Introduction

In this unit, students begin by applying the properties of real numbers to expressions, equalities, and inequalities, including absolute value inequalities and compound inequalities. Throughout the unit, students explore the relationship between linear equations and their graphs.

These explorations include modeling data with scatter plots and lines of regression, as well as linear programming and solving systems of equations. The unit concludes with instruction about operations on matrices and using matrices to solve systems of equations.

Assessment Options

Unit 1 Test  Pages 237–238 of the Chapter 4 Resource Masters may be used as a test or review for Unit 1. This assessment contains both multiple-choice and short answer items.

TestCheck and Worksheet Builder

This CD-ROM can be used to create additional unit tests and review worksheets.
“Buying a home,’ says Housing and Urban Development Secretary Andrew Cuomo, ‘is the most expensive, most complicated and most intimidating financial transaction most Americans ever make.’” In this project, you will be exploring how functions and equations relate to buying a home and your income.

Log on to www.algebra2.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 1.
# Chapter 1

## Solving Equations and Inequalities

### Chapter Overview and Pacing

#### LESSON OBJECTIVES

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Description</th>
<th>Regular</th>
<th>Block</th>
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</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Expressions and Formulas (pp. 6–10)</td>
<td>1</td>
<td>optional</td>
</tr>
<tr>
<td></td>
<td>• Use the order of operations to evaluate expressions.</td>
<td></td>
<td>0.5</td>
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<tr>
<td></td>
<td>• Use formulas.</td>
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<tr>
<td>1-2</td>
<td>Properties of Real Numbers (pp. 11–19)</td>
<td>2 (with 1-2 Follow-Up)</td>
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<tr>
<td></td>
<td>• Classify real numbers.</td>
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<tr>
<td></td>
<td>• Use the properties of real numbers to evaluate expressions.</td>
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<tr>
<td></td>
<td><strong>Follow-Up:</strong> Investigating Polygons and Patterns</td>
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<tr>
<td>1-3</td>
<td>Solving Equations (pp. 20–27)</td>
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<tr>
<td></td>
<td>• Translate verbal expressions into algebraic expressions and equations, and vice versa.</td>
<td></td>
<td>1 (with 1-2 Follow-Up)</td>
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<tr>
<td></td>
<td>• Solve equations using the properties of equality.</td>
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<tr>
<td>1-4</td>
<td>Solving Absolute Value Equations (pp. 28–32)</td>
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<td></td>
<td>• Evaluate expressions involving absolute values.</td>
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<td></td>
<td>• Solve absolute value equations.</td>
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<td>1-5</td>
<td>Solving Inequalities (pp. 33–39)</td>
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<tr>
<td></td>
<td>• Solve inequalities.</td>
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<td></td>
<td>• Solve real-world problems involving inequalities.</td>
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<td>1-6</td>
<td>Solving Compound and Absolute Value Inequalities (pp. 40–46)</td>
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<td>• Solve compound inequalities.</td>
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<td></td>
<td>• Solve absolute value inequalities.</td>
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<td><strong>Study Guide and Practice Test (pp. 47–51)</strong></td>
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<td><strong>Standardized Test Practice (pp. 52–53)</strong></td>
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**TOTAL** 9 3 4.5 1.5

Pacing suggestions for the entire year can be found on pages T20–T21.
## Chapter Resource Manager

### CHAPTER 1 RESOURCE MASTERS

<table>
<thead>
<tr>
<th>Materials</th>
<th>Study Guide and Intervention</th>
<th>Practice (Skills and Average)</th>
<th>Reading to Learn Mathematics</th>
<th>Enrichment</th>
<th>Assessment</th>
<th>Applications</th>
<th>5-Minute Check Transparencies</th>
<th>Interactive Chalkboard</th>
<th>Alge2PASS: Tutorial Plus (lessons)</th>
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<tbody>
<tr>
<td>1–2</td>
<td>3–4</td>
<td>5</td>
<td>6</td>
<td>SC 1, SM 91–96</td>
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<td>51</td>
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<td>algebra tiles, index cards</td>
<td>(Follow:Up: ruler or geometry software)</td>
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<td>13–14</td>
<td>15–16</td>
<td>17</td>
<td>18</td>
<td>51, 53</td>
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<td>21–22</td>
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<td>24</td>
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*Key to Abbreviations: GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual*
Expressions and Formulas

An algebraic expression usually contains at least one variable and may also contain numbers and operations. The order of operations is a mathematical convention for deciding which operations are performed before others in an algebraic expression. That order is: evaluate powers; multiply and divide from left to right; and add and subtract from left to right. There is one more part to the convention: any grouping symbol (parentheses, brackets, braces, fraction bar) takes first priority. To evaluate an expression means to replace each variable with its given value and then follow the order of operations to simplify. A formula is an equation in which one variable is set equal to an algebraic expression.

Properties of Real Numbers

The set \( \mathbb{N} \) of natural numbers is \{1, 2, 3, \ldots\}; add zero and the result is the set \( \mathbb{W} \) of whole numbers. The set \( \mathbb{Z} \) of integers is \{\ldots, -2, -1, 0, 1, 2, \ldots\} and the numbers in the set \( \mathbb{Q} \) of rational numbers have the form \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \neq 0 \). The rationals, along with the set \( \mathbb{I} \) of irrational numbers, make up the set \( \mathbb{R} \) of real numbers. There is a one-to-one correspondence between the real numbers and the points on a line in that each real number corresponds to exactly one point on a line and each point on a line corresponds to exactly one real number.

Properties of real numbers are used to justify the steps of solving equations and describing mathematical relationships. These include the commutative and associative properties of addition and the commutative and associative properties of multiplication. Another property, the distributive property, relates addition and multiplication. The real numbers include an identity element for the operation of addition, an identity element for the operation of multiplication, an additive inverse for every real number, and a multiplicative inverse for every real number except 0.

Solving Equations

A mathematic sentence with an equal sign between two algebraic or arithmetic expressions is called an equation. To solve an equation requires a series of equations, equivalent to the given equation, that result in a final equation that isolates the variable on one side. That final equation presents the solution to the original equation. However, solutions should always be substituted into the original equation to check for correctness.

The rules for writing equivalent equations are called Properties of Equality. We can write the equation...
1-4 Solving Absolute Value Equations

The absolute value of a number is its distance from zero. Described algebraically, the definition of absolute value is \(|a| = a\) if \(a \geq 0\) and \(|a| = -a\) if \(a < 0\). The absolute value symbols are a grouping symbol like parentheses or a fraction bar. For example, to evaluate \(2 \cdot |15 - 31|\), first calculate inside the symbols. So, \(2 \cdot |15 - 31| = 2 \cdot |-16| = 2 \cdot (16) = 32\).

The equation \(|a - 6| = 4\) can be interpreted as the distance between \(a\) and \(6\) is 4 units. The value \(a - 6\) can be 4 or -4, so if \(a - 6 = 4\), then \(a = 10\). If \(a - 6 = -4\), then \(a = 2\). The solution is {2, 10}. “No solution” can be written as {} or \(\emptyset\), the symbols for the empty set.

1-5 Solving Inequalities

An inequality is a mathematical sentence with one of the symbols \(<, \leq, >, \text{ or } \geq\) between two expressions. Solving an inequality means writing a series of equivalent inequalities, ending with one that isolates the variable. The rules for writing equivalent inequalities are called properties of inequality. (The properties hold for all inequalities, but are usually expressed initially in terms of \(>\).) If \(a > b\), then we can write \(a + c > b + c\) and \(a - c > b - c\). Also, if \(a > b\) and \(c > 0\), then we can write \(ac > bc\) and \(\frac{a}{c} > \frac{b}{c}\). If \(c < 0\), we can write \(ac < bc\) and \(\frac{a}{c} < \frac{b}{c}\). In general, multiplying or dividing an inequality by a negative number reverses the order of the inequality. The Trichotomy Property states that for any two real numbers, either the values are equal or one value is greater than the other. In symbols, exactly one of these statements is true: \(a < b\), \(a = b\), or \(a > b\).

When the solution to an inequality is graphed, an open circle indicates a value that is not included and a closed circle indicates a value that is included. Open circles are used with \(<\) and \(>\), and closed circles are used with \(\leq\) and \(\geq\). Solutions to inequalities are often written using set-builder notation, so a solution such as \(x \geq 4\) would be written \([x| x \geq 4]\), read the set of values \(x\) such that \(x\) is greater than or equal to 4.

1-5 Solving Compound and Absolute Value Inequalities

There are important connections between compound inequalities and absolute value inequalities. An absolute value inequality using \(<\) or \(\leq\) is related to a compound inequality using the word and. For example, thinking of \(|a| < 7\) as \(|a - 0| < 7\), then the value of \(a\) is any number whose distance from 0 is less than 7 units.

|Possible values for \(a\)|
|---|---|---|---|---|---|
|\(-8\)| \(-6\)| \(-4\)| \(0\)| \(2\)| \(4\)| \(6\)| \(8\)|
|\(|a| < 7\) means \(-7 < a \text{ and } a < 7\) or \(-7 < a < 7\)|

An absolute value inequality using \(>\) or \(\geq\) is related to a compound inequality using the word or. For example, thinking of \(|b| > 5\) as \(|b - 0| > 5\), then the value of \(b\) is any number whose distance from 0 is more than 5.

|Possible values for \(b\)|
|---|---|---|---|---|
|\(-8\)| \(-6\)| \(-4\)| \(0\)| \(2\)| \(4\)| \(6\)| \(8\)|
|\(|b| > 5\) means \(b < -5 \text{ or } b > 5\)|

To solve absolute value inequalities, use two patterns. One pattern is to rewrite \(|A| < B\) as \(-B < A \text{ and } A < B\) (or \(-B < A < B\), so rewrite \(|2x - 5| < 18\) as \(-18 < 2x - 5\) and \(2x - 5 < 18\). The solution is \(-\frac{13}{2} < x < \frac{23}{2}\). The other pattern is to rewrite \(|A| > B\) as \(A < -B\) or \(A > B\), so rewrite the inequality \(|3x + 1| > 15\) as \(3x + 1 < -15\) or \(3x + 1 > 15\). The solution is \(x < -\frac{16}{3}\) or \(x > \frac{14}{3}\).
## TestCheck and Worksheet Builder

This networkable software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

## Key to Abbreviations

- **TWE** = Teacher Wraparound Edition
- **CRM** = Chapter Resource Masters

## Chapter 1: Solving Equations and Inequalities

<table>
<thead>
<tr>
<th>Type</th>
<th>Student Edition</th>
<th>Teacher Resources</th>
<th>Technology/Internet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ongoing</td>
<td>Prerequisite Skills, pp. 5, 10, 18, 27, 32, 39</td>
<td>5-Minute Check Transparencies Quizzes, <strong>CRM</strong> pp. 51–52, Mid-Chapter Test, <strong>CRM</strong> p. 53, Study Guide and Intervention, <strong>CRM</strong> pp. 1–2, 7–8, 13–14, 19–20, 25–26, 31–32</td>
<td>Alge2PASS: Tutorial Plus <a href="http://www.algebra2.com/self_check_quiz">www.algebra2.com/self_check_quiz</a> <a href="http://www.algebra2.com/extra_examples">www.algebra2.com/extra_examples</a></td>
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<tr>
<td>Mixed Review</td>
<td>pp. 18, 27, 32, 39, 46</td>
<td>Cumulative Review, <strong>CRM</strong> p. 54</td>
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</tr>
<tr>
<td>Error Analysis</td>
<td>Find the Error, pp. 24, 43, Common Misconceptions, p. 12</td>
<td>Find the Error, <strong>TWE</strong> pp. 24, 44, Unlocking Misconceptions, <strong>TWE</strong> pp. 15, 18, 22, Tips for New Teachers, <strong>TWE</strong> pp. 10, 27</td>
<td></td>
</tr>
<tr>
<td>Chapter Assessment</td>
<td>Study Guide, pp. 47–50, Practice Test, p. 51</td>
<td>Multiple-Choice Tests (Forms 1, 2A, 2B), <strong>CRM</strong> pp. 37–42, Free-Response Tests (Forms 2C, 2D, 3), <strong>CRM</strong> pp. 43–48, Vocabulary Test/Review, <strong>CRM</strong> p. 50</td>
<td>TestCheck and Worksheet Builder (see below), MindJogger Videoquizzes <a href="http://www.algebra2.com/vocabulary_review">www.algebra2.com/vocabulary_review</a> <a href="http://www.algebra2.com/chapter_test">www.algebra2.com/chapter_test</a></td>
</tr>
</tbody>
</table>

**Additional Intervention Resources**

- The Princeton Review’s *Cracking the SAT & PSAT*
- The Princeton Review’s *Cracking the ACT*
- ALEKS
**Intervention Technology**

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

<table>
<thead>
<tr>
<th>Algebra 2 Lesson</th>
<th>Alge2PASS Lesson</th>
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<tbody>
<tr>
<td>1-4</td>
<td>1  Solving Multi-Operational Equations IV</td>
</tr>
<tr>
<td>1-5</td>
<td>2  Solving Inequalities</td>
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</table>

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student’s needs. Subscribe at www.k12aleks.com.

**Intervention at Home**

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  - www.algebra2.com/extra_examples
  - www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  - www.algebra2.com/vocabulary_review
  - www.algebra2.com/chapter_test
  - www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

**Reading and Writing in Mathematics**

*Glencoe Algebra 2* provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

**Student Edition**

- Foldables Study Organizer, p. 5
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 8, 14, 24, 30, 37, 43)
- Writing in Math questions in every lesson, pp. 10, 17, 27, 31, 38, 45
- Reading Study Tip, pp. 11, 12, 34, 35
- WebQuest, p. 27

**Teacher Wraparound Edition**

- Foldables Study Organizer, pp. 5, 47
- Study Notebook suggestions, pp. 8, 15, 19, 24, 30, 37, 43
- Modeling activities, pp. 18, 32
- Speaking activities, pp. 10, 27
- Writing activities, pp. 39, 46
- Differentiated Instruction, (Verbal/Linguistic), p. 29
- ELL Resources, pp. 4, 9, 17, 26, 29, 31, 38, 45, 47

**Additional Resources**

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 1 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 1 Resource Masters*, pp. 5, 11, 17, 23, 29, 35)
- Vocabulary PuzzleMaker software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- Teaching Mathematics with Foldables provides suggestions for promoting cognition and language.
- Reading and Writing in the Mathematics Classroom
- WebQuest and Project Resources

For more information on Reading and Writing in Mathematics, see pp. T6–T7.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Algebra allows you to write expressions, equations, and inequalities that hold true for most or all values of variables. Because of this, algebra is an important tool for describing relationships among quantities in the real world. For example, the angle at which you view fireworks and the time it takes you to hear the sound are related to the width of the fireworks burst. A change in one of the quantities will cause one or both of the other quantities to change. In Lesson 1-1, you will use the formula that relates these quantities.

The chart below correlates the objectives for each lesson to the NCTM Standards 2000. There is also space for you to reference your state and/or local objectives.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>NCTM Standards</th>
<th>Local Objectives</th>
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<tbody>
<tr>
<td>1-1</td>
<td>1, 2, 4, 8, 9</td>
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<td>1-2</td>
<td>1, 8, 9</td>
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<td>1-2 Follow-Up</td>
<td>1, 3, 9, 10</td>
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<td>1, 2, 4, 6, 8, 9</td>
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<td>1-6</td>
<td>1, 2, 6, 9, 10</td>
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</table>

Key to NCTM Standards:
1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 1 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 1 test.
Prerequisite Skills  To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lessons 1-1 through 1-3  Operations with Rational Numbers

Simplify.

1.  $20 - 0.16$  $19.84$
2.  $12.2 + (-8.45)$  $3.75$
3.  $-3.01 - 14.5$  $-17.51$
4.  $-1.8 + 17$  $15.2$
5.  $\frac{1}{4} - \frac{2}{3}$  $-\frac{5}{12}$
6.  $\frac{3}{5} + (-6)$  $-\frac{22}{5}$
7.  $-\frac{7}{2} + \frac{5}{3}$  $-\frac{1}{6}$
8.  $-11\frac{5}{8} - \left(-\frac{45}{8}\right)$  $-\frac{71}{8}$
9.  $(0.15)(3.2)$  $0.48$
10.  $2 ÷ (-0.4)$  $-5$
11.  $(-1.21) ÷ (-1.1)$  $1.1$
12.  $(-9)(0.036)$  $-0.324$
13.  $-4 + \frac{3}{2}$  $-2\frac{1}{2}$
14.  $\left(\frac{3}{4}\right) \left(-\frac{3}{10}\right)$  $-\frac{3}{8}$
15.  $(-2\frac{3}{4})(-3\frac{1}{2})$  $8\frac{4}{5}$
16.  $7\frac{1}{8} ÷ (-2)$  $-3\frac{9}{16}$

For Lesson 1-1  Powers

Evaluate each power.

17.  $2^3$  $8$
18.  $5^3$  $125$
19.  $(-7)^2$  $49$
20.  $(-1)^3$  $-1$
21.  $(-0.8)^2$  $0.64$
22.  $-(1.2)^2$  $-1.44$
23.  $\left(\frac{3}{5}\right)^2$  $\frac{4}{9}$
24.  $\left(-\frac{4}{11}\right)^2$  $\frac{16}{121}$

For Lesson 1-5  Compare Real Numbers

Identify each statement as true or false.

