparencies
  - 5-Minute Check Transparency 11-7 Answer Key Transparencies
TRIGONOMETRIC RATIOS

The trigonometric ratios are a concept that students will use in other classes such as Geometry, Algebra II, and Trigonometry. Any time students are introduced to such important concepts, have them record the concepts in their Study Notebooks for future reference. Additionally, the act of recording the concepts will help the students remember them.

In-Class Example

**Teaching Tip** While it is true that a fraction with radicals in the denominator is not in simplest form, it is not necessary to rationalize the denominator if students are going to use a calculator to simplify the expression. Suggest that students use a calculator to find \( \sqrt{10} \) and \( \sqrt{30} \). Both produce the same result, and rationalization was not necessary.

1. Find the sine, cosine, and tangent of each acute angle of \( \triangle DEF \). Round to the nearest ten-thousandth.

\[
\begin{align*}
\sin D &= \frac{13}{\sqrt{8^2 + 19^2}} = 0.7293; \\
\cos D &= \frac{19}{\sqrt{8^2 + 19^2}} = 0.6842; \\
\tan D &= \frac{13}{19} = 0.6842; \\
\sin F &= \frac{18^2 + 8^2}{\sqrt{13^2 + 19^2}} = 0.6842; \\
\cos F &= \frac{13}{19} = 0.7293; \\
\tan F &= \frac{18^2 + 8^2}{13} = 0.9382
\end{align*}
\]

**Example 1** Sine, Cosine, and Tangent

Find the sine, cosine, and tangent of each acute angle of \( \triangle RST \). Round to the nearest ten-thousandth.

Write each ratio and substitute the measures. Use a calculator to find each value.

\[
\begin{align*}
\sin R &= \frac{\text{opposite leg}}{\text{hypotenuse}} \\
&= \frac{17}{18} \approx 0.9444 \\
&= \sqrt{18} \approx 3.39 \quad \text{or } 0.3287 \\
\cos R &= \frac{\text{adjacent leg}}{\text{hypotenuse}} \\
&= \frac{18}{17} \approx 0.9444 \\
&= \frac{\sqrt{18}}{17} \approx 0.3287 \\
\tan R &= \frac{\text{opposite leg}}{\text{adjacent leg}} \\
&= \frac{\sqrt{18}}{18} \approx 0.3287 \\
&= \frac{17}{\sqrt{18}} \approx 0.3480
\end{align*}
\]

You can use a calculator to find the values of trigonometric functions or to find the measure of an angle. On a graphing calculator, press the trigometric function key, and then enter the value. On a non-graphing scientific calculator, enter the value, and then press the function key. In either case, be sure your calculator is in degree mode. Consider \( \cos 50^\circ \).

**Graphing Calculator**

\[\text{KEYSTROKES: } \cos 50 \quad \rightarrow \quad 0.6427876097\]

**Non-graphing Scientific Calculator**

\[\text{KEYSTROKES: } 50 \quad \cos \quad 0.6427876097\]

**Example 2** Find the Sine of an Angle

Find \( \sin 35^\circ \) to the nearest ten-thousandth.

\[\text{KEYSTROKES: } \sin 35 \quad \rightarrow \quad 0.5735764364\]

Rounded to the nearest ten-thousandth, \( \sin 35^\circ \approx 0.5736 \).

**Example 3** Find the Measure of an Angle

Find the measure of \( \angle J \) to the nearest degree.

Since the lengths of the opposite and adjacent sides are known, use the tangent ratio.

\[
\tan J = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{6}{9} = \frac{2}{3}
\]

Now use the [TAN\(^{-1}\)] on a calculator to find the measure of the angle whose tangent ratio is \( \frac{6}{9} \).

\[\text{KEYSTROKES: } 2nd \quad \text{TAN}^{-1} \quad 6 \quad \div \quad 9 \quad \rightarrow \quad 33.69006753\]

To the nearest degree, the measure of \( \angle J \) is 34°.

**Differentiated Instruction**

**Auditory/Musical** Divide the class into small groups. Challenge each group to devise a rap verse, limerick, poem, or a song that they can use to remember the trigonometric ratios.
SOLVE TRIANGLES You can find the missing measures of a right triangle if you know the measure of two sides of a triangle or the measure of one side and one acute angle. Finding all of the measures of the sides and the angles in a right triangle is called solving the triangle.

Example 4 Solve a Triangle

Find all of the missing measures in \( \triangle ABC \).

You need to find the measures of \( \angle B \), \( \angle A \), and \( \angle C \).

Step 1 Find the measure of \( \angle B \). The sum of the measures of the angles in a triangle is 180.

\[
180^\circ - 90^\circ - 38^\circ = 52^\circ
\]

The measure of \( \angle B \) is 52°.

Step 2 Find the value of \( x \), which is the measure of the side opposite \( \angle A \). Use the sine ratio.

\[
sin 38^\circ = \frac{x}{12} \quad \text{Definition of sine}
\]

\[
0.6157 = \frac{x}{12} \quad \text{Evaluate sin 38}\degree.
\]

\[
7.4 = x \quad \text{Multiply by 12.}
\]

\( \overline{BC} \) is about 7.4 inches long.