25.  $-5 < -7$  false
26.  $6 > -8$  true
27.  $-2 \geq -2$  true
28.  $-3 \geq -3.01$  true
29.  $-9.02 < -9.2$  false
30.  $\frac{1}{5} < \frac{1}{8}$  false
31.  $\frac{2}{5} \geq \frac{16}{40}$  true
32.  $\frac{3}{4} > 0.8$  false

Note-Taking and Charting Main Ideas  Use this Foldable study guide for student notes about equations and inequalities. Note-taking is a skill that is based upon listening or reading for main ideas and then recording those ideas for future reference. In the columns of their Foldable, have students take notes about the processes and procedures that they learn. Encourage students to apply what they know and what they learn as they analyze and solve equations and inequalities.
**1-1 Expressions and Formulas**

**Focus**

5-Minute Check Transparency 1-1 Use as a quiz or review of prerequisite skills.

Mathematical Background notes are available for this lesson on p. 4C.

Building on Prior Knowledge

In previous courses, students have performed operations on integers and used the order of operations. In this lesson, they should realize that using formulas requires these skills.

**How are formulas used by nurses?**

Ask students:
- What are the units for the flow rate \( F \)? drops per minute
- Why is 12 hours multiplied by 60? to convert the time from hours to minutes
- Medicine What might happen if the flow rate is too fast or slow? Too fast: the fluid might not be absorbed by the patient’s body as expected; too slow: the medication might not be effective.

**Vocabulary**
- order of operations
- variable
- algebraic expression
- formula

**What You’ll Learn**

- Use the order of operations to evaluate expressions.
- Use formulas.

**ORDER OF OPERATIONS**

A numerical expression such as \( \frac{15 - 1}{2} \) must have exactly one value. In order to find that value, you must follow the order of operations.

**Key Concept**

Order of Operations

1. Evaluate expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars, as in \( \frac{5 + 7}{2} \).
2. Evaluate all powers.
3. Do all multiplications and/or divisions from left to right.
4. Do all additions and/or subtractions from left to right.

Grouping symbols can be used to change or clarify the order of operations. When calculating the value of an expression, begin with the innermost set of grouping symbols.

**Example 1 Simplify an Expression**

Find the value of \( (2(10 - 4))^2 + 3 \div 5 \).

\[
(2(10 - 4))^2 + 3 \div 5 = (2(6))^2 + \frac{3}{5} = \frac{2(36)}{5} = \frac{72}{5} = 15
\]

The value is 15.

---

**Resource Manager**

**Workbook and Reproducible Masters**

*Chapter 1 Resource Masters*
- Study Guide and Intervention, pp. 1–2
- Skills Practice, p. 3
- Practice, p. 4
- Reading to Learn Mathematics, p. 5
- Enrichment, p. 6

*Science and Mathematics Lab Manual*, pp. 91–96

*School-to-Career Masters*, p. 1

*Transparencies*

5-Minute Check Transparency 1-1

Real-World Transparency 1

Answer Key Transparencies

**Technology**

Interactive Chalkboard
Scientific calculators follow the order of operations.

**Graphing Calculator Investigation**

**Order of Operations**

**Think and Discuss 2, 4, 5. See margin.**

1. Simplify $8 - 2 \times 4 + 5$ using a graphing calculator. 5
2. Describe the procedure the calculator used to get the answer.
3. Where should parentheses be inserted in $8 - 2 \times 4 + 5$ so that the expression has each of the following values?
   - a. $-10$ around $4 + 5$
   - b. $29$ around $8 - 2$
   - c. $-5$ around $2 \times 4 + 5$
4. Evaluate $18^2 \div (2 \times 3)$ using your calculator. Explain how the answer was calculated.
5. If you remove the parentheses in Exercise 4, would the solution remain the same? Explain.

**Variables** are symbols, usually letters, used to represent unknown quantities. Expressions that contain at least one variable are called algebraic expressions. You can evaluate an algebraic expression by replacing each variable with a number and then applying the order of operations.

**Example 2 Evaluate an Expression**

Evaluate $x^2 - y(x + y)$ if $x = 8$ and $y = 1.5$.

\[
x^2 - y(x + y) = 8^2 - 1.5(8 + 1.5) \quad \text{Replace} \ x \ \text{with} \ 8 \ \text{and} \ \text{y with} \ 1.5.
\]
\[
= 64 - 1.5(8 + 1.5) \quad \text{Find} \ 8^2.
\]
\[
= 64 - 1.5(9.5) \quad \text{Add} \ 8 \ \text{and} \ 1.5.
\]
\[
= 64 - 14.25 \quad \text{Multiply} \ 1.5 \ \text{and} \ 9.5.
\]
\[
= 49.75 \quad \text{Subtract} \ 14.25 \ \text{from} \ 64.
\]

The value is 49.75.

**Example 3 Expression Containing a Fraction Bar**

Evaluate $\frac{a^2 + 2bc}{c^2 - 5}$ if $a = 2$, $b = -4$, and $c = -3$.

The fraction bar acts as both an operation symbol, indicating division, and as a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

\[
\frac{a^2 + 2bc}{c^2 - 5} = \frac{2^2 + 2(-4)(-3)}{(-3)^2 - 5} \quad a = 2, \ b = -4, \ \text{and} \ \ c = -3
\]
\[
= \frac{4 + (-8)(-3)}{9 - 5} \quad \text{Evaluate} \ \text{the numerator and the denominator separately.}
\]
\[
= \frac{4 + 24}{9 - 5} \quad \text{Multiply} \ -8 \ \text{by} \ -3.
\]
\[
= \frac{28}{4} \quad \text{or} \ 8 \quad \text{Simplify} \ \text{the numerator and the denominator.} \ \text{Then divide.}
\]

The value is 8.

www.algebra2.com/extra_examples

**Answers**

**Graphing Calculator Investigation**

2. The calculator multiplies 2 by 4, subtracts the result from 8, and then adds 5.

4. 54; The calculator found the square of 18 and divided it by the product of 2 and 3.

5. No; you would square 18 and then divide it by 2. The result would then be multiplied by 3.
FORMULAS

A **formula** is a mathematical sentence that expresses the relationship between certain quantities. If you know the value of every variable in the formula except one, you can find the value of the remaining variable.

### Example 4: Use a Formula

**GEOMETRY** The formula for the area of a trapezoid is \( A = \frac{1}{2}h(b_1 + b_2) \), where \( h \) represents the height, and \( b_1 \) and \( b_2 \) represent the measures of the bases. Find the area of the trapezoid shown below.

![Trapezoid diagram]

Substitute each value given into the formula. Then evaluate the order of operations.

\[
A = \frac{1}{2}h(b_1 + b_2)
\]

Area of a trapezoid

\[
= \frac{1}{2}(10)(16 + 52)
\]

Replace \( h \) with 10, \( b_1 \) with 16, and \( b_2 \) with 52.

\[
= \frac{1}{2}(10)(68)
\]

Add 16 and 52.

\[
= 5(68)
\]

Divide 10 by 2.

\[
= 340
\]

Multiply 5 by 68.

The area of the trapezoid is 340 square inches.

---

**About the Exercises...**

**Organization by Objective**
- **Order of Operations**: 16–37
- **Formulas**: 38–54

**Odd/Even Assignments**
Exercises 16–49 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Alert!** Exercise 53 involves research on the Internet or other reference materials.

**Assignment Guide**
- **Basic**: 17–33 odd, 37–47 odd, 53, 55–66
- **Average**: 17–53 odd, 55–66
- **Advanced**: 16–54 even, 55–58, (optional: 59–66)

**GUIDED PRACTICE KEY**

<table>
<thead>
<tr>
<th>Exercises</th>
<th>Examples</th>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>4–9</td>
<td>1, 3</td>
<td>4, 5</td>
</tr>
<tr>
<td>10–12</td>
<td>2</td>
<td>6, 7</td>
</tr>
<tr>
<td>13–15</td>
<td>4</td>
<td>8, 9</td>
</tr>
</tbody>
</table>

**Check for Understanding**

**Concept Check**

1. **Describe** how you would evaluate the expression \( a + b[(c + d) ÷ e] \) given values for \( a, b, c, d, \) and \( e \).
2. **OPEN ENDED** Give an example of an expression where subtraction is performed before division and the symbols ( ), [ ], or { } are not used.
3. **Determine** which expression below represents the amount of change someone would receive from a $50 bill if they purchased 2 children’s tickets at $4.25 each and 3 adult tickets at $7 each at a movie theater. Explain.

\[
a. 50 \div (2 \times 4.25 + 3 \times 7) \quad b. 50 - (2 \times 4.25 + 3 \times 7) \\
c. (50 - 2 \times 4.25) + 3 \times 7 \quad d. 50 - (2 \times 4.25) + (3 \times 7)
\]

**b; See margin for explanation.**

**Guided Practice**

Find the value of each expression.

4. \( 8(3 + 6) \)
5. \( 10 - 8 ÷ 2 \)
6. \( 14 \cdot 2 - 5 \)
7. \( [9 + 3(5 - 7)] ÷ 3 \)
8. \( [6 - (12 - 8)]^2 ÷ 5 \)
9. \( \frac{17(2 + 26)}{4} \)

Evaluate each expression if \( x = 4, y = -2, \) and \( z = 6. \)

10. \( x - x + y \)
11. \( x + (y - 1)^3 \)
12. \( x + [3(y + z) - y] \)

---

**Visual/Spatial** Suggest that students first rewrite an expression they are to evaluate and then write the value for each variable on top of that variable before they start to evaluate the expression. Students may find it helpful to use colored pencils to color code the values for the different variables in an expression.
Application

**BANKING** For Exercises 13–15, use the following information.

Simple interest is calculated using the formula $I = ptr$, where $p$ represents the principal in dollars, $r$ represents the annual interest rate, and $t$ represents the time in years. Find the simple interest $I$ given each of the following values.

13. $p = $1800, $r = 6\%$, $t = 4$ years $\text{\$432}$
14. $p = $5000, $r = 3.75\%$, $t = 10$ years $\text{\$1875}$
15. $p = $31,000, $r = 2\%$, $t = 18$ months $\text{\$1162.50}$

**Practice and Apply**

1. Find the value of each expression.
   16. $18 + 6 = 3 = 20$
   17. $7 - 20 = 3 = 5$
   18. $(3 + 8) - 4 = 29$
   19. $2(6^2 - 9) = 54$
   20. $2 + 8(5) + 2 - 3 = 19$
   21. $[38 - (8 - 3)] = 3 = 11$
   22. $1 - [30 + [7 + (3 - 4)] = 7$
   23. $\frac{1}{3}(4 - 7) = 15$
   24. $\frac{3}{5}(6 - 22) = -52$
   25. $0.3(1.5 + 24 + 0.5) = 15.3$
   26. $\frac{1}{5} - 20(81.9 + 9) = 0$
   27. $\frac{35}{12}(52 - 25) = 25.1$

2. BICYCLING The amount of pollutants saved by riding a bicycle rather than driving car is calculated by adding the organic gases, carbon monoxide, and nitrous oxides emitted. To find the pounds of pollutants created by starting a typical car 10 times and driving it for 50 miles, find the value of the expression $\frac{(52.84 \times 10) + (5.955 \times 50)}{454}$.

3. Find the value of $\text{ab}$ if $n = 3\%$, $a = 2000$, and $b = \frac{1}{5}$.

**More about Bicycles**

In order to increase awareness and acceptance of bicycling throughout the country, communities, corporations, clubs, and individuals are invited to join in sponsoring bicycling activities during the month of May, National Bike Month.

Source: League of American Bicyclists

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**Significant Digits**

All measurements are approximations. The significant digits of an approximate number are those which indicate the results of a measurement. For example, the time of an event, measured to the nearest second, is 123.4 seconds. The measurement 123.4 has 4 significant digits, 1, 2, 3, and 4.

1. Number and area between significant digits are significant.
2. Example: $0.0034$ has 2 significant digits, 3 and 4.
3. Zeros at the end of a decimal fraction are significant. The measurement $0.00340$ has 3 significant digits, 3, 4, and 0.
4. Undetermined zeros in whole numbers are significant. The measurement $30,000$ has 1 significant digit, 3.
5. Some of the above rules are not always true. For example, $0.005$ has 1 significant digit, 5.

Enrichment, p. 6

1. There is an alternative method for grouping symbols. Brackets are used outside of parentheses. These are used outside of brackets. Identify the measurement expression(s) in each of the following expressions.
   a. $3(2 + 5) - (3 - 2)$
   b. $3 \times (4 - 2) - (5 - 6)$
   c. $3(2 + 5) - (3 - 2)$
   d. $3 \times (4 - 2) - (5 - 6)$
   e. $3(2 + 5) - (3 - 2)$
   f. $3 \times (4 - 2) - (5 - 6)$

2. Read the following instructions. Then group symbols to show how the instructions can be put in the form of a mathematical expression.

   a. Multiply the difference of 13 and 5 by the sum of 9 and 21. Add the result to 10. Then divide the result by 2. Write the expression that a nurse would use to calculate the flow rate per minute. The formula for the volume of a sphere is $V = \frac{4}{3} \pi r^3$, where $V$ is the volume and $r$ is the radius. Express the volume of a sphere in terms of its radius. Use $\pi$ as 3.14. 20 drops per milliliter. Do not find the value in any particular order of operations. If everyone did not use the same order of operations, different people might get different answers.

**Helping You Remember**

1. Think of a phrase or sentence to help you remember the order of operations. Sample answer: Please excuse my dear aunt Sally (parentheses, exponents, multiplication and division, addition and subtraction).

Lesson 1-1 Expressions and Formulas 9
52. MEDICINE Suppose a patient must take a blood pressure medication that is dispensed in 125-milligram tablets. The dosage is 15 milligrams per kilogram of body weight and is given every 8 hours. If the patient weighs 25 kilograms, how many tablets would be needed for a 30-day supply? Use the formula \( n = \frac{24d}{[8(b \times 15 + 125)]} \), where \( n \) is the number of tablets, \( d \) is the number of days the supply should last, and \( b \) is the body weight of the patient in kilograms. 30

53. MONEY In 1950, the average price of a car was about $2000. This may sound inexpensive, but the average income in 1950 was much less than it is now. To compare dollar amounts over time, use the formula \( V = \frac{A}{S} \), where \( A \) is the old dollar amount, \( S \) is the starting year’s Consumer Price Index (CPI), \( C \) is the converting year’s CPI, and \( V \) is the current value of the old dollar amount. Buying a car for $2000 in 1950 was like buying a car for how much money in 2000? $8266.03

54. FIREWORKS Suppose you are about a mile from a fireworks display. You count 5 seconds between seeing the light and hearing the sound of the firework display. You estimate the viewing angle is about 4°. Using the information at the left, estimate the width of the firework display. 400 ft

55. CRITICAL THINKING Write expressions having values from one to ten using exactly four 4s. You may use any combination of the operation symbols +, −, ×, ÷, and/or grouping symbols, but no other numbers are allowed. An example of such an expression with a value of zero is \((4 + 4) − (4 + 4)\). See margin.

56. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How are formulas used by nurses?
Include the following in your answer:
- an explanation of why a formula for the flow rate of an IV is more useful than a table of specific IV flow rates, and
- a description of the impact of using a formula, such as the one for IV flow rate, incorrectly.

57. Find the value of \(1 + 3(5 − 17) \div 2 \times 6\). C

58. The following are the dimensions of four rectangles. Which rectangle has the same area as the triangle at the right? D

59. \(\sqrt{9} \quad 3\)

60. \(\sqrt{16} \quad 4\)

61. \(\sqrt{100} \quad 10\)

62. \(\sqrt{169} \quad 13\)

63. \(-\sqrt{4} \quad -2\)

64. \(-\sqrt{25} \quad -5\)

65. \(\frac{4}{\sqrt{9}} \quad \frac{2}{3}\)

66. \(\frac{36}{\sqrt{49}} \quad \frac{6}{7}\)

**Open-Ended Assessment**

**Speaking** Ask students to state various formulas they remember using in previous courses, and to explain what each variable represents (for example, \(P = 2(l + w)\) to find the perimeter of a rectangle, where \(l\) is the length and \(w\) is the width). Then have a volunteer suggest appropriate values for the variables in the formula. Ask the class as a whole to evaluate the given formula using the suggested values.

**Intervention** Students may be reluctant to take time to show all the steps they use when evaluating an expression, such as showing the substituted values before doing the computations. Help them see that these steps enable them to self-diagnose errors and to prevent calculation errors that might keep them from getting correct values.

**Getting Ready for Lesson 1-2**

**PREREQUISITE SKILL** Lesson 1-2 presents the properties of real numbers and the subsets of the real numbers, including irrationals. Remind students that the square root of a number is irrational if that number is not a perfect square. Exercises 59–66 should be used to determine your students’ familiarity with evaluating square roots.

**Answers**

55. Sample answer:

\[
\begin{align*}
4 - 4 + 4 &= 1 \\
4 + 4 + 4 &= 2 \\
(4 + 4) + 4 &= 3 \\
4 \times (4 - 4) + 4 &= 4 \\
(4 \times 4 + 4) + 4 &= 5 \\
(4 + 4) + 4 &= 6
\end{align*}
\]

\[
\begin{align*}
44 \div 4 - 4 &= 7 \\
(4 + 4) \times (4 + 4) &= 8 \\
4 + 4 + 4 &= 9 \\
(44 - 4) + 4 &= 10
\end{align*}
\]

56. Nurses use formulas to calculate a drug dosage given a supply dosage and a doctor’s drug order. They also use formulas to calculate IV flow rates. Answers should include the following.
- A table of IV flow rates is limited to those situations listed, while a formula can be used to find any IV flow rate.
- If a formula used in a nursing setting is applied incorrectly, a patient could die.
Properties of Real Numbers

What You’ll Learn
- Classify real numbers.
- Use the properties of real numbers to evaluate expressions.

How is the Distributive Property useful in calculating store savings?
Manufacturers often offer coupons to get consumers to try their products. Some grocery stores try to attract customers by doubling the value of manufacturers’ coupons. You can use the Distributive Property to calculate these savings.

REAL NUMBERS All of the numbers that you use in everyday life are real numbers. Each real number corresponds to exactly one point on the number line, and every point on the number line represents exactly one real number.

Real numbers can be classified as either rational or irrational.

Vocabulary
- real numbers
- rational numbers
- irrational numbers

Vocabulary
- real numbers
- rational numbers
- irrational numbers

Study Tip
Reading Math
A ratio is the comparison of two numbers by division.

Key Concept
Rational Numbers
- **Words** A rational number can be expressed as a ratio \( \frac{m}{n} \), where \( m \) and \( n \) are integers and \( n \) is not zero. The decimal form of a rational number is either a terminating or repeating decimal.
- **Examples** \( \frac{1}{6}, 1.9, 2.575757..., -3, \sqrt{4}, 0 \)

Irrational Numbers
- **Words** A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.
- **Examples** \( \sqrt{5}, \pi, 0.010010001... \)

The sets of natural numbers, \( \{1, 2, 3, 4, 5, \ldots\} \), whole numbers, \( \{0, 1, 2, 3, 4, \ldots\} \), and integers, \( \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\} \) are all subsets of the rational numbers. The whole numbers are a subset of the rational numbers because every whole number \( n \) is equal to \( \frac{n}{1} \).
Power

REAL NUMBERS

Teaching Tip Point out that a non-terminating decimal whose digits show a pattern but which has no repeating group of digits, such as the number 0.010010001… given in the Key Concepts examples on p. 11, is irrational. Another example is the number 1.23223222333…

In-Class Example

1. Name the sets of numbers to which each number belongs.
   a. \( \frac{2}{3} \) Q, R
   b. 9.999… Q, R
   c. \( \sqrt{6} \) I, R
   d. \( \sqrt{100} \) N, W, Z, Q, R
   e. –23.3 Q, R

   Reading Tip Ask students whether fraction and rational number mean the same thing. (No; 4 is not a fraction but it is a rational number. Fraction refers to the form of a number: \( \frac{8}{4} \) is in the form of a fraction but it is a whole number in value.)

PROPERTIES OF REAL NUMBERS

Reading Tip Help students remember the names of properties by connecting the term commutative with “commuting, or moving from one position to another,” and by connecting the term associative with “the people you associate with, or your group.”

Study Tip

Common Misconception
Do not assume that a number is irrational because it is expressed using the square root symbol. Find its value first.

Study Tips offer students helpful information about the topics they are studying.

Example 1 Classify Numbers

Name the sets of numbers to which each number belongs.

a. \( \sqrt{16} \)
   \( \sqrt{16} = 4 \) naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

b. \( -185 \)
   integers (Z), rationals (Q), and reals (R)

c. \( \sqrt{20} \)
   irrationals (I) and reals (R)

\( \sqrt{20} \) lies between 4 and 5 so it is not a whole number.

d. \( -\frac{7}{8} \)
   rationals (Q) and reals (R)

e. 0.45
   rationals (Q) and reals (R)

The bar over the 45 indicates that those digits repeat forever.

Properties of Real Numbers

The real number system is an example of a mathematical structure called a field. Some of the properties of a field are summarized in the table below.

Key Concepts

For any real numbers \( a, b, \) and \( c: \)

<table>
<thead>
<tr>
<th>Property</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>( a + b = b + a )</td>
<td>( a \cdot b = b \cdot a )</td>
</tr>
<tr>
<td>Associative</td>
<td>( (a + b) + c = a + (b + c) )</td>
<td>( (a \cdot b) \cdot c = a \cdot (b \cdot c) )</td>
</tr>
<tr>
<td>Identity</td>
<td>( a + 0 = a = 0 + a )</td>
<td>( a \cdot 1 = a = 1 \cdot a )</td>
</tr>
<tr>
<td>Inverse</td>
<td>( a + (-a) = 0 = (-a) + a )</td>
<td>If ( a \neq 0 ), then ( a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a ).</td>
</tr>
<tr>
<td>Distributive</td>
<td>( a(b + c) = ab + ac ) and ( (b + c)a = ba + ca )</td>
<td></td>
</tr>
</tbody>
</table>
**Example 2** Identify Properties of Real Numbers

Name the property illustrated by each equation.

a. \((5 + 7) + 8 = 8 + (5 + 7)\)
   
   **Commutative Property of Addition**

   The Commutative Property says that the order in which you add does not change the sum.

b. \(3(4x) = (3 \cdot 4)x\)
   
   **Associative Property of Multiplication**

   The Associative Property says that the way you group three numbers when multiplying does not change the product.

**Example 3** Additive and Multiplicative Inverses

Identify the additive inverse and multiplicative inverse for each number.

a. \(-1\frac{3}{4}\)
   
   Since \(-1\frac{3}{4} + \left(1\frac{3}{4}\right) = 0\), the additive inverse of \(-1\frac{3}{4}\) is \(1\frac{3}{4}\).

b. \(1.25\)
   
   Since \(1.25 + (-1.25) = 0\), the additive inverse of 1.25 is \(-1.25\).

   The multiplicative inverse of 1.25 is \(\frac{1}{1.25}\) or 0.8.

   **CHECK** Notice that \(1.25 \times 0.8 = 1\). √

You can model the Distributive Property using algebra tiles.

**Algebra Activity**

**Distributive Property**

- A 1 tile is a square that is 1 unit wide and 1 unit long. Its area is 1 square unit. An x tile is a rectangle that is 1 unit wide and x units long. Its area is x square units.

- To find the product \(3(x + 1)\), model a rectangle with a width of 3 and a length of \(x + 1\). Use your algebra tiles to mark off the dimensions on a product mat. Then make the rectangle with algebra tiles.

- The rectangle has 3 x tiles and 3 1 tiles. The area of the rectangle is \(x + x + 1 + 1 + 1\) or \(3x + 3\). Thus, \(3(x + 1) = 3x + 3\).

**Model and Analyze**

Tell whether each statement is true or false. Justify your answer with algebra tiles and a drawing. 1–4. See pp. 53A–53B for drawings.

1. \(4(x + 2) = 4x + 2\) false
2. \(3(2x + 4) = 6x + 7\) false
3. \(2(3x + 5) = 6x + 10\) true
4. \((4x + 1)5 = 4x + 5\) false

You can model the Distributive Property using algebra tiles.

**Materials:** algebra tiles, product mat

- Have students verify with their tiles that the length of an x tile is not a multiple of the side length of a 1 tile.

- Suggest that students can verify they have modeled an expression like \(2(3x + 5)\) correctly if they read the expression as “2 rows of 3 x tiles and 5 1 tiles.” If they arrange their models like the one shown in the book, the rows of tiles can be “read” from left to right just as when reading the text.
A rational number is the ratio of two integers. Since $\sqrt{3}$ is not an integer, $\frac{\sqrt{3}}{2}$ is not a rational number.

2. A rational number is the ratio of two integers. Since $\sqrt{3}$ is not an integer, $\frac{\sqrt{3}}{2}$ is not a rational number.

3. Zero does not have a multiplicative inverse since $\frac{1}{0}$ is undefined.

4. Explain why $\frac{\sqrt{3}}{2}$ is not a rational number. See margin.

5. Disprove the following statement by giving a counterexample. A counterexample is a specific case that shows that a statement is false. Explain. Every real number has a multiplicative inverse.

The properties of real numbers can be used to simplify algebraic expressions.

Example 4 Use the Distributive Property to Solve a Problem

Food Service

A restaurant adds a 20% tip to the bills of parties of 6 or more people. Suppose a server waits on five such tables. The bill without the tip for each party is listed in the table. How much did the server make in tips during this shift?

<table>
<thead>
<tr>
<th>Party 1</th>
<th>Party 2</th>
<th>Party 3</th>
<th>Party 4</th>
<th>Party 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>185.45</td>
<td>205.20</td>
<td>195.05</td>
<td>245.80</td>
<td>262.00</td>
</tr>
</tbody>
</table>

There are two ways to find the total amount of tips received.

Method 1

Multiply each dollar amount by 20% or 0.2 and then add.

$T = 0.2(185.45) + 0.2(205.20) + 0.2(195.05) + 0.2(245.80) + 0.2(262)$

$= 37.09 + 41.04 + 39.01 + 49.16 + 52.40$

$= 218.70$

Method 2

Add the bills of all the parties and then multiply the total by 0.2.

$T = (0.2)(1093.50)$

$= 218.70$

The server made $218.70 during this shift.

Notice that both methods result in the same answer.

The Distributive Property is often used in real-world applications.

Example 5 Simplify an Expression

Simplify $2(5m + n) + 3(2m - 4n)$.

$2(5m + n) + 3(2m - 4n) =$

Distributive Property

$= 10m + 2n + 6m - 12n$ Multiply

$= (10 + 6)m + (-12)n$ Distributive Property

$= 16m - 10n$ Simplify.

Check for Understanding

Concept Check

1. OPEN ENDED Give an example of each type of number. Sample answers given.
   a. natural 2  
   b. whole 5  
   c. integer -11  
   d. rational 1.3  
   e. irrational $\sqrt{2}$  
   f. real $-1.3$

2. Explain why $\frac{\sqrt{3}}{2}$ is not a rational number. See margin.

3. Disprove the following statement by giving a counterexample. A counterexample is a specific case that shows that a statement is false. Explain.
   Every real number has a multiplicative inverse.

Daily Intervention notes help you help students when they need it most.

Differentiated Instruction

Kinesthetic To model the Distributive Property, write $7(8 + 6)$ on the board. Then have a student distribute an index card with 7 on it to a student holding an index card with 8 written on it and also distribute an index card with 7 on it to a student holding an index card with 6 written on it. Ask each student holding 2 cards to name their product. Have the student who distributed the 7s find the sum of the products. Complete the equation on the board: $7(8 + 6) = 7(8) + 7(6)$.
Homework Help to refer if they need students which every lesson is

1. 3. To indicate both roots of the equation $x^2 = 9$, the mathematical notation is $x = \pm \sqrt{9}$ or $x = \pm 3$.

Extra Practice See page 628.

$\star$ indicates increased difficulty

### Practice and Apply

**GUIDED PRACTICE KEY**

<table>
<thead>
<tr>
<th>Exercises</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>4–6</td>
<td>1</td>
</tr>
<tr>
<td>7–9</td>
<td>2</td>
</tr>
<tr>
<td>10–12</td>
<td>3</td>
</tr>
<tr>
<td>13–16</td>
<td>5</td>
</tr>
<tr>
<td>17, 18</td>
<td>4</td>
</tr>
</tbody>
</table>

Name the sets of numbers to which each number belongs.

4. $-4$ Z, Q, R
5. $45$ N, W, Z, Q, R
6. $6.23$ Q, R

Name the property illustrated by each equation.

7. $\frac{2}{3} \cdot \frac{2}{1} = 1$ Mult. Iden.
8. $(a + 4) + 2 = a + (4 + 2)$ Add. Iden.
9. $4x + 0 = 4x$ Add. Iden.
10. $-8 \quad \frac{-1}{8}$
11. $\frac{1}{3} \cdot \frac{-1}{3} = 1$ Mult. Iden.
12. $1.5 \cdot \frac{-1.5}{2} = \frac{2}{3}$ Add. Iden.

Simplify each expression.

13. $3x + 4y - 5x = -2x + 4y$
14. $9p - 2n + 4p + 2n = 13p$
15. $3(5c + 4d) + 6(d - 2c) = 3c + 18d$
16. $\frac{1}{2}(16 - 4a) - \frac{3}{4}(12 + 20a) = -17a - 1$

Application

**BAND BOOSTERS** For Exercises 17 and 18, use the information below and in the table.

Ashley is selling chocolate bars for $1.50 each to raise money for the band.

17. Write an expression to represent the total amount of money Ashley raised during this week.
18. Evaluate the expression from Exercise 17 using the Distributive Property. $\$175.50$

### Extra Practice

See page 628.

Homework Help charts show students which examples to which refer if they need additional practice. Extra Practice for every lesson is provided on pages 828–861.

www.algebra2.com/self_check_quiz

**Unlocking Misconceptions**

**Positive Root** Remind students that $\sqrt{9}$ means only the positive root, if one exists, so $\sqrt{9} = 3$. To indicate both roots of the equation $x^2 = 9$, the mathematical notation is $x = \pm \sqrt{9}$ or $x = \pm 3$.

### Study Notebook

- Have students—
  - add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 1.
  - copy the Venn diagram on p. 12, and add at least three examples for each set.
  - include any other item(s) that they find helpful in mastering the skills in this lesson.

### About the Exercises...

**Organization by Objective**

- **Real Numbers**: 19–27, 59–62
- **Properties of Real Numbers**: 28–58, 63–69

Exercises 19–26, 28–39, and 43–62 are structured so that students practice the same concepts whether they are assigned odd or even problems.

### Assignment Guide


**Advanced**: 20–38 even, 40–42, 44–62 even, 66–82 (optional: 83–86)

**All**: Practice Quiz 1 (1–10)

### Answers

Answers

65. \[3\left(2\frac{1}{4}\right) + 2\left(1\frac{1}{8}\right)\]
   \[= 3\left(2 + \frac{1}{4}\right) + 2\left(1 + \frac{1}{8}\right)\]
   Definition of a mixed number
   \[= 3(2) + 3\left(\frac{1}{4}\right) + 2(1) + 2\left(\frac{1}{8}\right)\]
   Distributive Property
   \[= 6 + \frac{3}{4} + 2 + \frac{1}{4}\]
   Multiply.
   \[= \left(6 + 2\right) + \left(\frac{3}{4} + \frac{1}{4}\right)\]
   Commutative Property of Addition
   \[= 8 + \frac{3}{4} + \frac{1}{4}\]
   Add.
   \[= 8 + \left(\frac{3}{4} + \frac{1}{4}\right)\]
   Associative Property of Addition
   \[= 8 + 1\text{ or }9\]
   Add.

71. Answers should include the following.
   • Instead of doubling each coupon value and then adding these values together, the Distributive Property could be applied allowing you to add the coupon values first and then double the sum.
   • If a store had a 25% off sale on all merchandise, the Distributive Property could be used to calculate these savings. For example, the savings on a $15 shirt, $40 pair of jeans, and $25 pair of slacks could be calculated as $0.25(15) + 0.25(40) + 0.25(25)$ or as $0.25(15 + 40 + 25)$ using the Distributive Property.

59. true
   60. false; \(-3\)

63. \[6.5(4.5) + 4.25 + 5.25 + 6.5 + 5\] or \[6.5(4.5) + 6.5(4.25) + (6.5)(5.25) + 6.5(6.5) + 6.5(5)\]

65. BAKING Mitena is making two types of cookies. The first recipe calls for \(2\frac{1}{4}\) cups of flour, and the second calls for \(1\frac{1}{8}\) cups of flour. If Mitena wants to make 3 batches of the first recipe and 2 batches of the second recipe, how many cups of flour will she need? Use the properties of real numbers to show how Mitena could compute this amount mentally. Justify each step. See margin.

Online Lesson Plans
USA TODAY Education's Online site offers resources and interactive features connected to each day’s newspaper. Experience TODAY, USA TODAY’s daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.
66. Illustrate the Distributive Property by writing two expressions for the area of the basketball court. \(50(47 + 47)\); \((50 + 47)(47)\)

67. Evaluate the expression from Exercise 66 using the Distributive Property. What is the area of an NCAA basketball court? \(4700 \text{ ft}^2\)

**SCHOOL SHOPPING** For Exercises 68 and 69, use the graph at the right.

68. Illustrate the Distributive Property by writing two expressions to represent the amount that the average student spends shopping for school at specialty stores and department stores.

69. Evaluate the expression from Exercise 68 using the Distributive Property. \$62.15

**CRITICAL THINKING** Is the Distributive Property also true for division? In other words, does \(b / c = b \div a \neq 0\)? If so, give an example and explain why it is true. If not true, give a counterexample.

70. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. *See margin.*

How is the Distributive Property useful in calculating store savings?