Step 3 Find the value of \( y \), which is the measure of the side adjacent to \( \angle A \). Use the cosine ratio.

\[
cos 38^\circ = \frac{y}{12} \quad \text{Definition of cosine}
\]

\[
0.7880 = \frac{y}{12} \quad \text{Evaluate cos 38}\degree.
\]

\[
9.5 = y \quad \text{Multiply by 12.}
\]

\( \overline{AC} \) is about 9.5 inches long.

So, the missing measures are 52°, 7.4 in., and 9.5 in.

Trigonometric ratios are often used to find distances or lengths that cannot be measured directly. In these situations, you will sometimes use an angle of elevation or an angle of depression. An angle of elevation is formed by a horizontal line of sight and a line of sight above it. An angle of depression is formed by a horizontal line of sight and a line of sight below it.

Example 4 Verifying Right Triangles

You can use the Pythagorean Theorem to verify that the sides are sides of a right triangle.

www.algebra1.com/extra_examples/sol
Teaching Tip Problems involving angles of depression are solved in the same manner as problems with angles of elevation. It is just a different orientation of the angle.

**INDIRECT MEASUREMENT**

In the diagram, Barone is flying his model airplane 400 feet above him. An angle of depression is formed by a horizontal line of sight and a line of sight below it. Find the angles of depression at points A and B to the nearest degree.

The angle of depression at point A is 45° and the angle of depression at point B is 37°.

2. The angle measured by the hypsometer is not the angle of elevation. It is the other acute angle formed in the triangle. So, to find the measure of the angle of elevation, subtract the reading on the hypsometer from 90° since the sum of the measures of the two acute angles in a right triangle is 90°.

**Example 5 Angle of Elevation**

**INDIRECT MEASUREMENT** At point A, Umeko measured the angle of elevation to point P to be 27 degrees. At another point B, which was 600 meters closer to the cliff, Umeko measured the angle of elevation to point P to be 31.5 degrees. Determine the height of the cliff.

**Explore** Draw a diagram to model the situation. Two right triangles, ΔBPC and ΔAPC, are formed. You know the angle of elevation for each triangle. To determine the height of the cliff, find the length of PC, which is shared by both triangles.

**Plan** Let \( y \) represent the distance from the top of the cliff \( P \) to its base \( C \). Let \( x \) represent \( BC \) in the first triangle and let \( x + 600 \) represent \( AC \).

**Solve** Write two equations involving the tangent ratio.

\[
\tan 31.5° = \frac{y}{x} \quad \text{and} \quad \tan 27° = \frac{y}{600 + x}
\]

\[
x \tan 31.5° = y \quad \text{and} \quad (600 + x)\tan 27° = y
\]

**Algebra Activity**

**Make a Hypsometer**

- Tie one end of a piece of string to the middle of a straw. Tie the other end of string to a paper clip.
- Tape a protractor to the side of the straw. Make sure that the string hangs freely to create a vertical or plumb line.
- Find an object outside that is too tall to measure directly, such as a basketball hoop, a flagpole, or the school building.
- Look through the straw to the top of the object you are measuring. Find the angle measure where the string and protractor intersect. Determine the angle of elevation by subtracting this measurement from 90°.
- Measure the distance from your eye level to the ground and from your foot to the base of the object you are measuring.

**Analyze**

1. See students’ work.
2. Make a sketch of your measurements. Use the equation \( \tan \text{angle of elevation} = \frac{\text{height of object} - x}{\text{distance of object}} \), where \( x \) represents distance from the ground to your eye level, to find the height of the object.
3. Why do you have to subtract the angle measurement on the hypsometer from 90° to find the angle of elevation?

**Compare** Your answer with someone who measured the same object. Did your heights agree? Why or why not? See students’ work.

**Example**

**Algebra Activity**

**Materials** string, drinking straw, paper clip, protractor, tape, meter sticks or tape measure

Remind students that the angle of elevation is being measured from their eye level, not the ground. They must account for this distance when calculating the height of the object that they are measuring with their hypsometer.
Lesson 11-7
Trigonometric Ratios

Since both expressions are equal to \( y \), use substitution to solve for \( x \).

\[
\begin{align*}
x \tan 31.5^\circ &= (600 + x) \tan 27^\circ & \text{Substitute.} \\
x \tan 31.5^\circ &= 600 \tan 27^\circ + x \tan 27^\circ & \text{Distribute.} \\
x \tan 31.5^\circ - x \tan 27^\circ &= 600 \tan 27^\circ & \text{Isolate } x. \\
x &= \frac{600 \tan 27^\circ}{\tan 31.5^\circ - \tan 27^\circ} & \text{Divide.} \\
x &= 2960 \text{ feet} & \text{Use a calculator.}
\end{align*}
\]
Use this value for \( x \) and the equation \( x \tan 31.5^\circ = y \) to solve for \( y \).

\[
x \tan 31.5^\circ = y \quad \text{Original equation} \\
2960 \tan 31.5^\circ = y \quad \text{Replace } x \text{ with } 2960. \\
1814 = y \quad \text{Use a calculator.}
\]
The height of the cliff is about 1814 feet.

Examine Examine the solution by finding the angles of elevation.

\[
\begin{align*}
\tan B &= \frac{y}{x} \\
\tan B &= \frac{1814}{2960} \\
B &= 31.5^\circ \\
\tan A &= \frac{y}{600 + x} \\
\tan A &= \frac{1814}{600 + 2960} \\
A &= 27^\circ
\end{align*}
\]
The solution checks.