*Include the following in your answer:
- an explanation of how the Distributive Property could be used to calculate the coupon savings listed on a grocery receipt,
- an example of how the Distributive Property could be used to calculate the savings from a clothing store sale where all items were discounted by the same percent.

72. If \(a\) and \(b\) are natural numbers, then which of the following must also be a natural number? B

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(a - b)</th>
<th>I only</th>
<th>II only</th>
<th>III only</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. a</td>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II. a</td>
<td>b</td>
<td>II only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III. a</td>
<td>exist</td>
<td>only I and II only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>II and III only</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

73. If \(x = 1.4\), find the value of \(27(x + 1.2) - 26(x + 1.2)\). C

<table>
<thead>
<tr>
<th>(x)</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>82.6</td>
<td>1.2</td>
<td>58</td>
<td>65</td>
<td>17</td>
</tr>
</tbody>
</table>

**Enrichment, p. 12**

**Properties of a Group**

A set of numbers form a group with respect to an operation if that operation has the closure property (the Union Property), the Associative Property, the Additive Identity, and the multiplicative Inverse for each element of the set.

**ASSOCIATION**

How does the set \(H, e, a, b, c, \ldots\) form a group with respect to addition? C

1. **Closure Property** For all numbers in the set, \(a + b\) is also in the set. \(1 + 2, 2 + 3\), and \(9 + 10\), where \(e = 0\) is also in the set.

2. **Associative Property** For all numbers in the set, \((a + b) + c = a + (b + c)\) is also in the set.

3. **Additive Identity** In the set, \(0\) is also in the set such that \(a + 0 = a\) is also in the set.

4. **Inverse Property** For each number \(a\) in the set, there exists \(-a\) such that \(a + (-a) = 0\) is also in the set.

**Solving**

Name the set of numbers to which each number belongs.

**SCHOOL SHOPPING** For Exercises 68 and 69, use the graph at the right.

**Critical Thinking** Is the Distributive Property also true for division? In other words, does \(b / c = b \div a \neq 0\)? If so, give an example and explain why it is true. If not true, give a counterexample.

**Writing in Math** Answer the question that was posed at the beginning of the lesson. *See margin.*

How is the Distributive Property useful in calculating store savings?

**Including the following in your answer:**
- an explanation of how the Distributive Property could be used to calculate the coupon savings listed on a grocery receipt,
- an example of how the Distributive Property could be used to calculate the savings from a clothing store sale where all items were discounted by the same percent.

**Standardized Test Practice exercises were closely parallel to those on actual state proficiency tests and college entrance exams.**

**Skills Practice, p. 9 and Practice, p. 10 (shown)**

**Reading to Learn Mathematics, p. 11**

**Pre-Activity** How is the Distributive Property useful in calculating store savings?

Read the introduction to Lessons 1-5 at the top of page 13 in your textbook.

- **Write the Associative Property of Addition in complete sentences.**
- **Write the Commutative Property of Multiplication in complete sentences.**
- **Write the Associative Property of Multiplication in complete sentences.**
- **Write the Commutative Property of Multiplication in complete sentences.**

**Enrichment, p. 12**

**Properties of a Group**

A set of numbers form a group with respect to an operation if that operation has the closure property (the Union Property), the Associative Property, the Additive Identity, and the multiplicative Inverse for each element of the set.

**Association**

How does the set \(H, e, a, b, c, \ldots\) form a group with respect to addition? C

1. **Closure Property** For all numbers in the set, \(a + b\) is also in the set. \(1 + 2, 2 + 3\), and \(9 + 10\), where \(e = 0\) is also in the set.

2. **Associative Property** For all numbers in the set, \((a + b) + c = a + (b + c)\) is also in the set.

3. **Additive Identity** In the set, \(0\) is also in the set such that \(a + 0 = a\) is also in the set.

4. **Inverse Property** For each number \(a\) in the set, there exists \(-a\) such that \(a + (-a) = 0\) is also in the set.

**Solving**

Name the set of numbers to which each number belongs.
Open-Ended Assessment

Modeling Ask students to give examples of each of the properties (identity, inverse, commutative, associative, and distributive) and examples for each set of numbers (reals, rationals, irrationals, integers, wholes, and naturals).

Getting Ready for Lesson 1-3

PREREQUISITE SKILL Lesson 1-3 presents translating verbal expressions into algebraic expressions and using the properties of equality to solve equations. After solving an equation, the solution is checked in the original equation by evaluating the expression on each side after replacing the variable with its numerical value. Use Exercises 83–86 to determine your students’ familiarity with evaluating expressions.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 1-1 and 1-2. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 1-1 and 1-2) is available on p. 51 of the Chapter 1 Resource Masters.

Extending the Lesson

For Exercises 74–77, use the following information.

For Exercises 74–77, use the following information.

The product of any two whole numbers is always a whole number. So, the set of whole numbers is said to be closed under multiplication. This is an example of the Closure Property. State whether each statement is true or false. If false, give a counterexample.

75. False; 0 = 1 = 1, which is not a whole number.
76. The set of integers is closed under multiplication. true
77. The set of whole numbers is closed under subtraction. true
78. The set of rational numbers is closed under addition. true
79. The set of whole numbers is closed under division. True, 2 / 3, which is not a whole number.

Maintain Your Skills

Mixed Review

Find the value of each expression. (Lesson 1-1)

78. 9(4 − 3)² 9
79. 5 + 9 ÷ 3(3) − 8 6

Evaluate each expression if a = −5, b = 0.25, c = 1/2, and d = 4. (Lesson 1-1)

80. a + 2b − c −5
81. b + 3(a + d)³ −2.75

82. GEOMETRY The formula for the surface area SA of a rectangular prism is SA = 2lw + 2wh + 2hw, where l represents the length, w represents the width, and h represents the height. Find the surface area of the rectangular prism. (Lesson 1-1) 358 in²

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate each expression if a = 2, b = −3/4, and c = 1.8.

83. 8b − 5 −11
84. 2/b + 1 7/10
85. 1.5c − 7 −4.3
86. −9(a − 6) 36

Two Quizzes in each chapter review skills and concepts presented in previous lessons.

Practice Quiz 1 Lessons 1-1 and 1-2

Find the value of each expression. (Lesson 1-1)

1. 18 − 12 ÷ 3 14
2. −4 + 9(7 − 2³) −9
3. 18 ÷ 3 × 4 6
4. Evaluate a³ + b² + c³ if a = −2, b = 1/3, and c = −12. (Lesson 1-1) −1
5. ELECTRICITY Find the amount of current I (in amperes) produced if the electromotive force E is 2.5 volts, the circuit resistance R is 1.05 ohms, and the resistance r within a battery is 0.2 ohm. Use the formula I = E / (R + r). (Lesson 1-1) 2 amperes

Name the sets of numbers to which each number belongs. (Lesson 1-2)

6. 3.5 Q, R
7. √100 N, W, Z, Q, R
8. Name the property illustrated by bc + (−bc) = 0. (Lesson 1-2) Add. Inv.
9. Name the additive inverse and multiplicative inverse of −7/6. (Lesson 1-2) 1/7, 6/7
10. Simplify 4(14x − 10y) − 6(x + 4y). (Lesson 1-2) 50x − 64y

Unlocking Misconceptions

Associative or Commutative Students sometimes use inappropriate visual cues to name properties. For example, they may think that an expression can only have two terms to be an example of commutativity. Suggest that students look first at the change from one expression to the other and ask themselves if it is a change in grouping (associativity) or in position (commutativity).
Investigating Polygons and Patterns

Collect the Data

Use a ruler or geometry drawing software to draw six large polygons with 3, 4, 5, 6, 7, and 8 sides. The polygons do not need to be regular. Convex polygons, ones whose diagonals lie in the interior, will be best for this activity.

1. Copy the table below and complete the column labeled Diagonals by drawing the diagonals for all six polygons and record your results.

<table>
<thead>
<tr>
<th>Figure Name</th>
<th>Sides (n)</th>
<th>Diagonals</th>
<th>Diagonals From One Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>quadrilateral</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>pentagon</td>
<td>5</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>hexagon</td>
<td>6</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>heptagon</td>
<td>7</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>octagon</td>
<td>8</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

Analyze the Data

2. Describe the pattern shown by the number of diagonals in the table above. See pp. 53A–53B

3. Complete the last column in the table above by recording the number of diagonals from one vertex to the number of sides for each polygon. \( n - 3 \)

4. If a polygon has \( n \) sides, how many vertices does it have? \( n \)

5. How many vertices does one diagonal connect? 2

Make a Conjecture

7. Write a formula in terms of \( n \) for the number of diagonals of a polygon of \( n \) sides. (Hint: Consider your answers to Exercises 2, 3, and 4.) \( \frac{n(n - 3)}{2} \)

8. Draw a polygon with 10 sides. Test your formula for the decagon. See pp. 53A–53B.

9. Explain how your formula relates to the number of vertices of the polygon and the number of diagonals that can be drawn from each vertex. See pp. 53A–53B.

Extend the Activity

10. Draw 3 noncollinear dots on your paper. Determine the number of lines that are needed to connect each dot to every other dot. Continue by drawing 4 dots, 5 dots, and so on and finding the number of lines to connect them. See pp. 53A–53B.

11. Copy and complete the table at the right. See table.

<table>
<thead>
<tr>
<th>Dots (x)</th>
<th>Connection Lines (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>28</td>
</tr>
</tbody>
</table>

12. Use any method to find a formula that relates the number of dots, \( x \), to the number of lines, \( y \). \( y = \frac{x(x - 1)}{2} \) or \( y = 0.5x^2 - 0.5x \)

13. Explain why the formula works. See pp. 53A–53B.

Teaching Algebra with Manipulatives

- p. 213 (student recording sheet)

Glencoe Mathematics Classroom Manipulative Kit

- ruler

Study Notebook

You may wish to have students summarize this activity and what they learned from it.
5-Minute Check Transparency 1-3  Use as a quiz or review of Lesson 1-2.

Mathematical Background notes are available for this lesson on p. 4C.

Building on Prior Knowledge

In Lesson 1-2, students evaluated expressions with real numbers. In this lesson, they apply this skill to writing expressions and solving equations.

How can you find the most effective level of intensity for your workout?

Ask students:

• How can the expression $6 \times P \div (220 - A)$ be written as a ratio? $\frac{6P}{220 - A}$

• To achieve a 100% intensity level, the numerator and denominator of the ratio you just found must be equal. At what 10-second pulse count would you achieve a 100% intensity level? Answers will vary.

• Fitness Find your 10-second pulse count $P$ after running in place for 30 seconds. What is your level of intensity for this value of $P$? Answers will vary.

VERBAL EXPRESSIONS TO ALGEBRAIC EXPRESSIONS

Verbal expressions can be translated into algebraic or mathematical expressions using the language of algebra. Any letter can be used as a variable to represent a number that is not known.

Example 1  Verbal to Algebraic Expression

Write an algebraic expression to represent each verbal expression.

a. 7 less than a number $n - 7$

b. three times the square of a number $3x^2$

c. the cube of a number increased by $p^3 + 4p$

d. twice the sum of a number and 5 $2(y + 5)$

A mathematical sentence containing one or more variables is called an open sentence. A mathematical sentence stating that two mathematical expressions are equal is called an equation.

Example 2  Algebraic to Verbal Sentence

Write a verbal sentence to represent each equation.

a. $10 = 12 - 2$ Ten is equal to 12 minus 2.

b. $n + (-8) = -9$ The sum of a number and $-8$ is $-9$.

c. $\frac{n}{6} = n^2$ A number divided by 6 is equal to that number squared.

Open sentences are neither true nor false until the variables have been replaced by numbers. Each replacement that results in a true sentence is called a solution of the open sentence.

Resource Manager

Workbook and Reproducible Masters

Chapter 1 Resource Masters
• Study Guide and Intervention, pp. 13–14
• Skills Practice, p. 15
• Practice, p. 16
• Reading to Learn Mathematics, p. 17
• Enrichment, p. 18
• Assessment, pp. 51, 53

Graphing Calculator and Spreadsheet Masters, p. 27
School-to-Career Masters, p. 2
Teaching Algebra With Manipulatives Masters, pp. 214–215

Transparencies

5-Minute Check Transparency 1-3
Answer Key Transparencies

Technology

Interactive Chalkboard
PROPERTIES OF EQUALITY To solve equations, we can use properties of equality. Some of these equivalence relations are listed in the table below.

<table>
<thead>
<tr>
<th>Key Concept</th>
<th>Symbols</th>
<th>Properties of Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>For any real number ( a ), ( a = a ).</td>
<td>(-7 + n = -7 + n)</td>
</tr>
<tr>
<td>Symmetric</td>
<td>For all real numbers ( a ) and ( b ), if ( a = b ), then ( b = a ).</td>
<td>(\text{If } 3 = 5x - 6, \text{ then } 5x = 6 - 3)</td>
</tr>
<tr>
<td>Transitive</td>
<td>For all real numbers ( a ), ( b ), and ( c ), if ( a = b ) and ( b = c ), then ( a = c ).</td>
<td>(\text{If } 2x + 1 = 7 \text{ and } 7 = 5x - 8, \text{ then } 2x + 1 = 5x - 8)</td>
</tr>
<tr>
<td>Substitution</td>
<td>If ( a = b ), then ( a ) may be replaced by ( b ) and ( b ) may be replaced by ( a ).</td>
<td>(\text{If } (4 + 5)m = 18, \text{ then } 9m = 18)</td>
</tr>
</tbody>
</table>

**Example 3 Identify Properties of Equality**

Name the property illustrated by each statement.

a. If \( 3m = 5n \) and \( 5n = 10p \), then \( 3m = 10p \).
   - Transitive Property of Equality

b. If \(-11a + 2 = -3a\), then \(-3a = -11a + 2\).
   - Symmetric Property of Equality

Sometimes an equation can be solved by adding the same number to each side or by subtracting the same number from each side or by multiplying or dividing each side by the same number.

<table>
<thead>
<tr>
<th>Key Concept</th>
<th>Properties of Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition and Subtraction Properties of Equality</td>
<td></td>
</tr>
</tbody>
</table>
   - **Symbols** For any real numbers \( a \), \( b \), and \( c \), if \( a = b \), then \( a + c = b + c \) and \( a - c = b - c \).
   - **Examples** If \( x - 4 = 5 \), then \( x - 4 + 4 = 5 + 4 \).
     - If \( n + 3 = -11 \), then \( n + 3 - 3 = -11 - 3 \).

| Multiplication and Division Properties of Equality |
   - **Symbols** For any real numbers \( a \), \( b \), and \( c \), if \( a = b \), then \( a \cdot c = b \cdot c \) and, if \( c \neq 0 \), \( \frac{a}{c} = \frac{b}{c} \).
   - **Examples** If \( m = 4 \), then \( 4 \cdot \frac{m}{4} = 4 \cdot 6 \).
     - If \(-3y = 6\), then \( \frac{-3y}{-3} = \frac{6}{-3} \).

**Example 4 Solve One-Step Equations**

Solve each equation. Check your solution.

a. \( a + 4.39 = 76 \)
   
   \[
   \begin{align*}
   &a + 4.39 = 76 \\
   &a + 4.39 - 4.39 = 76 - 4.39 \\
   &a = 71.61
   \end{align*}
   \]

The solution is 71.61. (continued on the next page)

**Teaching Tip** Suggest that students ask themselves these questions: “What is being shown on the left side of the equation in In-Class Example 4a at the right?” 5.48 is subtracted from \( s \). “What is the opposite or inverse of subtracting 5.48?” Adding 5.48. “What must be done to both sides of the equation \( s - 5.48 = 0.02 \) to get the variable \( s \) alone on one side of the equation?” Add 5.48 to both sides and simplify the resulting equation.

**In-Class Examples**

1. Write an algebraic expression to represent each verbal expression.
   a. 3 more than a number \( x + 3 \)
   b. six times the cube of a number \( 6x^3 \)
   c. the square of a number decreased by the product of 5 and the number \( x^2 - 5x \)
   d. twice the difference of a number and 6 \( 2(x - 6) \)

2. Write a verbal sentence to represent each equation.
   a. \( 14 + 9 = 23 \) The sum of 14 and 9 is 23.
   b. \( 6 = -5 + x \) Six is equal to -5 plus a number.
   c. \( 7y - 2 = 19 \) Seven times a number minus 2 is 19.

3. Name the property illustrated by each statement.
   a. If \( xy = 28 \) and \( x = 7 \), then \( 7y = 28 \). **Substitution Property of Equality**
   b. \( a - 2.03 = a - 2.03 \) **Reflexive Property of Equality**

**Reading Tip** Help students remember the name of the Reflexive Property by relating \( a = a \) to seeing your reflection in a mirror.

4. Solve each equation. Check your solution.
   a. \( s - 5.48 = 0.02 \) 5.5
   b. \( 18 = \frac{1}{2}t \) 36
In-Class Examples

5 Solve
\[ 53 = 3(y - 2) - 2(3y - 1). \]
\[ \boxed{-19} \]

6 GEOMETRY The area of a trapezoid is \( A = \frac{1}{2}(b_1 + b_2)h \), where \( A \) is the area, \( b_1 \) is the length of one base, \( b_2 \) is the length of the other base, and \( h \) is the height of the trapezoid. Solve the formula for \( h \).

\[ h = \frac{2A}{b_1 + b_2} \]

Study Tip

Multiplication and Division Properties of Equality

Example 4b could also have been solved using the Division Property of Equality. Note that dividing each side of the equation by \( \frac{3}{5} \) is the same as multiplying each side by \( \frac{5}{3} \).