Check for Understanding

Concept Check

1. Explain how to determine which trigonometric ratio to use when solving for an unknown measure of a right triangle. See margin.

2. OPEN ENDED Draw a right triangle and label the measure of the hypotenuse and the measure of one acute angle. Then solve for the remaining measures.

3. Compare the measure of the angle of elevation and the measure of the angle of depression for two objects. What is the relationship between their measures? They are equal.

Guided Practice

For each triangle, find \( \sin Y \), \( \cos Y \), and \( \tan Y \) to the nearest ten thousandth.

4. \( \sin Y = 0.8, \cos Y = 0.6, \tan Y = 1.3333 \)

5. \( \sin Y = 0.3846, \cos Y = 0.9231, \tan Y = 0.4167 \)

Use a calculator to find the value of each trigonometric ratio to the nearest ten thousandth.

6. \( \sin 60^\circ = 0.8660 \)

7. \( \cos 75^\circ = 0.2588 \)

8. \( \tan 10^\circ = 0.1763 \)

Use a calculator to find the measure of each angle to the nearest degree.

9. \( \sin W = 0.9848, W = 80^\circ \)

10. \( \cos X = 0.6157, X = 52^\circ \)

11. \( \tan C = 0.3249, C = 18^\circ \)

Answers

1. If you know the measure of the hypotenuse, use sine or cosine, depending on whether you know the measure of the adjacent side or the opposite side. If you know the measures of the two legs, use tangent.

2. Sample answer:

\[
\begin{align*}
A &= 180^\circ - (90^\circ + 50^\circ) \text{ or } 40^\circ \\
\sin 50^\circ &= \frac{AC}{10} \\
\cos 50^\circ &= \frac{BC}{10} \\
AC &= 7.66 \\
BC &= 6.43
\end{align*}
\]
For each triangle, find the measure of the indicated angle to the nearest degree.


Solve each right triangle. State the side lengths to the nearest tenth and the angle measures to the nearest degree.

15. 16. 17.

DRIVING The percent grade of a road is the ratio of how much the road rises or falls in a given horizontal distance. If a road has a vertical rise of 40 feet for every 1000 feet horizontal distance, calculate the percent grade of the road and the angle of elevation the road makes with the horizontal. 4% grade, about 2.3°

Application

18. For each triangle, find \( \sin R \), \( \cos R \), and \( \tan R \) to the nearest ten thousandth.

19–24. See margin.

Use a calculator to find the value of each trigonometric ratio to the nearest ten thousandth.

25. \( \sin 30° \) 0.5
26. \( \sin 80° \) 0.9848
27. \( \cos 45° \) 0.7071
28. \( \cos 48° \) 0.6691
29. \( \tan 32° \) 0.6249
30. \( \tan 15° \) 0.2679
31. \( \tan 67° \) 2.3559
32. \( \sin 53° \) 0.7986
33. \( \cos 12° \) 0.9781

Use a calculator to find the measure of each angle to the nearest degree.

34. \( \cos V = 0.5000 \) 60°
35. \( \cos Q = 0.7658 \) 40°
36. \( \sin K = 0.9781 \) 78°
37. \( \sin A = 0.8827 \) 62°
38. \( \tan S = 1.2401 \) 51°
39. \( \tan H = 0.6473 \) 33°
40. \( \sin V = 0.3832 \) 23°
41. \( \cos M = 0.9793 \) 12°
42. \( \tan L = 3.6541 \) 75°
For each triangle, find the measure of the indicated angle to the nearest degree.

43. 44. 45.

46. 47. 48.

49. 50. 51.

Solve each right triangle. State the side lengths to the nearest tenth and the angle measures to the nearest degree.

52. A 53. C 54.

55. 60.

58. \( \triangle A = 53^\circ \)

A submarine is traveling parallel to the surface of the water 626 meters below the surface. The sub begins a constant ascent to the surface so that it will emerge on the surface after traveling 4420 meters from the point of its initial ascent.

61. What angle of ascent did the submarine make? about 8.1°

62. What horizontal distance did the submarine travel during its ascent? 4375 m

---

Modern Art

The painting below, neatly titled "Right Triangles," was painted by the artist known only as "Jessica." Using the information about right triangles, see if you can determine the dimensions of the painting. (Note: The triangle that includes \( \angle J \) is isosceles.)
Open-Ended Assessment
Writing  Ask students to sketch a right triangle, label the sides and angles with their measures, and find the trigonometric ratios for the two acute angles.

Assessment Options
Quiz (Lesson 11-7) is available on p. 700 of the Chapter 11 Resource Masters.

Answers
66. Let \( \sin A = \frac{a}{c} \) and let \( \cos A = \frac{b}{c} \), where \( a \) and \( b \) are legs of a right triangle and \( c \) is the hypotenuse.

Then \( \sin^2 A + \cos^2 A = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} \).

Since the Pythagorean Theorem states that \( a^2 + b^2 = c^2 \), the expression becomes \( \frac{a^2}{c^2} + \frac{b^2}{c^2} = 1 \) or \( 1 \).

Thus \( \sin^2 A + \cos^2 A = 1 \).

67. If you know the distance between two points and the angles from these two points to a third point, you can determine the distance to the third point by forming a triangle and using trigonometric ratios. Answers should include the following.

- If you measure your distance from the mountain and the angle of elevation to the peak of the mountain from two different points, you can write an equation using trigonometric ratios to determine its height, similar to Example 5.
- You need to know the altitude of the two points you are measuring.

68. See margin.

66. CRITICAL THINKING  An important trigonometric identity is \( \sin^2 A + \cos^2 A = 1 \).

Use substitution to solve each system of equations.

67. Writing in Math  Answer the question that was posed at the beginning of the lesson. See margin.

How are trigonometric ratios used in surveying?

Include the following in your answer:

- any explanation of how trigonometric ratios are used to measure the height of a mountain, and
- any additional information you need to know about the point from which you are measuring in order to find the altitude of a mountain.

For Exercises 68 and 69, use the figure at the right.

68. \( RT \) is equal to \( TS \). What is \( RS \)?

\( \square \, 2\sqrt{6} \quad \square \, 2\sqrt{3} \quad \square \, 4\sqrt{3} \quad \square \, 2\sqrt{2} \)

69. What is the measure of \( \angle Q \)?

\( \square \, 25^\circ \quad \square \, 30^\circ \quad \square \, 45^\circ \quad \square \, 60^\circ \)

Maintain Your Skills

Mixed Review

For each set of measures given, find the measures of the missing sides if \( \triangle KLM \sim \triangle NOP \). (Lesson 11-6)

70. \( k = 5, \ell = 3, m = 6, n = 10 \quad a = 6, p = 12 \)

71. \( \ell = 9, m = 3, n = 12, p = 4.5 \quad k = 8, a = 13.5 \)

Find the possible values of \( a \) if the points with the given coordinates are the indicated distance apart. (Lesson 11-5)

72. \( (9, 28), (a, -8); d = 39 \quad -6 \text{ or } 24 \)

73. \( (3, a), (10, -1); d = \sqrt{65} \quad -5 \text{ or } 3 \)

Find each product. (Lesson 8-6)

74. \( c^2(c^2 + 3c) = c^4 + 3c^3 \)

75. \( s(4s^2 - 9s + 12) = 4s^3 - 9s^2 + 12s \)

76. \( xy^2(2x^2 + 5xy - 7y^2) = 2x^3y^2 + 5x^2y^3 - 7xy^4 \)

Use substitution to solve each system of equations. (Lesson 7-2)

77. \( a = 3b + 2 (11, 3) \quad 4a - 7b = 23 \)

78. \( p + q = 10 (3, 7) \quad 3p - 2q = -5 \)

79. \( 3r + 6s = 0 (-2, 1) \quad -4r - 10s = -2 \)
The Language of Mathematics

The language of mathematics is a specific one, but it borrows from everyday language, scientific language, and world languages. To find a word’s correct meaning, you will need to be aware of some confusing aspects of language.

### Confusing Aspect

<table>
<thead>
<tr>
<th>Confusing Aspect</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>Some words are used in English and in mathematics, but have distinct meanings.</td>
<td>factor, leg, prime, power, rationalize</td>
</tr>
<tr>
<td>Some words are used in English and in mathematics, but the mathematical meaning is more precise.</td>
<td>difference, even, similar, slope</td>
</tr>
<tr>
<td>Some words are used in science and in mathematics, but the meanings are different.</td>
<td>divide, radical, solution, variable</td>
</tr>
<tr>
<td>Some words are only used in mathematics.</td>
<td>decimal, hypotenuse, integer, quotient</td>
</tr>
<tr>
<td>Some words have more than one mathematical meaning.</td>
<td>base, degree, range, round, square</td>
</tr>
<tr>
<td>Sometimes several words come from the same root word.</td>
<td>polygon and polynomial, radical and radicand</td>
</tr>
<tr>
<td>Some mathematical words sound like English words.</td>
<td>cosine and cosign, sine and sign, sum and some</td>
</tr>
<tr>
<td>Some words are often abbreviated, but you must use the whole word when you read them.</td>
<td>cos for cosine, sin for sine, tan for tangent</td>
</tr>
</tbody>
</table>

Words in boldface are in this chapter.

### Reading to Learn 1–3. See pp. 639A–639B.

1. How do the mathematical meanings of the following words compare to the everyday meanings?
   - a. factor
   - b. leg
   - c. rationalize

2. State two mathematical definitions for each word. Give an example for each definition.
   - a. degree
   - b. range
   - c. round

3. Each word below is shown with its root word and the root word’s meaning. Find three additional words that come from the same root.
   - a. domain, from the root word domus, which means house
   - b. radical, from the root word radix, which means root
   - c. similar, from the root word similis, which means like

### Getting Started

Have students think of words that have more than one meaning. As students respond, make a list of the words on the chalkboard along with their multiple definitions.

### Teach

- Place students in small groups. Assign each group several of the words listed in this activity to define. Have students use their textbooks, dictionaries, or the Internet to find the definitions. Then, as you discuss each group of words, ask the groups to give the definitions.

### Assess

#### Study Notebook

Ask students to summarize what they have learned about the language of mathematics.