CHECK

\[ a + 4.39 = 76 \quad \text{Original equation} \]
\[ 71.61 + 4.39 = 76 \quad \text{Substitute 71.61 for } a. \]
\[ 76 = 76 \quad \text{Simplify.} \]

B.

\[ \frac{-3}{5}d = 18 \]
\[ \frac{-3}{5}d = 18 \quad \text{Original equation} \]
\[ \frac{-3}{5}(18) \quad \text{Multiply each side by } \frac{5}{3}, \text{ the multiplicative inverse of } \frac{-3}{5}. \]
\[ d = -30 \quad \text{Simplify}. \]

The solution is \(-30\).

CHECK

\[ \frac{-3}{5}d = 18 \quad \text{Original equation} \]
\[ \frac{-3}{5}(-30) = 18 \quad \text{Substitute } -30 \text{ for } d. \]
\[ 18 = 18 \quad \text{Simplify}. \]

Sometimes you must apply more than one property to solve an equation.

Example 5 Solve a Multi-Step Equation

Solve \( 2(2x + 3) - 3(4x - 5) = 22 \).

\[ 2(2x + 3) - 3(4x - 5) = 22 \quad \text{Original equation} \]
\[ 4x + 6 - 12x + 15 = 22 \quad \text{Distributive and Substitution Properties} \]
\[ -8x + 21 = 22 \quad \text{Commutative, Distributive, and Substitution Properties} \]
\[ -8x = 1 \quad \text{Subtraction, Distributive, and Substitution Properties} \]
\[ x = -\frac{1}{8} \quad \text{Division and Substitution Properties} \]

The solution is \(-\frac{1}{8}\).

You can use properties of equality to solve an equation or formula for a specified variable.

Example 6 Solve for a Variable

GEOMETRY The surface area of a cone is \( S = \pi r \ell + \pi r^2 \), where \( S \) is the surface area, \( \ell \) is the slant height of the cone, and \( r \) is the radius of the base. Solve the formula for \( \ell \).

\[ S = \pi r \ell + \pi r^2 \quad \text{Surface area formula} \]
\[ S - \pi r^2 = \pi r \ell + \pi r^2 - \pi r^2 \quad \text{Subtract } \pi r^2 \text{ from each side.} \]
\[ S - \pi r^2 = \pi r \ell \quad \text{Simplify.} \]
\[ \frac{S - \pi r^2}{\pi r} = \frac{\pi r \ell}{\pi r} \quad \text{Divide each side by } \pi r. \]
\[ \ell = \quad \text{Simplify.} \]

Unlocking Misconceptions

- **Solving Equations** Students may want to simplify, collect terms, and use the properties of equality to perform an operation on each side of an equation all in one or two steps. Help them see that it is more efficient to write down each step in the solution process than to have to solve the equation again because of a computational error.

- **Checking Solutions** Explain that checking solutions in order to discover possible errors is a vital procedure when you use math on the job.
Many standardized test questions can be solved by using properties of equality.

### Example 7: Apply Properties of Equality

**Multiple-Choice Test Item**

If \(3n - 8 = \frac{9}{5}\), what is the value of \(3n - 3\)?

- A \(\frac{34}{5}\)
- B \(\frac{49}{15}\)
- C \(-\frac{16}{5}\)
- D \(-\frac{27}{5}\)

**Read the Test Item**

You are asked to find the value of the expression \(3n - 3\). Your first thought might be to find the value of \(n\) and then evaluate the expression using this value. Notice, however, that you are not required to find the value of \(n\). Instead, you can use the Addition Property of Equality on the given equation to find the value of \(3n - 3\).

**Solve the Test Item**

\[
\begin{align*}
3n - 8 &= \frac{9}{5} \\
3n - 8 + 8 &= \frac{9}{5} + 8 \\
3n &= \frac{9}{5} + \frac{40}{5} \\
3n &= \frac{49}{5} \\
3n - 3 &= \frac{49}{5} - \frac{15}{5} \\
3n - 3 &= \frac{34}{5}
\end{align*}
\]

The answer is A.

To solve a word problem, it is often necessary to define a variable and write an equation. Then solve by applying the properties of equality.

### Example 8: Write an Equation

**HOME IMPROVEMENT** Josh and Pam have bought an older home that needs some repair. After budgeting a total of $1685 for home improvements, they started by spending $425 on small improvements. They would like to replace six interior doors next. What is the maximum amount they can afford to spend on each door?

**Explore** Let \(c\) represent the cost to replace each door.

**Plan** Write and solve an equation to find the value of \(c\).

\[
\begin{align*}
\text{The number of doors} &\quad \times \quad \text{the cost to replace each door} &\quad + \quad \text{previous expenses} &\quad = \quad \text{the total cost} \\
6 &\quad \times \quad c &\quad + \quad 425 &\quad = \quad 1685
\end{align*}
\]

**Solve**

\[
\begin{align*}
6c + 425 &= 1685 \\
6c + 425 - 425 &= 1685 - 425 \\
6c &= 1260 \\
6c &= 1260/6 \\
c &= 210
\end{align*}
\]

They can afford to spend $210 on each door.

**Examine** The total cost to replace six doors at $210 each is 6($210) or $1260. Add the other expenses of $425 to that, and the total home improvement bill is $1685. Thus, the answer is correct.

---

**Test-Taking Tip**

If a problem seems to require lengthy calculations, look for a shortcut. There is probably a quicker way to solve it. Try using properties of equality.

---

**Teaching Tip**

Students, especially those with math anxiety, tend to omit the planning step. Encourage students to see that this step helps them find a way to write an equation, even if they only do the planning mentally.
Check for Understanding

1. OPEN ENDED Write an equation whose solution is \(-7\).
2. Determine whether the following statement is sometimes, always, or never true. Explain.
Dividing each side of an equation by the same expression produces an equivalent equation.
3. FIND THE ERROR Crystal and Jamal are solving \(C = \frac{5}{9}(F - 32)\) for \(F\).

Crystal
\[ C = \frac{5}{9}(F - 32) \]
\[ C + 32 = \frac{5}{9}F \]
\[ \frac{9}{5}(C + 32) = F \]

Jamal
\[ C = \frac{5}{9}(F - 32) \]
\[ \frac{9}{5}C = F - 32 \]
\[ \frac{9}{5}C + 32 = F \]

Who is correct? Explain your reasoning. Jamal; see margin for explanation.

Guided Practice

Write an algebraic expression to represent each verbal expression.

4. five increased by four times a number \(5 + 4n\)
5. twice a number decreased by the cube of the same number \(2n - n^3\)

Write a verbal expression to represent each equation.

6. \(9n - 3 = 6\) 9 times a number decreased by 3 is 6.
7. \(5 + 3x = 2x\) 5 plus 3 times the square of a number is twice that number.

Find the Error exercises help students identify and address common errors before they occur.

Practice and Apply

Solve each equation or formula for the specified variable.

16. \(y = 9 + \frac{2n}{4}\) for \(y\) \[ y = 9 + \frac{2n}{4} \]

17. \(I = \frac{p}{rt}\) for \(p\) \[ I = \frac{p}{rt} \]

18. If \(4x + 7 = 18\), what is the value of \(12x + 21\)? \(D\) 83

Standardized Test Practice

\(\star\) indicates increased difficulty

Practice and Apply

Write an algebraic expression to represent each verbal expression.

19. the sum of 5 and three times a number \(5 + 3n\)
20. seven more than the product of a number and 10 \(10n + 7\)
21. four less than the square of a number \(n^2 - 4\)
22. the product of the cube of a number and \(-6\) \(-6n^3\)
23. five times the sum of 9 and a number \(5(9 + n)\)
24. twice the sum of a number and 8 \(2(n + 8)\)

Extra Practice

\(\star\) 25. the square of the quotient of a number and 4 \(\left(\frac{n}{4}\right)^2\)
\(\star\) 26. the cube of the difference of a number and 7 \((n - 7)^3\)
GEOMETRY For Exercises 27 and 28, use the following information. The formula for the surface area of a cylinder with radius $r$ and height $h$ is $\pi r h + 2\pi r^2$.

27. Translate this verbal expression of the formula into an algebraic expression. $2\pi r h + 2\pi r^2$

28. Write an equivalent expression using the Distributive Property. $2\pi r (h + r)$

Write a verbal expression to represent each equation.

1. 7 minus half a number divided by 4 is equal to twice 3.
2. A number divided by 4 is equal to twice the product of the radius and height.
3. A number divided by 4 is equal to twice the sum of that number and 1.
4. If $x - 2 = -8$, then $3(y - 2) = 3(-8)$. Multiplication (=)

Solve each equation. Check your solution.

41. $2p + 15 = 29$ 42. $14 - 3n = -10$
43. $7a - 3a + 2a - a = 16$ 44. $x + 9x - 6a + 4x = 20$
45. $\frac{1}{9} - \frac{2}{3}y = \frac{1}{18}$ 46. $\frac{5}{8} + \frac{3}{4}z = \frac{1}{16}$
47. $27 = -9(y + 5)$ 48. $-7(p + 8) = 21$
49. $3f - 2 = 4f + 5$ 49. $3f - 2 = 4f + 5$
50. $3d + 7 = 6d + 5$ 51. $2d + 1 = 7 - 1.7n$
52. $1.7x - 8 = 2.7x + 4$ 53. $2(2x + 3) - 2(z - 1) = 78$
54. $4(k + 3) + 2 = 4.5(k + 1)$
55. $\frac{3}{11}a - 1 = \frac{7}{11}a + 9 - \frac{55}{2}$ 56. $\frac{2}{5}x + \frac{3}{7} = 1 - \frac{4}{2x}$

Solve each equation or formula for the specified variable.

57. $d = rt$, for $r$ 58. $x = \frac{-b}{2a}$, for $a$
59. $V = \frac{1}{3}\pi r^2 h$, for $h$ 60. $A = \frac{1}{2}h(a + b)$, for $b$
61. $\frac{a(b - 2)}{x - 3} = x$, for $b$ 62. $x = \frac{y}{y + 4}$, for $y$

Define a variable, write an equation, and solve the problem.

63. BOWLING Jon and Morgan arrive at Sunnybrook Lanes with $16.75. Find the maximum number of games they can bowl if they each rent shoes. $a = \text{number of games}$; $2(1.50) + a(2.50) = 16.75$; $5

www.algebra2.com/self_check_quiz

About the Exercises...
Organization by Objective

- **Verbal Expressions to Algebraic Expressions**: 19–34
- **Properties of Equality**: 35–74

Exercises 19–26 and 29–70 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**


**Average**: 19–25 odd, 27–28, 29–69 odd, 75–89

**Advanced**: 20–26 even, 30–70 even, 71–83 (optional: 84–89)

The Assignment Guides provide suggestions for exercises that are appropriate for basic, average, or advanced students. Many of the homework exercises are paired, so that students can do the odds one day and the evens the next day.
Study Guide and Intervention, p. 13 (shown) and p. 14

**Vernacular Expressions to Algebraic Expressions**

Translate the word phrases into algebraic expressions:

<table>
<thead>
<tr>
<th>Word Phrase</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>The product of a number and 5</td>
<td>$5x$</td>
</tr>
<tr>
<td>The quotient of a number and 2</td>
<td>$\frac{x}{2}$</td>
</tr>
<tr>
<td>The sum of a number and 3</td>
<td>$x + 3$</td>
</tr>
<tr>
<td>The difference of a number and 5</td>
<td>$x - 5$</td>
</tr>
</tbody>
</table>

**Concepts**

Write an algebraic expression to represent each verbal expression.

1. The sum of six times a number and 20
2. Four times the sum of a number and 3
3. Four times the number 10 and four times a number
4. The product of 3 and the sum of 11 and a number
5. Four times the square of a number increased by five times the same number
6. 12 more than the product of 7 and a number

**Writing a verbal expression to represent each equation.**

Sample answers are given.

1. $5x + 20 = 25$
2. $4(x + 3) = 24$
3. $10y + 4y = 100 + 4n$
4. $3c = 3 + 11c$
5. Four times the number increased by five times the same number
6. $7n = 23$

**Write a verbal expression to represent each equation.**

Sample answers are given.

1. $5x = 25$
2. $4(x + 3) = 24$
3. $10y + 4y = 100 + 4n$
4. $3c = 3 + 11c$
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**Write a verbal expression to represent each equation.**

Sample answers are given.

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**Write a verbal expression to represent each equation.**

Sample answers are given.

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4. $3c = 3 + 11c$
5. Four times the number increased by five times the same number
6. $7n = 23$

Skills Practice, p. 15 and Practice, p. 16 (shown)

**Write an algebraic expression to represent each word phrase.**

1. Four times the sum of the cube of a number and three times the number and 5
2. Twice the sum of the cube of a number and three times the number
3. Can A Can B
4. 1.2
5. Solve each equation. Check your solution.
6. Subtract. (−)
7. Define a variable, write an equation, and solve the problem.
8. Geometry (a) Find your target heart rate while jogging. B. (b) Write an equation that shows how to calculate your target heart rate
9. Read the Lesson
10. a. How are algebraic expressions and equations alike?
11. b. How are algebraic expressions and equations different?
12. Sample answer: Both contain variables, constants, and operation signs.
13. Sample answer: Equations contain equal signs; expressions do not.

Reading to Learn Mathematics, p. 17

**ELL**

Pro-Activity: What can you find the most effective level of intensity for your workouts?

Read the introduction to Lesson 1-3 at the top of page 20 in your textbook. Then find your target heart rate while jogging. B. Write an equation that shows how to calculate your target heart rate.

1. How are algebraic expressions and equations alike?
2. a. How are algebraic expressions and equations different?
3. Sample answer: Equations contain equal signs; expressions do not.

Reading the Lesson

1. a. How are algebraic expressions and equations alike?
2. b. How are algebraic expressions and equations different?
3. Sample answer: Equations contain equal signs; expressions do not.

Enrichment, p. 18

**Venn Diagrams**

Relationships among sets can be shown using Venn diagrams. Study the diagrams below. The figure represents set A, and B, which are subsets of set A.

Answers

72. Central: 690 mi.; Union: 1085 mi
73. The Central Pacific had to lay their track over the Rocky Mountains, while the Union Pacific mainly built track over flat prairie.
74. **MONEY** Allison is saving money to buy a video game system. In the first week, her savings were $8 less than \( \frac{2}{5} \) the price of the system. In the second week, she saved 50 cents more than \( \frac{1}{2} \) the price of the system. She was still $37 short. Find the price of the system.

**S295**

75. **CRITICAL THINKING** Write a verbal expression to represent the algebraic expression \( 3(x - 5) + 4x(x + 1) \). **See margin.**

76. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 53A–53B.**

How can you find the most effective level of intensity for your workout? Include the following in your answer:

• an explanation of how to find the age of a person who is exercising at an 80% level of intensity \( I \) with a pulse count of 27, and

• a description of when it would be desirable to solve a formula like the one given for a specified variable.

77. If \(-6x + 10 = 17\), then \(3x - 5 = \) **B**

\[\begin{align*}
\text{A} & : -\frac{3}{6} \\
\text{B} & : -\frac{17}{2} \\
\text{C} & : 2 \\
\text{D} & : \frac{19}{3} \\
\text{E} & : \frac{5}{3}
\end{align*}\]

78. In triangle \(PQR\), \(QS\) and \(SR\) are angle bisectors and angle \(P = 74^\circ\). How many degrees are there in angle \(QSR\)? **D**

\[\begin{align*}
\text{A} & : 106 \\
\text{B} & : 121 \\
\text{C} & : 125 \\
\text{D} & : 127 \\
\text{E} & : 143
\end{align*}\]

---

**Standardized Test Practice**

**Mixed Review** (Lesson 1-2)

79. \(2x + 9y + 4z - y - 8x\)

\[-6x + 8y + 4z\]

---

80. \(4(2a + 5b) - 3(4b - a)\)

\(11a + 8b\)

---

**Evaluate each expression if** \(a = 3, b = -2,\) and \(c = 1.2\). (Lesson 1-1)

81. \(a - [b(a - c)]\)

\(6.6\)

---

82. \(c^2 - ab\)

\(7.44\)

---

**GEOMETRY** The formula for the surface area \(S\) of a regular pyramid is \(S = \frac{1}{2}Pl + B\), where \(P\) is the perimeter of the base, \(l\) is the slant height, and \(B\) is the area of the base. Find the surface area of the square-based pyramid shown at the right. (Lesson 1-1)

\(105\) \(\text{cm}^2\)

---

**PREREQUISITE SKILL** Identify the additive inverse for each number or expression. (To review additive inverses, see Lesson 1-2.)