### ELL

English Language Learners may benefit from writing key concepts from this activity in their Study Notebooks in their native language and then in English.
Round 1

Concepts (5 questions)

- This alphabetical list of vocabulary terms in Chapter 11 includes a page reference where each term was introduced.
- Assessment A vocabulary test/review for Chapter 11 is available on p. 698 of the Chapter 11 Resource Masters.

Lesson-by-Lesson Review

For each lesson,
- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

The Vocabulary PuzzleMaker software improves students’ mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Vocabulary and Concept Check

- angle of depression (p. 625)
- hypotenuse (p. 605)
- rationalizing the denominator (p. 588)
- angle of elevation (p. 625)
- leg (p. 605)
- similar triangles (p. 616)
- conjugate (p. 589)
- Pythagorean triple (p. 606)
- sine (p. 623)
- cosine (p. 623)
- radical equation (p. 598)
- tangent (p. 623)
- radical expression (p. 586)
- solve a triangle (p. 625)
- radicand (p. 586)
- trigonometric ratios (p. 623)

State whether each sentence is true or false. If false, replace the underlined word, number, expression, or equation to make a true sentence.

1. The binomials \(-3 + \sqrt{7}\) and \(3 - \sqrt{7}\) are conjugates. false, \(-3 - \sqrt{7}\)

2. In the expression \(-4\sqrt{5}\), the radicand is 5. true

3. The sine of an acute angle of a right triangle is the measure of the opposite leg divided by the measure of the hypotenuse. true

4. The longest side of a right triangle is the hypotenuse. true

5. After the first step in solving \(\sqrt{3x + 19} = x + 3\), you would have \(3x + 19 = x^2 + 9\). false, \(3x + 19 = x^2 + 6x + 9\)

6. The two sides that form the right angle in a right triangle are called the legs of the triangle. true

7. The expression \(\frac{2x\sqrt{3x}}{\sqrt{6y}}\) is in simplest radical form. false, \(\frac{x\sqrt{2xy}}{y}\)

8. A triangle with sides having measures of 25, 20, and 15 is a right triangle. true

Lesson-by-Lesson Review

11-1 Simplifying Radical Expressions

Concept Summary

- A radical expression is in simplest form when no radicands have perfect square factors other than 1, no radicands contain fractions, and no radicals appear in the denominator of a fraction.

Example

Simplify \(\frac{3}{\sqrt{5} - \sqrt{2}}\).

\[
\frac{3}{\sqrt{5} - \sqrt{2}} = \frac{3}{\sqrt{5} - \sqrt{2}} \cdot \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}}
\]

Multiply by \(\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}}\) to rationalize the denominator.

\[
= \frac{3(\sqrt{5}) + 3\sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}
\]

\[
= \frac{15 + 3\sqrt{2}}{25 - 2}
\]

\[
= \frac{15 + 3\sqrt{2}}{23}
\]

Simplify.

www.algebra1.com/vocabulary_review

Foldables Study Organizer

Have students flip back through their Foldables to make sure they have included information for every lesson.

Encourage students to refer to their Foldables while completing the Study Guide and Review and use them in preparing for the Chapter Test.

9. \[ \sqrt{\frac{60}{27}} \]

10. \[ \sqrt{\frac{44p^2}{b^5}} \]

11. \[ (3 - 2\sqrt{12})^2 \]

12. \[ \frac{27 - 9\sqrt{2}}{7} \]

13. \[ \frac{2\sqrt{2}}{3\sqrt{5} + 5\sqrt{3}} \]

14. \[ \frac{\sqrt{3p^4}}{8q^{10}} \]

15. \[ 5 - 3\sqrt{6} + 6\sqrt{3} + 5\sqrt{3} \]

16. \[ \frac{\sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3}}{2} \]

17. \[ 2\sqrt{3} - \sqrt{5}(\sqrt{10} + 4\sqrt{6}) \]

18. \[ 2\sqrt{3} + 8\sqrt{5} - 3\sqrt{5} + 3\sqrt{3} \]

19. \[ 4\sqrt{2} + 6\sqrt{3} + 36\sqrt{3} \]

20. \[ 5\sqrt{18} - 3\sqrt{112} - 3\sqrt{98} \]

21. \[ -6\sqrt{2} - 12\sqrt{7} \]

22. \[ 18\sqrt{10} + 30 + 6\sqrt{2} + 2\sqrt{5} \]

23. \[ \frac{\sqrt{3} - \sqrt{2}(2\sqrt{2} + \sqrt{3})}{\sqrt{6} - 1} \]

24. \[ 6\sqrt{5} + 2\sqrt{3\sqrt{2} + \sqrt{5}} \]

**Operations with Radical Expressions**

**Concept Summary**

- Radical expressions with like radicands can be added or subtracted.
- Use the FOIL Method to multiply radical expressions.