84. \(5\) \(-5\)

85. \(-3\) \(3\)

86. \(2.5\) \(-2.5\)

87. \(\frac{1}{4}\) \(-\frac{1}{4}\)

88. \(-3x\) \(3x\)

89. \(5 - 6y\) \(-5 + 6y\)

---

**Getting Ready for the Next Lesson**

**Assessment Options**

**Quiz (Lesson 1-3)** is available on p. 51 of the *Chapter 1 Resource Masters*.

**Mid-Chapter Test (Lessons 1-1 through 1-3)** is available on p. 53 of the *Chapter 1 Resource Masters*. 

---

**Answer**

75. The product of 3 and the difference of a number and 5 added to the product of four times the number and the sum of the number and 1

\(3(x - 5) + 4x(x + 1)\)
### 5-Minute Check Transparency 1-4
Use as a quiz or review of Lesson 1-3.

Mathematical Background notes are available for this lesson on p. 4D.

### Building on Prior Knowledge
In Lesson 1-3, students wrote expressions and solved equations. In this lesson, they apply those skills to equations involving absolute values.

#### How can an absolute value equation describe the magnitude of an earthquake?

Ask students:
- In the absolute value equation $|E - 6.1| = 0.3$, what does the variable $E$ represent? the actual magnitude of the earthquake
- What is the meaning of the number 0.3 in the equation? the uncertainty of the estimated magnitude
- What would the equation be for the magnitude of an earthquake estimated at 5.8 on the Richter scale? $|E - 5.8| = 0.3$

### Absolute Value Expressions
The absolute value of a number is its distance from 0 on the number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol $|x|$ is used to represent the absolute value of a number $x$.

### Key Concept

| Words | For any real number $a$, if $a$ is positive or zero, the absolute value of $a$ is $a$. If $a$ is negative, the absolute value of $a$ is the opposite of $a$. |
| Symbols | For any real number $a$, $|a| = a$ if $a \geq 0$, and $|a| = -a$ if $a < 0$. |
| Model | $| -3 | = 3$ and $|3| = 3$ |

When evaluating expressions that contain absolute values, the absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

#### Example 1 Evaluate an Expression with Absolute Value

Evaluate $1.4 + |5y - 7|$ if $y = -3$.

$1.4 + |5y - 7| = 1.4 + |5(-3) - 7|$

Replace $y$ with $-3$.

$= 1.4 + |-15 - 7|$

Simplify $5(-3)$ first.

$= 1.4 + |-22|$

Subtract 7 from $-15$.

$= 1.4 + 22$

Add.

$= 23.4$

The value is 23.4.
**Absolute Value Equations**

Some equations contain absolute value expressions. The definition of absolute value is used in solving these equations. For any real numbers $a$ and $b$, where $b \geq 0$, if $|a| = b$, then $a = b$ or $-a = b$. This second case is often written as $a = -b$.

**Example 2** Solve an Absolute Value Equation

Solve $|x - 18| = 5$. Check your solutions.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = b$</td>
<td>$a = -b$</td>
</tr>
<tr>
<td>$x - 18 = 5$</td>
<td>$x - 18 = -5$</td>
</tr>
<tr>
<td>$x - 18 + 18 = 5 + 18$</td>
<td>$x - 18 + 18 = -5 + 18$</td>
</tr>
<tr>
<td>$x = 23$</td>
<td>$x = 13$</td>
</tr>
</tbody>
</table>

**CHECK**

$|x - 18| = 5$

$|23 - 18| = 5$

$|5| = 5$

$5 = 5$ ✔

The solutions are 23 or 13. Thus, the solution set is {13, 23}.

Because the absolute value of a number is always positive or zero, an equation like $|x| = -5$ is never true. Thus, it has no solution. The solution set for this type of equation is the empty set, symbolized by {} or $\emptyset$.

**Example 3** No Solution

Solve $|5x - 6| + 9 = 0$.

$|5x - 6| + 9 = 0$ Original equation

$|5x - 6| = -9$ Subtract 9 from each side.

This sentence is never true. So the solution set is $\emptyset$.

It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equation is used, the answers may not be actual solutions of the original equation.

**Example 4** One Solution

Solve $|x + 6| = 3x - 2$. Check your solutions.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = b$</td>
<td>$a = -b$</td>
</tr>
<tr>
<td>$x + 6 = 3x - 2$</td>
<td>$x + 6 = -(3x - 2)$</td>
</tr>
<tr>
<td>$6 = 2x - 2$</td>
<td>$x + 6 = -3x + 2$</td>
</tr>
<tr>
<td>$8 = 2x$</td>
<td>$4x + 6 = 2$</td>
</tr>
<tr>
<td>$4 = x$</td>
<td>$4x = -4$</td>
</tr>
<tr>
<td>$x = -1$</td>
<td>$x = -1$</td>
</tr>
</tbody>
</table>

There appear to be two solutions, 4 or $-1$.

(continued on the next page)

www.algebra2.com/extra_examples

Lesson 1-4  Solving Absolute Value Equations  29
### Concept Check

1. Explain why if the absolute value of a number is always nonnegative, \(|a|\) can equal \(-a\).

2. Write an absolute value equation for each solution set graphed below.

   a. \[
   \begin{align*}
   x + 4 &= 17 \quad \text{(4 units)} \\
   x - 9 &= 20 \quad \text{(2 units)}
   \end{align*}
   \]

   b. \[
   \begin{align*}
   x + 6 &= 3x - 2 \\
   4 + 6 &\leq 3(4) - 2 \\
   10 &\leq 12 - 2
   \end{align*}
   \]

   or \[
   \begin{align*}
   x + 6 &\leq 3(-1) - 2 \\
   10 &\leq -3 - 2
   \end{align*}
   \]

   Since \(5 \neq -5\), the only solution is 4. Thus, the solution set is \(\{4\}\).

3. Determine whether the following statement is sometimes, always, or never true. Explain. See margin.

   For all real numbers \(a\) and \(b\), \(a \neq 0\), the equation \(|ax + b| = 0\) will have one solution.

4. OPEN ENDED Write and evaluate an absolute value expression with increased difficulty.

   Sample answer: \(4 - 6\); 2

### Guided Practice

**Guided Practice Key**

<table>
<thead>
<tr>
<th>GUIDED PRACTICE KEY</th>
<th>Exercises</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8–13</td>
<td>2–4</td>
<td></td>
</tr>
<tr>
<td>14–16</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**Application**

**FOOD** For Exercises 14–16, use the following information.

A meat thermometer is used to assure that a safe temperature has been reached to destroy bacteria. Most meat thermometers are accurate to within plus or minus 2\(^\circ\)F. Source: U.S. Department of Agriculture

14. The ham you are baking needs to reach an internal temperature of 160\(^\circ\)F. If the thermometer reads 160\(^\circ\)F, write an equation to determine the least and greatest temperatures of the meat. \(x - 160 = 2\)

15. Solve the equation you wrote in Exercise 14. least: 158\(^\circ\)F; greatest: 162\(^\circ\)F

16. To what temperature reading should you bake a ham to ensure that the minimum internal temperature is reached? Explain. 162\(^\circ\)F; This would ensure a minimum internal temperature of 160\(^\circ\)F.

**Practice and Apply**

Evaluate each expression if \(a = -5\), \(b = 6\), and \(c = 2.8\).

17. \(-3a\) | 15  
18. \(-4b\) | 24  
19. \(|a + 5|\) | 0  
20. \(2 - b\) | 4  
21. \(|2b - 15|\) | 3  
22. \(|4a + 7|\) | 13  
23. \(|18 - 5c|\) | -4  
24. \(-|c - a|\) | -7.8  
25. \(6 - |3c + 7|\) | -9.4  
26. \(9 - |2b + 8|\) | 5  
27. \(|a - 10| + |2a|\) | 55  
28. \(|a - b| - |10c - a|\) | -22  

52. Answers should include the following.

- This equation needs to show that the difference of the estimate \(E\) from the originally stated magnitude of 6.1 could be plus 0.3 or minus 0.3, as shown in the graph below. Instead of writing two equations, \(E - 6.1 = 0.3\) and \(E - 6.1 = -0.3\), absolute value symbols can be used to account for both possibilities, \(|E - 6.1| = 0.3\).

- Using an original magnitude of 5.9, the equation to represent the estimated extremes would be \(|E - 5.9| = 0.3\).
47. COFFEE
Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F, plus or minus five degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee. $|x - 200| = 5$; maximum: 205°F; minimum: 195°F

48. MANUFACTURING
A machine is used to fill each of several bags with 16 ounces of sugar. After the bags are filled, another machine weighs them. If the bag weighs 0.3 ounce more or less than the desired weight, the bag is rejected. Write an equation to find the heaviest and lightest bag the machine will approve. $|x - 16| = 0.3$; heaviest: 16.3 oz, lightest: 15.7 oz

49. METEOROLOGY
The atmosphere of Earth is divided into four layers based on temperature variations. The troposphere is the layer closest to the planet. The average upper boundary of the layer is about 13 kilometers above Earth’s surface. This height varies with latitude and with the seasons by much as 5 kilometers. Write and solve an equation describing the maximum and minimum heights of the upper boundary of the troposphere. $|x - 13| = 5$; maximum: 18 km, minimum: 8 km

CRITICAL THINKING
For Exercises 50 and 51, determine whether each statement is sometimes, always, or never true. Explain your reasoning.

50. If $a$ and $b$ are real numbers, then $|a + b| = |a| + |b|$.
51. If $a$, $b$, and $c$ are real numbers, then $|a + b| = |a| + |b|$.

52. WRITING IN MATH
Answer the question that was posed at the beginning of the lesson. See margin.

How can an absolute value equation describe the magnitude of an earthquake?

Include the following in your answer:
- A verbal and graphical explanation of how $|E - 6.1| = 0.3$ describes the possible extremes in the variation of the earthquake’s magnitude, and
- An equation to describe the extremes for a different magnitude.

53. Which of the graphs below represents the solution set for $|x - 3| - 4 = 0$?

- [Graph A]
- [Graph B]
- [Graph C]
- [Graph D]

www.algebra2.com/self_check_quiz
Open-Ended Assessment

Modeling  Have students draw a number-line diagram like the one shown in Example 2 to model the equation \(|x - 3| = 7\) and another number line to model the equation \(|y| = 7\). You might suggest that students think of the equation \(|y| = 7\) as \(|y - 0| = 7\).

Each lesson ends with Open-Ended Assessment strategies for closing the lesson. These include writing, modeling, and speaking.

Getting Ready for Lesson 1-5

PREREQUISITE SKILL  Lesson 1-5 presents solving inequalities using steps similar to those for solving equations. Exercises 74–79 should be used to determine your students’ familiarity with solving equations.

Maintain Your Skills

Mixed Review

Write an algebraic expression to represent each verbal expression. (Lesson 1-3)

59. twice the difference of a number and 11 2(n – 11)
60. the product of the square of a number and 5 5n²
61. 3x + 6 = 22 \(\frac{16}{3}\)
62. \(7p - 4 = 3(4 + 5p)\) -2
63. \(\frac{5}{2}y - 3 = \frac{3}{2}y + 1\) 14

Name the property illustrated by each equation. (Lesson 1-2)

64. \((5 + 9) + 13 = 13 + (5 + 9)\) Comm. (+)
65. \(m(4 - 3) = m \cdot 4 - m \cdot 3\) Dist.
66. \(\frac{1}{4} \cdot 4 = 1\) Mult. Inv.
67. \(5x + 0 = 5x\) Add. Iden.

Determine whether each statement is true or false. If false, give a counterexample. (Lesson 1-2)

68. Every real number is a rational number. false; \(\sqrt{3}\)
69. Every natural number is an integer. true
70. Every irrational number is a real number. true
71. Every rational number is an integer. false; 1.2

GEOMETRY  For Exercises 72 and 73, use the following information.
The formula for the area \(A\) of a triangle is \(A = \frac{1}{2}bh\), where \(b\) is the measure of the base and \(h\) is the measure of the height. (Lesson 1-1)

72. \(\frac{1}{2}(x + 3)(x + 5)\)
73. Write an expression to represent the area of the triangle above.
74. Evaluate the expression you wrote in Exercise 72 for \(x = 23\). 364 ft²

Getting Ready for the Next Lesson

PREREQUISITE SKILL  Solve each equation. (To review solving equations, see page 20.)

74. \(14y - 3 = 25\) 2
75. \(4.2x + 6.4 = 40\) 8
76. \(7w + 2 = 3w - 6\) -2
77. \(2(a - 1) = 8a - 6\) 2
78. \(48 + 5y = 96 - 3y\) 6
79. \(\frac{2x + 3}{5} = \frac{3}{10}\) -\(\frac{3}{4}\)

54. Find the value of \(-9 - |4| - 3|5 - 7|\). A
   \(\begin{array}{cc}
   A & -19 \\
   B & -11 \\
   C & -7 \\
   D & 11 \\
   \end{array}\)

For Exercises 55–58, consider the equation \(|x + 1| + 2 = |x + 4|\).

55. To solve this equation, we must consider the case where \(x + 4 \geq 0\) and the case where \(x + 4 < 0\). Write the equations for each of these cases.
56. Notice that each equation you wrote in Exercise 55 has two cases. For each equation, write two other equations taking into consideration the case where \(x + 1 \geq 0\) and the case where \(x + 1 < 0\).
57. Solve each equation you wrote in Exercise 56. Then, check each solution in the original equation, \(|x + 1| + 2 = |x + 4|\). What are the solution(s) to this absolute value equation? \((-1, 5)\)
58. MAKE A CONJECTURE  For equations with one set of absolute value symbols, two cases must be considered. For an equation with two sets of absolute value symbols, four cases must be considered. How many cases must be considered for an equation containing three sets of absolute value symbols? 8
5-Minute Check
Transparency 1-5
Use as a quiz or review of Lesson 1-4.

Mathematical Background notes are available for this lesson on p. 4D.

Building on Prior Knowledge
In Lessons 1-3 and 1-4, students solved equations. In this lesson, students use similar steps to solve inequalities.

How can inequalities be used to compare phone plans?
Kuni is trying to decide between two rate plans offered by a wireless phone company.

<table>
<thead>
<tr>
<th>Monthly Access Fee</th>
<th>Plan 1</th>
<th>Plan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$35.00</td>
<td>$55.00</td>
</tr>
<tr>
<td>Minutes Included</td>
<td>150</td>
<td>400</td>
</tr>
<tr>
<td>Additional Minutes</td>
<td>40¢</td>
<td>35¢</td>
</tr>
</tbody>
</table>

To compare these two rate plans, we can use inequalities. The monthly access fee for Plan 1 is less than the fee for Plan 2, $35 < $55. However, the additional minutes fee for Plan 1 is greater than that of Plan 2, 0.40 > 0.35.

SOLVE INEQUALITIES  For any two real numbers, a and b, exactly one of the following statements is true.

\[
\begin{align*}
& a < b \\
& a = b \\
& a > b
\end{align*}
\]

This is known as the Trichotomy Property or the property of order.

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.

Key Concept  Properties of Inequality

<table>
<thead>
<tr>
<th>Addition Property of Inequality</th>
<th>Properties of Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Words For any real numbers, a, b, and c:</td>
<td>• Example</td>
</tr>
<tr>
<td>If ( a &gt; b ), then ( a + c &gt; b + c ).</td>
<td>3 &lt; 5</td>
</tr>
<tr>
<td>If ( a &lt; b ), then ( a + c &lt; b + c ).</td>
<td>( 3 + (-4) &lt; 5 + (-4) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subtraction Property of Inequality</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Words For any real numbers, a, b, and c:</td>
<td>Example</td>
</tr>
<tr>
<td>If ( a &gt; b ), then ( a - c &gt; b - c ).</td>
<td>( -2 &gt; -7 )</td>
</tr>
<tr>
<td>If ( a &lt; b ), then ( a - c &lt; b - c ).</td>
<td>( -6 &lt; -15 )</td>
</tr>
</tbody>
</table>

These properties are also true for \( \leq \) and \( \geq \).

These properties can be used to solve inequalities. The solution sets of inequalities in one variable can then be graphed on number lines. Use a circle with an arrow to the left for \( < \) and an arrow to the right for \( > \). Use a dot with an arrow to the left for \( \leq \) and an arrow to the right for \( \geq \).
**Example 1** Solve an Inequality Using Addition or Subtraction

Solve \(7x - 5 > 6x + 4\). Graph the solution set on a number line.

**Original inequality**

\[7x - 5 > 6x + 4\]

**Add \(-6x\) to each side.**

\[x - 5 > 4\]

Add 5 to each side.

\[x > 9\]

Any real number greater than 9 is a solution of this inequality.

The graph of the solution set is shown at the right.

**CHECK** Substitute 9 for \(x\) in \(7x - 5 > 6x + 4\). The two sides should be equal.

Then substitute a number greater than 9. The inequality should be true.

Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a negative number requires that the order of the inequality be reversed. For example, to reverse inequality by a change the truth of the inequality. However, multiplying or dividing each side of an inequality by a negative number requires that the order of the inequality be reversed.

### Key Concept

#### Properties of Inequality

**Multiplication Property of Inequality**

- **Words** For any real numbers, \(a\), \(b\), and \(c\), where

  - \(c\) is positive: \(a > b\) then \(ac > bc\).
  - \(c\) is negative: \(a > b\) then \(ac < bc\).

  - \(c\) is positive: \(a < b\) then \(ac < bc\).
  - \(c\) is negative: \(a < b\) then \(ac > bc\).

**Examples**

- \(-2 < 3\)
- \(-8 < 12\)
- \(5 > -1\)
- \((-3)(5) < (-3)(1)\)
- \(-15 < 3\)

**Division Property of Inequality**

- **Words** For any real numbers, \(a\), \(b\), and \(c\), where

  - \(c\) is positive: \(a > b\) then \(a / c > b / c\).
  - \(c\) is negative: \(a < b\) then \(a / c > b / c\).

- **Examples**

  - \(-18 < -9\)
  - \(-18 < -9\)
  - \(3 > 3\)
  - \(-6 < -3\)
  - \(12 > 8\)
  - \(12 > 8\)
  - \(-2 < -2\)
  - \(-6 < -4\)

These properties are also true for \(\leq\) and \(\geq\).

The solution set of an inequality can be expressed by using **set-builder notation**. For example, the solution set in Example 1 can be expressed as \(\{x \mid x > 9\}\).
Example 2 Solve an Inequality Using Multiplication or Division

Solve \(-0.25y \geq 2\). Graph the solution set on a number line.

\[-0.25y \geq 2\]

Original inequality

\[-0.25 \leq \frac{2}{-0.25}\]

Divide each side by \(-0.25\), reversing the inequality symbol.

\[y \leq -8\]

Simplify.

The solution set is \(\{y | y \leq -8\}\).

The graph of the solution set is shown below.

A dot means that this point is included in the solution set.

Study Tip

When solving an inequality:
- If you arrive at a false statement, such as \(3 > 5\), then the solution set for that inequality is the empty set, \(\emptyset\).
- If you arrive at a true statement such as \(3 > -1\), then the solution set for that inequality is the set of all real numbers.

Example 3 Solve a Multi-Step Inequality

Solve \(-m \leq \frac{m + 4}{9}\). Graph the solution set on a number line.

\[-m \leq \frac{m + 4}{9}\]

Original inequality

\[-9m \leq m + 4\]

Multiply each side by 9.

\[-10m \leq 4\]

Add \(-m\) to each side.

\[m \geq -\frac{4}{10}\]

Divide each side by \(-10\), reversing the inequality symbol.

\[m \geq -\frac{2}{5}\]

Simplify.

The solution set is \([-\frac{2}{5}, \infty)\) and is graphed below.

In-Class Examples

2 Solve \(12 \geq -0.3p\). Graph the solution set on a number line.

\(|p| \geq -40\)

3 Solve \(-x > \frac{x - 7}{2}\). Graph the solution set on a number line.

\((-\infty, \frac{7}{3})\)

Teaching Tip

Remind students that when solving an inequality, in order to keep each intermediate inequality equivalent to the original, they must show both the division by a negative number and the reversal of the inequality sign in the same step.

Concept Check

Ask students to name three different ways to show the solution of an inequality. Four possible responses: as a graph on a number line, as an inequality, using set-builder notation, using interval notation.

Differentiated Instruction

Intrapersonal

Have students discuss the differences between solving an equation and solving an inequality and then how the solution processes are the same.
REAL-WORLD PROBLEMS WITH INEQUALITIES

**In-Class Example**

**Teaching Tip** To understand the situation given in Example 4, some students may find it helpful to make a sketch representing the elevator, the boxes, and the person.

4. **CONSUMER COSTS** Alida has at most $10.50 to spend at a convenience store. She buys a bag of potato chips and a can of soda for $1.55. If gasoline at this store costs $1.35 per gallon, how many gallons of gasoline can Alida buy for her car, to the nearest tenth of a gallon? no more than 6.6 gal

**Answers**

Graphing Calculator Investigation

1. The graph is of the line $y = 1$, for $x \geq -1$.

**Answers (p. 37)**

4. $(-\infty, 1.5)$
6. $[3, +\infty)$
8. $(-\infty, -7)$
10. $(-\infty, -24)$
5. $(-\infty, \frac{5}{3}]$
7. $(6, +\infty)$
9. $(15, +\infty)$

**REAL-WORLD PROBLEMS WITH INEQUALITIES**

Inequalities can be used to solve many verbal and real-world problems.

**Example 4** Write an Inequality

**DELIVERIES** Craig is delivering boxes of paper to each floor of an office building. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each elevator trip?

**Explore** Let $b =$ the number of boxes Craig can safely take on each trip. A maximum capacity of 2000 pounds means that this weight must be less than or equal to 2000.

**Plan** The total weight of the boxes is $64b$. Craig’s weight plus the total weight of the boxes must be less than or equal to 2000. Write an inequality.

**Solve**

$160 + 64b \leq 2000$

$\frac{160 - 160}{64} + 64b \leq 2000 - 160$

$64b \leq 1840$

$\frac{64b}{64} \leq \frac{1840}{64}$

$b \leq 28.75$

**Examine** Since he cannot take a fraction of a box, Craig can take no more than 28 boxes per trip and still meet the safety requirements of the elevator.

You can use a graphing calculator to find the solution set for an inequality.

**Graphing Calculator Investigation**

**Solving Inequalities**

The inequality symbols in the TEST menu on the TI-83 Plus are called relational operators. They compare values and return 1 if the test is true or 0 if the test is false.

You can use these relational operators to find the solution set of an inequality in one variable.

**Think and Discuss**

1. See margin.
2. Using the TRACE function, investigate the graph. What values of $x$ are on the graph? What values of $y$ are on the graph? all real numbers; 0 and 1
3. Based on your investigation, what inequality is graphed? $x \geq -1$
4. Solve $11x + 3 \geq 2x - 6$ algebraically. How does your solution compare to the inequality you wrote in Exercise 3? The solutions are the same.

**Graphing Calculator Investigation**

**Solving Inequalities** After students enter $11x + 3$, have them press 2 [MATH] 4 to insert the $\geq$ symbol before entering $2x - 6$. The values of $x$ for which 0 is returned (where the inequality is false) are not visible on the screen because they overlay part of the $x$-axis. To help students realize this fact, have them use the Trace feature to travel from positive values of $x$ to increasingly negative values of $x$ along the graph shown in the window.
1. Explain why it is not necessary to state a division property for inequalities.

Sample answer: 

2. Write an inequality using the > symbol whose solution set is graphed below.

Sample answer: 

3. OPEN ENDED Write an inequality for which the solution set is the empty set.

Sample answer: \( x + 2 < x + 1 \)

Solve each inequality. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line.

4. \( a + 2 < 3.5 \) \( \{a | a < 1.5\} \)

5. \( 5 \geq 3x \) \( \{x | x \leq \frac{5}{3}\} \)

6. \( 11 - c \geq 8 \) \( \{c | c \geq 3\} \)

7. \( 4y + 7 > 31 \) \( \{y | y > 6\} \)

8. \( 2w + 19 < 5 \) \( \{w | w < -7\} \)

9. \( -0.6p < -9 \) \( \{p | p > 15\} \)

10. \( \frac{n}{12} + 15 \leq 13 \) \( \{n | n \leq -24\} \)

Define a variable and write an inequality for each problem. Then solve.

12. The product of 12 and a number is greater than 36. \( 12n > 36; n > 3 \)

13. Three less than twice a number is at most 5. \( 2n - 3 \leq 5; n \leq 4 \)

14. SCHOOL The final grade for a class is calculated by taking 75% of the average test score and adding 25% of the score on the final exam. If all scores are out of 100 and a student has a 76 test average, what score does the student need to make on the final exam to have a final grade of at least 80? at least 92

* indicates increased difficulty

**Homework Help**

For Exercises See Examples
15–40 1–3
41–51 4

**Extra Practice**

See page 629.


21. \( k \mid k \geq -3.5 \)

23. \( m \mid m > -4 \)

27. \( n \mid n \geq 1.75 \)

28. \( \{w | w > -1\} \)

29. \( \{x | x > -279\} \)

30. \( \{d | d > -18\} \)

31. \( \{d | d > -3\} \)

32. \( \{z | z > 2.6\} \)

34. \( \{a | a \geq \frac{5}{7}\} \)

Solve each inequality. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line.

15. \( n + 4 \geq 7 \) \( \{n | n \geq 11\} \)

16. \( b - 3 \leq 15 \) \( \{b | b \leq 18\} \)

17. \( 5x < 35 \) \( \{x | x < 7\} \)

18. \( \frac{a}{2} < -4 \) \( \{a | a < -8\} \)

19. \( \frac{x}{3} \geq -9 \) \( \{x | x \geq -27\} \)

20. \( -8p \geq 24 \) \( \{p | p \leq -3\} \)

21. \( 13 - 4k \leq 27 \)

24. \( 6b + 11 \geq 15 \) \( \{b | b \geq \frac{2}{3}\} \)

25. \( 2(4t + 9) \leq 18 \) \( \{t | t \leq 3\} \)

26. \( 90 \geq 5(2r + 6) \) \( \{r | r \leq 6\} \)

27. \( 14 - 8n \leq 0 \)

28. \( 24 - 5w \leq 8 \)

29. \( 0.2x + 5.58 < 0 \)

30. \( 1.5 - 0.25c \leq 6 \)

31. \( 6d + 3 \geq 5d - 2 \)

32. \( 9c + 2 \geq 4x + 15 \)

33. \( 2(g + 4) < 3g \) \( g \mid g < 2 \)

34. \( 3(a + 4) \leq 3(3a + 4) \) \( \{a | a \leq 4\} \)

35. \( y < -\frac{y}{9} + \frac{2}{9} \) \( \{y | y < \frac{2}{3}\} \)

36. \( \frac{1 - 4p}{2} < 0.2 \) \( \{p | p > 0\} \)

37. \( \frac{4x + 2}{6} < \frac{2x + 1}{5} \) \( \phi \)

38. \( 12(1 - n) \leq -6n \) \( \{n | n \leq -\frac{3}{2}\} \)

39. PART-TIME JOB David earns $5.60 an hour working at Box Office Videos. Each week, 25% of his total pay is deducted for taxes. If David wants his take-home pay to be at least $105 a week, solve the inequality \( 5.6x - 0.25(5.6x) \geq 105 \) to determine how many hours he must work. at least 25 h

40. STATE FAIR Juan’s parents gave him $35 to spend at the State Fair. He spends $13.25 for food. If rides at the fair cost $1.50 each, solve the inequality \( 1.5r + 13.25 \leq 35 \) to determine how many rides he can afford. 14 rides

**Answers**

15. \([-11, +\infty)\)

16. \([-\infty, 18]\)

17. \([-\infty, 7]\)

18. \([-8, +\infty)\)

19. \([-\infty, 27]\)

20. \([-\infty, -3]\)

21. \([-3.5, +\infty)\)

22. \([-\infty, -5)\)

23. \([-\infty, -4)\)

24. \(\left[\frac{2}{3}, +\infty)\right)\)

25. \([-\infty, 0]\)

26. \([-\infty, 6]\)

27. \([1.75, +\infty)\)

28. \(\left[-\frac{1}{20}, +\infty)\right\)

29. \([-\infty, -279]\)

30. \([5, +\infty)\)

31. \([-\infty, 5)\)

32. \([-\infty, 2.6)\)

33. \([-\infty, -2)\)

34. \(\left[\frac{5}{7}, +\infty)\right\)

35. \(\left[-\infty, \frac{1}{5}\right)\)

36. \((0, +\infty)\)

37. \(\phi\)

38. \(-\frac{3}{2}\)
In 1995, 55% of children ages three to five were enrolled in center-based child care programs. Parents cared for 26% of children, relatives cared for 19% of children, and non-relatives cared for 17% of children.

Source: National Center for Education Statistics

### Child Care

#### Exercise 47. CHILD CARE

By Ohio law, when children are napping, the number of children per child care staff member may be as many as twice the maximum listed at the right. Write and solve an inequality to determine how many staff members are required to be present in a room where 17 children are napping and the youngest child is 18 months old.

\[ 2(Tm) + 17; m \leq \frac{17}{14}, \text{ at least 2} \]

#### Child Care Staff Members

<table>
<thead>
<tr>
<th>Maximum Number of Children</th>
<th>Per Child Care Staff Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

Source: Ohio Department of Job and Family Services

### CAR SALES

For Exercises 48 and 49, use the following information.

Mrs. Lucas earns a salary of $24,000 per year plus 1.5% commission on her sales. If the average price of a car she sells is $30,500, about how many cars must she sell to make an annual income of at least $40,000?

**Exercise 48.** Write an inequality to describe this situation.

\[ $24,000 + 0.015(30,500a) \geq 40,000 \]

**Exercise 49.** Solve the inequality and interpret the solution.

\[ n \geq 34.97; \text{ She must sell at least 35 cars}. \]

### TEST GRADES

For Exercises 50 and 51, use the following information.

Ahmik’s scores on the first four of five 100-point history tests were 85, 91, 89, and 94.

**Exercise 50.** If a grade of at least 90 is an A, write an inequality to find the score Ahmik must receive on the fifth test to have an A test average. **See margin.**

**Exercise 51.** Solve the inequality and interpret the solution. \[ s \geq 91; \text{ Ahmik must score at least 91 on her next test to have an A test average.} \]

### CRITICAL THINKING

Which of the following properties hold for inequalities? Explain your reasoning or give a counterexample.

- Reflexive
- Symmetric
- Transitive

**Exercise 52a.** It holds only for \( \leq \) or \( \geq \); \( 2 < 2 \). **52b.** \( 1 < 2 \) but \( 2 \not< 1 \)

### WRITING IN MATH

Answer the question that was posed at the beginning of the lesson. **See pp. 53A–53B.**

**How can inequalities be used to compare phone plans?**

Include the following in your answer:

- an inequality comparing the number of minutes offered by each plan, and
- an explanation of how Kuni might determine when Plan 1 might be cheaper than Plan 2 if she typically uses more than 150 but less than 400 minutes.

---

**Equivalence Relations**

A relation \( R \) on a set \( A \) is an equivalence relation if it has the following properties:

- Reflexive Property: For every element \( a \) of set \( A \), \( aRa \) holds.
- Symmetric Property: For all elements \( a \) and \( b \) of set \( A \), if \( aRb \), then \( bRa \).
- Transitive Property: For all elements \( a, b, \) and \( c \) of set \( A \), if \( aRb \) and \( bRc \), then \( a Rc \).

Equality is an equivalence relation on the set \( A \) of all real numbers, since it is reflexive, symmetric, and transitive. Therefore, it is an equivalence relation.

In each of the following, a relation and a set are given. Write yes if the relation is an equivalence relation on the given set. If it is not, tell which of the properties it fails to exhibit.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
</table>

**Answer**

\[ 50. \quad 85 + 91 + 89 + 94 + s \geq 90 \]

---

**Reading to Learn Mathematics, p. 29**

**ELL**

**How can inequalities be used to compare phone plans?**

Read the introduction to Lesson 5 on the top of page 38 in your textbook.

- Write an inequality comparing the number of minutes per month included in the two phone plans.
- Suppose that in one month you are still uncertain about which phone plan best suits your needs. How would you plan to solve this problem?

**Plan 1**

509

**Plan 2**

515

Which plan should you choose? **Plan 2**

**Reading the Lesson**

- There are several different ways to write a true inequality. Write each of the following in interval notation.
  - \( a < b < c \)
  - \( a < b \) and \( b < c \)
- If you have a true inequality, all numbers between the two numbers are also true for \( a \) and \( c \).
- If you have a false inequality, at least one of the two numbers must be false.

**Helping You Remember**

- One way to remember something is to explain it to another person. A naive student says that solving inequalities is forgetting to reverse the inequality symbol when multiplying or dividing both sides of an inequality by a negative number. Suppose that your classmate is having trouble understanding this rule. How would you explain this rule to your classmate?