**Examples**

1. Simplify \( \sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3} \).

\[
\begin{align*}
\sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3} &= \sqrt{6} - \sqrt{3^2 \cdot 6} + 3\sqrt{2^2 \cdot 3} + 5\sqrt{3} \\
&= \sqrt{6} - 3\sqrt{6} + 3\sqrt{2} \cdot \sqrt{3} + 5\sqrt{3} \\
&= \sqrt{6} - 3\sqrt{6} + 3\sqrt{6} + 5\sqrt{3} \\
&= \sqrt{6} - 3\sqrt{6} + 6\sqrt{6} + 5\sqrt{3} \\
&= -2\sqrt{6} + 11\sqrt{3} \\
&= \text{Simplify radicands.}
\end{align*}
\]

2. Find \( (2\sqrt{3} - \sqrt{5})(\sqrt{10} + 4\sqrt{6}) \).

\[
\begin{align*}
(2\sqrt{3} - \sqrt{5})(\sqrt{10} + 4\sqrt{6}) &= \text{First terms} + \text{Outer terms} + \text{Inner terms} + \text{Last terms} \\
&= 2\sqrt{3}\sqrt{10} + 2\sqrt{3}(4\sqrt{6}) + (-\sqrt{5}\sqrt{10}) + (-\sqrt{5}(4\sqrt{6}) \\
&= 2\sqrt{30} + 8\sqrt{18} - \sqrt{50} - 4\sqrt{30} \\
&= 2\sqrt{30} + 8\sqrt{3^2 \cdot 2} - \sqrt{5^2 \cdot 2} - 4\sqrt{30} \\
&= 2\sqrt{30} + 24\sqrt{2} - 5\sqrt{2} - 4\sqrt{30} \\
&= -2\sqrt{30} + 19\sqrt{2} \\
&= \text{Prime factorization} \\
&= \text{Multiply.} \\
&= \text{Combine like terms.} \\
&= \text{Simplify.}
\end{align*}
\]
11-3 Radical Equations

Concept Summary
- Solve radical equations by isolating the radical on one side of the equation.
- Square each side of the equation to eliminate the radical.

Example
Solve $\sqrt{5 - 4x} - 6 = 7$.

Original equation

$\sqrt{5 - 4x} = 13$  Add 6 to each side.

$5 - 4x = 169$  Square each side.

$-4x = 164$  Subtract 5 from each side.

$x = -41$  Divide each side by $-4$.

Exercises  Solve each equation. Check your solution.  See Examples 2 and 3 on page 599.

25. $10 + 2\sqrt{b} = 0$  no solution  26. $\sqrt{a} + 4 = 6$  32.  27. $\sqrt{7x - 1} = 5$  36  28. $\frac{4a}{3} - 2 = 0$  3  29. $\sqrt{x} + 4 = x - 8$  12  30. $\sqrt{3x - 14} + x = 6$  5

11-4 The Pythagorean Theorem

Concept Summary
- If $a$ and $b$ are the measures of the legs of a right triangle and $c$ is the measure of the hypotenuse, then $c^2 = a^2 + b^2$.
- If $a$ and $b$ are measures of the shorter sides of a triangle, $c$ is the measure of the longest side, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Example
Find the length of the missing side.

$c^2 = a^2 + b^2$  Pythagorean Theorem

$25^2 = 15^2 + b^2$  $c = 25$ and $a = 15$

$625 = 225 + b^2$  Evaluate squares.

$400 = b^2$  Subtract 225 from each side.

$20 = b$  Take the square root of each side.

Exercises  If $c$ is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round answers to the nearest hundredth.  See Example 2 on page 606.  31–36. See margin.

31. $a = 30, b = 16, c = ?$  32. $a = 6, b = 10, c = ?$  33. $a = 10, c = 15, b = ?$

34. $b = 4, c = 56, a = ?$  35. $a = 18, c = 30, b = ?$  36. $a = 1.2, b = 1.6, c = ?$

Determine whether the following side measures form right triangles.  See Example 4 on page 607.

37. 9, 16, 20  no  38. 20, 21, 29  yes  39. 9, 40, 41  yes  40. $18, \sqrt{24}, 30$  no
The Distance Formula

Concept Summary
- The distance \( d \) between any two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is given by
  \[
  d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
  \]

Example
Find the distance between the points with coordinates \((-5, 1)\) and \((1, 5)\).

\[
\begin{align*}
  d &= \sqrt{(1 - (-5))^2 + (5 - 1)^2} \\
  &= \sqrt{6^2 + 4^2} \\
  &= \sqrt{36 + 16} \\
  &= \sqrt{52} \text{ or about 7.21 units}
\end{align*}
\]

Exercises
Find the distance between each pair of points whose coordinates are given. Express in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary. See Example 1 on page 611.

41. \((9, 2), (1, 13)\)
42. \((4, 2), (7, 9)\)
43. \((4, -6), (-2, 7)\)
44. \((2\sqrt{5}, 9), (4\sqrt{5}, 3)\)
45. \((4, 8), (-7, 12)\)
46. \((-2, 6), (5, 11)\)

\[
\begin{align*}
  42. \sqrt{58} &\approx 7.62 \\
  43. \sqrt{205} &\approx 14.32
\end{align*}
\]

Similar Triangles

Concept Summary
- Similar triangles have congruent corresponding angles and proportional corresponding sides.
  - If \( \triangle ABC \sim \triangle DEF \), then \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \).

Example
Find the measure of side \( a \) if the two triangles are similar.

\[
\frac{10}{5} = \frac{6}{a} \quad \text{Corresponding sides of similar triangles are proportional.}
\]

\[
10a = 30 \\
\therefore a = 3 \quad \text{Divide each side by 10.}
\]
11-7 Trigonometric Ratios

Concept Summary
Three common trigonometric ratios are sine, cosine, and tangent.

- \( \sin A = \frac{BC}{AB} \)
- \( \cos A = \frac{AC}{AB} \)
- \( \tan A = \frac{BC}{AC} \)

Example
Find the sine, cosine, and tangent of \( \angle A \). Round to the nearest ten thousandth.

\[
\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{20}{25} = 0.8000 \\
\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{15}{25} = 0.6000 \\
\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{20}{15} = 1.3333
\]

Exercises For \( \triangle ABC \), find each value of each trigonometric ratio to the nearest ten thousandth. See Example 1 on page 624.

55. \( \cos B = 0.5283 \) 
56. \( \tan A = 0.6222 \) 
57. \( \sin B = 0.8491 \) 
58. \( \cos A = 0.8491 \) 
59. \( \tan B = 1.6071 \) 
60. \( \sin A = 0.5283 \)

Use a calculator to find the measure of each angle to the nearest degree. See Example 3 on page 624.

61. \( \tan M = 0.8043, \angle M = 39^\circ \) 
62. \( \sin T = 0.1212, \angle T = 7^\circ \) 
63. \( \cos B = 0.9781, \angle B = 12^\circ \) 
64. \( \cos F = 0.7443, \angle F = 42^\circ \) 
65. \( \sin A = 0.4540, \angle A = 27^\circ \) 
66. \( \tan Q = 5.9080, \angle Q = 80^\circ \)
Vocabulary and Concepts

Match each term and its definition.
1. measure of the opposite leg divided by the measure of the hypotenuse   \[ \text{b} \]
2. measure of the adjacent leg divided by the measure of the hypotenuse   \[ \text{a} \]
3. measure of the opposite leg divided by the measure of the adjacent leg   \[ \text{c} \]

Skills and Applications

Simplify.
1. \[ 4\sqrt{3} + 3\sqrt{7} \]
2. \[ 9 + \sqrt{2} - 3\sqrt{3} - \sqrt{6} \]
3. \[ 2\sqrt{27} + \sqrt{63} - 4\sqrt{3} \]
4. \[ \sqrt{6} + \frac{2}{\sqrt{3}} \]
5. \[ \sqrt{6} \]
6. \[ \sqrt{112x^2 y^6} \]
7. \[ \sqrt{\frac{4}{30}} \]
8. \[ \sqrt{64 + 12\sqrt{6} + 6\sqrt{2}} \]
9. \[ (1 - \sqrt{3}/3 + \sqrt{2}) \]

Solve each equation. Check your solution.
10. \[ \sqrt{10x} = 20 \]
11. \[ \sqrt{4x + 1} = 11 \]
12. \[ \sqrt{4x} + 1 = 5 \]
13. \[ x = \sqrt{-6x - 8} \] no solution
14. \[ x = 5x + 14 \]
15. \[ \sqrt{4x - 3} = 6 - x \]

If \( c \) is the measure of the hypotenuse of a right triangle, find each missing measure.
16. \[ \sqrt{164} \approx 12.81 \]
17. \[ \sqrt{72} \approx 8.49 \]
18. \[ \sqrt{120} \approx 10.95 \]
19. \[ \sqrt{4} \]
20. \[ \sqrt{1} \]
21. \[ \sqrt{3} \]

If necessary, round to the nearest hundredth.
16. \[ a = 8, b = 10, c = ? \]
17. \[ a = 6\sqrt{2}, c = 12, b = ? \]
18. \[ b = 13, c = 17, a = ? \]

Find the distance between each pair of points whose coordinates are given.
19. \[ (4, 7), (4, -2) \]
20. \[ (-1, 1), (1, -5) \]

For each set of measures given, find the measures of the missing sides if \( \triangle ABC \sim \triangle JKH \).
22. \[ c = 20, h = 15, k = 16, j = 12 \]
23. \[ c = 12, b = 13, a = 6, h = 10 \]
24. \[ k = 5, c = 6.5, b = 7.5, a = 4.5 \]
25. \[ h = 1\frac{1}{2}, c = 4\frac{1}{2}, k = 2\frac{1}{4}, a = 3 \]

Solve each right triangle. State the side lengths to the nearest tenth and the angle measures to the nearest degree.
26. \[ m\angle A = 44^\circ \]
27. \[ m\angle B = 46^\circ \]
28. \[ m\angle A = 36^\circ \]
29. \[ m\angle B = 54^\circ \]
30. \[ m\angle C = 25.8^\circ \]
31. \[ a = 20 \]
32. \[ b = 7.4 \]
33. \[ c = 6.7 \]

Portfolio Suggestion

Introduction In this chapter, students are introduced to ways in which professionals such as architects and surveyors can use math to solve problems.

Ask Students Pick a profession such as architecture, surveying, or engineering and research to find out how triangles, ratios, or trigonometric ratios are used in the field. Write a short report in which you explain how these concepts are used, along with examples.
These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the Chapter 11 Resource Masters.

### Additional Practice
See pp. 703–704 in the Chapter 11 Resource Masters for additional standardized test practice.

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**Test-Taking Tip**

Questions 7, 21 and 22

Be sure that you know and understand the Pythagorean Theorem. References to right angles, the diagonal of a rectangle, or the hypotenuse of a triangle indicate that you may need to use the Pythagorean Theorem to find the answer to an item.

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**ExamView® Pro**

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.
Part 2  Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. A line is parallel to the line represented by the equation \( \frac{1}{2}y + \frac{3}{2}x + 4 = 0 \). What is the slope of the parallel line? (Lesson 5-6) \(-3\)

11. Graph the solution of the system of linear inequalities \( 2x - y > 2 \) and \( 3x + 2y < -4 \). (Lesson 6-6) See margin.

12. The sum of two integers is 66. The second integer is 18 more than half of the first. What are the integers? (Lesson 7-2) \(32 \) and \(34\)

13. The function \( h(t) = -16t^2 + v_0t + h_0 \) describes the height in feet above the ground \( h(t) \) of an object thrown vertically from a height of \( h_0 \) feet, with an initial velocity of \( v_0 \) feet per second, if there is no air friction and \( t \) is the time in seconds. How many seconds will it take for the ball to reach the ground? (Lesson 9-5) \(5\)


14. Find all values of \( x \) that satisfy the equation \( x^2 - 8x + 6 = 0 \). Approximate irrational numbers to the nearest hundredth. (Lesson 10-4) \(7.16\) and \(0.84\)

15. Simplify the expression \( \sqrt{3\sqrt{81}} \). (Lesson 11-1) \(3\)

16. Simplify the expression \( \left( \frac{x^3}{x} \right)^{\frac{1}{2}} \). (Lesson 11-1) \(x^{\frac{1}{2}}\) or \(x\sqrt{x}\)

17. The area of a rectangle is 64. The length is \(x^3\) and the width is \(\frac{x + 1}{x}\). What is \(x\)? (Lesson 11-3) \(8\) or \(-8\)

Answers

www.algebra1.com/standardized_test/sol

Part 3  Extended Response

Record your answers on a sheet of paper. Show your work.

18. A rectangle is \(5\sqrt{7} + \sqrt{3}\) centimeters long and \(7\sqrt{7} - 2\sqrt{3}\) centimeters wide. (Lesson 11-2)
   a. Find the perimeter of the rectangle in simplest form. \(24\sqrt{17} - 2\sqrt{3}\) cm
   b. Find the area of the rectangle in simplest form. \(239 - 3\sqrt{21}\) cm²

19. Haley hikes 3 miles north, 7 miles east, and then 6 miles north again. (Lesson 11-4)
   a. Draw a diagram showing the direction and distance of each segment of Haley’s hike. Label Haley’s starting point, her ending point, and the distance, in miles, of each segment of her hike. See margin.
   b. To the nearest tenth of a mile, how far (in a straight line) is Haley from her starting point? 11.4 mi
c. See margin.
   c. How did your diagram help you to find Haley’s distance from her starting point?
   d. Describe the direction and distance of Haley’s return trip back to her starting position if she used the same trail. Haley would hike 6 miles south, 7 miles west, and 3 miles south.

20. Suppose a coordinate grid is superimposed on a city. Ben lives at \((-5, -8)\), and Sophie lives at \((9, 1)\). Each grid unit is equal to 0.13 mile. (Lessons 11-4 and 11-5)
   a. Describe in grid units one way to travel from Sophie’s to Ben’s.
   b. Find the distance between their two houses. about 2.16 mi
   c. Whose house is closer to a bakery located at \((-1, 5)\)? Sophie’s

21. Janelle is swimming in the ocean and sees a cliff diver who is about to dive into the water. The diver is 70 feet above the water, and the angle of elevation from Janelle to the diver is 22°. (Lesson 11-7)
   a. How far is Janelle from the diver standing on top of the cliff? Round to the nearest tenth. 186.9 ft
   b. How far is Janelle from the diver when he enters the water? Round to the nearest tenth. 173.3 ft

19c. The sketch shows that the distance she is from her starting point is the length of the hypotenuse of a right triangle with legs 7 mi and 9 mi long.
Page 604, Follow-Up to Lesson 11-3
Graphing Calculator Investigation

1. $\{x | x \geq 0\}$; shifted up 1

2. $\{x | x \geq 0\}$; shifted down 3

3. $\{x | x \geq -2\}$; shifted left 2

4. $\{x | x \geq 5\}$; shifted right 5

5. $\{x | x \leq 0\}$; reflected across $y$-axis

6. $\{x | x \geq 0\}$; expanded

7. $\{x | x \geq 0\}$; reflected across $x$-axis

8. $\{x | x \leq 1\}$; reflected across $y$-axis, shifted right 1, up 6

9. $\{x | x \geq -2.5\}$; shifted left 2.5, down 4
44. You can determine the distance between two points by forming a right triangle. Drawing a line through each point parallel to the axes forms the legs of the triangle. The hypotenuse of this triangle is the distance between the two points. You can find the lengths of each leg by subtracting the corresponding x- and y-coordinates, then use the Pythagorean Theorem. Answers should include the following.

- You can draw lines parallel to the axes through the two points that will intersect at another point forming a right triangle. The length of a leg of a triangle is the difference in the x- or y-coordinates. The length of the hypotenuse is the distance between the points. Using the Pythagorean Theorem to solve for the hypotenuse, you have the Distance Formula.
- The points are on a vertical line so you can calculate distance by determining the absolute difference between the y-coordinates.

57.

58.

59.

60.

61.

62.