**Example Answer**

"When you multiply or divide both sides of an inequality by a negative number, you must reverse the inequality symbol. This is because multiplying or dividing by a negative number changes the direction of the inequality. For example, if \( a < b \) and \( c < 0 \), then \( ac > bc \)."
54. If $4 - 5n \geq -1$, then $n$ could equal all of the following EXCEPT D
   \[ \text{A} \quad \frac{-1}{5}, \quad \text{B} \quad \frac{1}{5}, \quad \text{C} \quad 1, \quad \text{D} \quad 2. \]
55. If $a < b$ and $c < 0$, which of the following are true? D
   I. $ac > bc$ \quad II. $a + c < b + c$ \quad III. $a - c > b - c$
   \[ \text{A} \quad \text{I only}, \quad \text{B} \quad \text{II only}, \quad \text{C} \quad \text{III only}, \quad \text{D} \quad \text{I, II, and III}. \]

Use a graphing calculator to solve each inequality.
56. $-5x - 8 < 7 \quad x > -3$  \quad 57. $-4(2x - 3) \leq 60 \quad x \geq -2$
58. $3(x + 3) \geq 2(x + 4) \quad x \geq -1$

**Mixed Review**

60. \[ \left\{ \begin{array}{l} 5 \\ 11 \\ 4 \\ -4 \end{array} \right\} \]
62. $b = \text{ online browsers each year; } 6b + 19.2 = 106.6; \text{ about 14.6 million browsers each year}$

Name the sets of numbers to which each number belongs. (Lesson 1-2)
63. $31 \quad 64. \quad -4.2 \quad 65. \sqrt{7}$
66. **BABY-SITTING** Jenny babysat for $5\frac{1}{2}$ hours on Friday night and 8 hours on Saturday. She charges $4.25 per hour. Use the Distributive Property to write two separate expressions that represent how much money Jenny earned. (Lesson 1-2)

67. $|x| = 7 \quad \{-7, 7\}$  \quad 68. $|x + 5| = 18 \quad \{13, -23\}$  \quad 69. $|5y - 8| = 12 \quad \{\frac{4}{5}, \frac{-4}{5}\}$
70. $|2x - 3| = 14 \quad \{11, 25\}$  \quad 71. $|2x + 6| = 10 \quad \{-11, -1\}$  \quad 72. $|x + 4| + 3 = 17 \quad \{-18, 10\}$

**USA TODAY Snapshots®**

Just looking, thank you
Online shoppers who browse, research or compare products, but don’t necessarily make a purchase, are increasing:

![Graph showing increasing number of online shoppers](source.png)

**Getting Ready for Lesson 1-6**

**PREREQUISITE SKILL** Solve each equation. Check your solutions. (Lesson 1-2)
67. $x = 7 \quad \{-7, 7\}$  \quad 68. $x + 5 = 18 \quad \{13, -23\}$  \quad 69. $5y - 8 = 12 \quad \{\frac{4}{5}, \frac{-4}{5}\}$
70. $2x - 3 = 14 \quad \{11, 25\}$  \quad 71. $2x + 6 = 10 \quad \{-11, -1\}$  \quad 72. $x + 4 + 3 = 17 \quad \{-18, 10\}$

**Practice Quiz 2** Lessons 1-3 through 1-5

1. Solve $2d + 5 = 8d + 2$. Check your solution. (Lesson 1-3) \[ 0.5 \]
2. Solve $s = \frac{1}{2}gt^2$ for $g$. (Lesson 1-3) \[ \frac{2s}{t^2} = g \]
3. Evaluate $|x - 3y|$ if $x = -8$ and $y = 2$. (Lesson 1-4) \[ 14 \]
4. Solve $3|3x + 2| = 51$. Check your solutions. (Lesson 1-4) \[ \{-\frac{19}{3}, 5\} \]
5. Solve $(m - 5) - 3(2m - 5) < 5m + 1$. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line. (Lesson 1-5) \[ \text{See margin.} \]

**Open-Ended Assessment**

**Writing** Have students write their own list of tips for solving inequalities, including when to reverse the inequality sign and how to tell when the graph begins with a circle or with a dot.

**Assessment Options**

**Practice Quiz 2** The quiz provides students with a brief review of the concepts and skills in Lessons 1-3 through 1-5. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

**Quiz (Lessons 1-4 and 1-5)** is available on p. 52 of the Chapter 1 Resource Masters.

**Answer (Practice Quiz 2)**

\[ \{m|m > \frac{4}{9}\} \text{ or } \left(\frac{4}{9}, +\infty\right) \]

---

**Online Lesson Plans**

USA TODAY Education’s Online site offers resources and interactive features connected to each day’s newspaper. Experience TODAY, USA TODAY’s daily lesson plan, is available online and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.
Focus

5-Minute Check Transparency 1-6 Use as a quiz or review of Lesson 1-5.

Mathematical Background notes are available for this lesson on p. 4D.

Building on Prior Knowledge

In Lesson 1-5 students solved inequalities, and in Lesson 1-4 they solved absolute value equations. In this lesson, they expand these skills to solving compound inequalities and absolute value inequalities.

How are compound inequalities used in medicine?

Ask students:

- If you are scheduled to have a glucose tolerance test at 10 A.M., at what hour should you begin fasting? sometime between 6 P.M. and midnight
- Medicine What does a glucose tolerance test measure? how well the body processes sugar (glucose)

Study Tip

Interval Notation
The compound inequality \(-1 \leq x < 2\) can be written as \([-1, 2)\), indicating that the solution set is the set of all numbers between \(-1\) and 2, including \(-1\), but not including 2.

Key Concept

“And” Compound Inequalities

- Words A compound inequality containing the word and is true if and only if both inequalities are true.
- Example \(x \geq -1\) and \(x < 2\)

Another way of writing \(x \geq -1\) and \(x < 2\) is \(-1 \leq x < 2\). Both forms are read “x is greater than or equal to –1 and less than 2.”

Example 1 Solve an “and” Compound Inequality

Solve \(13 < 2x + 7 \leq 17\). Graph the solution set on a number line.

Method 1
Write the compound inequality using the word and. Then solve each inequality.

\[
13 < 2x + 7 \quad \text{and} \quad 2x + 7 \leq 17
\]
\[
6 < 2x \quad \text{and} \quad 2x \leq 10
\]
\[
3 < x \quad \text{and} \quad x \leq 5
\]

Method 2
Solve both parts at the same time by subtracting 7 from each part. Then divide each part by 2.

\[
13 < 2x + 7 \quad \leq 17
\]
\[
6 < 2x \quad \leq 10
\]
\[
3 < x \quad \leq 5
\]
Graph the solution set for each inequality and find their intersection.

\[
\begin{align*}
& 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \\
& x > 3 \\
& 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \\
& x \leq 5 \\
& 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \\
& 3 < x \leq 5
\end{align*}
\]

The solution set is \( \{ x \mid 3 < x \leq 5 \} \).

The graph of a compound inequality containing or is the union of the solution sets of the two inequalities.

**Example 2** Solve an “or” Compound Inequality

Solve \( y - 2 > -3 \) or \( y + 4 \leq -3 \). Graph the solution set on a number line.

Solve each inequality separately:

\[
\begin{align*}
y - 2 & > -3 \\
y & > -1 \\
y + 4 & \leq -3 \\
y & \leq -7
\end{align*}
\]

The solution set is \( \{ y \mid y > -1 \text{ or } y \leq -7 \} \).

**ABSOLUTE VALUE INEQUALITIES** In Lesson 1-4, you learned that the absolute value of a number is its distance from 0 on the number line. You can use this definition to solve inequalities involving absolute value.

www.algebra2.com/extra_examples

Ron Millard  
Shawnee Mission South H.S., Overland Park, KS

“To help make further work with absolute value more understandable, I teach my students to solve absolute value inequalities by using the definition of absolute value. Using this method, the statement \( |3x - 12| \geq 6 \) is rewritten as \( 3x - 12 \geq 6 \) or \( -(3x - 12) \geq 6 \).”
3 Solve $3 > |d|$. Graph the solution set on a number line.

$$|d| - 3 < d < 3$$

-4 -3 -2 -1 0 1 2 3 4

4 Solve $3 < |d|$. Graph the solution set on a number line.

$$|d| < -3 	ext{ or } d > 3$$

-4 -3 -2 -1 0 1 2 3 4

Reading Tip: Make sure students understand the meaning of Examples 3 and 4 before they go on. Have them say the problem in words (for Example 3: “The distance of $a$ from zero without regard to direction is less than 4.”) and demonstrate where $a$ can be located on a number line.

5 Solve $|2x - 2| \geq 4$. Graph the solution set on a number line.

$$x| \leq -1 \text{ or } x \geq 3$$

-3 -2 -1 0 1 2 3 4 5

**Example 3** Solve an Absolute Value Inequality ($<$)

Solve $|a| < 4$. Graph the solution set on a number line.

You can interpret $|a| < 4$ to mean that the distance between $a$ and 0 on a number line is less than 4 units. To make $|a| < 4$ true, you must substitute numbers for $a$ that are fewer than 4 units from 0.

Notice that the graph of $|a| < 4$ is the same as the graph of $a > -4$ and $a < 4$.

All of the numbers between -4 and 4 are less than 4 units from 0. The solution set is $|a| = -4 < a < 4$.

**Example 4** Solve an Absolute Value Inequality ($>$)

Solve $|a| > 4$. Graph the solution set on a number line.

You can interpret $|a| > 4$ to mean that the distance between $a$ and 0 is greater than 4 units. To make $|a| > 4$ true, you must substitute values for $a$ that are greater than 4 units from 0.

Notice that the graph of $|a| > 4$ is the same as the graph of $a > 4$ or $a < -4$.

All of the numbers not between -4 and 4 are greater than 4 units from 0. The solution set is $|a| > 4$ or $a < -4$.

An absolute value inequality can be solved by rewriting it as a compound inequality.

**Key Concept** Absolute Value Inequalities

- **Symbols** For all real numbers $a$ and $b$, $b > 0$, the following statements are true.
  1. If $|a| < b$ then $-b < a < b$.
  2. If $|a| > b$ then $a > b$ or $a < -b$.

- **Examples**
  - If $2x + 1 < 5$, then $-5 < 2x + 1 < 5$.
  - If $2x + 1 > 5$, then $2x + 1 > 5$ or $2x + 1 < 5$.

These statements are also true for $= \leq$ and $\geq$, respectively.

**Example 5** Solve a Multi-Step Absolute Value Inequality

Solve $|3x - 12| \geq 6$. Graph the solution set on a number line.

$|3x - 12| \geq 6$ is equivalent to $3x - 12 \geq 6$ or $3x - 12 \leq -6$. Solve each inequality.

$3x - 12 \geq 6$ or $3x - 12 \leq -6$

$3x \geq 18$ or $3x \leq 6$

$x \geq 6$ or $x \leq 2$

The solution set is $|x| = 6$ or $x = 2$.

**Differentiated Instruction**

**Kinesthetic** Have students work in pairs to create a number line on the floor, perhaps using floor tiles and masking tape. Ask one partner to write or say an inequality such as $|x| < 5$ and then have the other partner walk from -5 to 5 on the number line to demonstrate the possible values for $x$. 
Example 6  Write an Absolute Value Inequality

Job Hunting  
To prepare for a job interview, Megan researches the position’s requirements and pay. She discovers that the average starting salary for the position is $38,500, but her actual starting salary could differ from the average by as much as $2450.

1. Write an absolute value inequality to describe this situation.
   Let \( x = \) Megan’s starting salary.
   
   \[
   |38,500 - x| \leq 2450
   \]

2. Solve the inequality to find the range of Megan’s starting salary.
   Rewrite the absolute value inequality as a compound inequality. Then solve for \( x \).
   \[
   -2450 \leq 38,500 - x \leq 2450
   \]
   
   \[
   -2450 - 38,500 \leq 38,500 - x - 38,500 \leq 2450 - 38,500
   \]
   
   \[
   -40,950 \leq -x \leq -36,050
   \]
   
   \[
   40,950 \geq x \geq 36,050
   \]
   
   The solution set is \( \{x | 36,050 \leq x \leq 40,950\} \). Thus, Megan’s starting salary will fall between $36,050 and $40,950, inclusive.

Check for Understanding

1. Write a compound inequality to describe the following situation.
   \( \text{Buy a present that costs at least } \$5 \text{ and at most } \$15 \). \( 5 \leq x \leq 15 \)

2. Open Ended  Write a compound inequality whose graph is the empty set. Sample answer: \( x < -3 \text{ and } x > 2 \)

3. Find the Error  Sabrina and Isaac are solving \( |3x + 7| > 2 \).

<table>
<thead>
<tr>
<th>Sabrina</th>
<th>Isaac</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3x + 7 &gt; 2 ) or ( 3x + 7 &lt; -2 )</td>
<td>( 3x + 7 &gt; 2 )</td>
</tr>
<tr>
<td>( 3x &gt; 2 - 7 ) or ( 3x &lt; -9 )</td>
<td>( -2 &lt; 3x + 7 &lt; 2 )</td>
</tr>
<tr>
<td>( x &gt; -5 ) or ( x &lt; -3 )</td>
<td>( -9 &lt; 3x &lt; -5 )</td>
</tr>
<tr>
<td>( x &gt; -\frac{5}{3} ) or ( x &lt; -\frac{5}{3} )</td>
<td>( -3 &lt; x &lt; \frac{5}{3} )</td>
</tr>
</tbody>
</table>

Who is correct? Explain your reasoning.

Guided Practice

Write an absolute value inequality for each graph. 4–5. See margin for graphs.

4. all numbers between \(-8\) and \(8\)  \( |n| < 8 \)

5. all numbers greater than \(-3\) and less than \(-3\)  \( |n| > 3 \)

6. all numbers less than \(-4\) and greater than \(-4\)  \( |n| \geq 4 \)

7. \( |n| < 2 \)

Answers

4. \[
\begin{align*}
  &-12 &-8 &-4 &0 &4 &8 \\
\end{align*}
\]

5. \[
\begin{align*}
  &-6 &-4 &-2 &0 &2 &4 \\
\end{align*}
\]
Solve each inequality. Graph the solution set on a number line.
8. \( y - 3 > 1 \) or \( y + 2 < 1 \)
9. \( 3 < d + 5 < 3 \)
10. \( |a| \geq 5 \) or \( a \geq 5 \) or \( a \leq -5 \)
11. \( g + 4 \leq 9 \) or \( g \leq 5 \)
12. \( 4k - 8 \leq 20 \) or \( k \geq 3 < 7 \)
13. \( |w| \geq 2 \) or all real numbers

Application
14. **Flooring** Deion estimates that he will need between 55 and 60 ceramic tiles to retile his kitchen floor. If each tile costs $6.25, write and solve a compound inequality to determine what the cost of the tile could be.

\[ 55 \leq c \leq 60 \]

Extra Practice
See page 829.

**Answers**

8. 8–13. See margin for graphs.

9. \( |y| > 4 \) or \( y < -1 \)

10. \( |a| \geq 5 \) or \( a \geq 5 \) or \( a \leq -5 \)

11. \( |g| + 4 \leq 9 \) or \( g \leq 5 \)

12. \( 4k - 8 < 20 \) or \( k < 3 < 7 \)

13. \( |w| \geq 2 \) or all real numbers

**Homework Help**

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.


15. all numbers greater than 5 or less than or equal to \( -5 \)

16. all numbers less than 7 and greater than \( -7 \)

17. all numbers between \( -4 \) and \( 4 \)

18. all numbers less than or equal to \( -6 \) or greater than or equal to \( 6 \)

19. all numbers greater than 8 or less than \( -8 \)

20. all numbers less than or equal to \( 1.2 \) and greater than or equal to \( -1.2 \)

21. \( |n| > 1 \)

22. \( |n| \leq 5 \)

23. \( |n| \geq 1.5 \)

24. \( |n| < 6 \)

25. \( |n + 1| > 1 \)

26. \( |n - 1| \leq 3 \)

**Betta Fish**

**Adult Male Size:** 3 inches

**Water pH:** 6.8–7.4

**Temperature:** 75–86°F

**Diet:** omnivore, prefers live foods

**Tank Level:** top dweller

**Difficulty of Care:** easy to intermediate

**Life Span:** 2–3 years

Source: www.about.com

**Practice and Apply**

27–44. See pp. 53A–53B for graphs.

27. \( p \leq 2 \) or \( p \geq 8 \)

28. \( 3p + 1 \leq 7 \) or \( 2p - 9 \geq 7 \)

29. \(-11 < -4x + 5 < 13 \) or \( x - 2 < x < 4 \)

30. \( |a| \geq 5 \) or \( a \geq 5 \) or \( a \leq -5 \)

31. \(-4 \leq 4f + 24 < 4 \) or \( f - 7 < f < -5 \)

32. \( a + 2 > 2 \) or \( a - 8 < 1 \)

33. \( |g| \leq 9 \) or \( g \leq 9 \) or \( g \leq 9 \)

34. \( |2n| \leq 8 \) or \( m \leq 4 \) or \( m \leq -4 \)

35. \( |3k| < 0 \)

36. \( 5y < 35 \) or \( y < 7 \)

37. \( b - 1 > 6 \) or \( b > 10 \) or \( b < 2 \)

38. \( 6r - 3 < 21 \) or \( r - 3 < r < 4 \)

39. \( |3w - 2| \leq 5 \) or \( w \leq \frac{7}{3} \) or \( w \leq 1 \)

40. \( 7x + 4 < 0 \)

41. \( n \geq n \) or all real numbers

42. \( n \leq n \) or all real numbers

43. \( 2n - 7 \leq 0 \) or \( n = \frac{7}{2} \)

44. \( n - 3 \leq n \) or \( n > 1.5 \)

45. **Betta Fish** A Siamese Fighting Fish, also known as a Betta fish, is one of the most recognized and colorful fish kept as a pet. Using the information at the left, write a compound inequality to describe the acceptable range of water pH levels for a male Betta. \( 6.8 < x < 7.4 \)
SPEED LIMITS For Exercises 46 and 47, use the following information. On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.

46. Write an inequality to represent the allowable speed for a car on an interstate highway. $45 \leq s \leq 65$

47. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway. $45 \leq s \leq 55$

48. HEALTH Hypothermia and hyperthermia are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both conditions are potentially dangerous and occur when a person’s body temperature fluctuates by more than 8° from the normal body temperature of 98.6°F. Write and solve an absolute value inequality to describe body temperatures that are considered potentially dangerous. $|t - 98.6| \geq 8; (b) b > 106.6$ or $b < 90.6$

MAIL For Exercises 49 and 50, use the following information. The U.S. Postal Service defines an oversized package as one for which the length $L$ of its longest side plus the distance $D$ around its thickest part is more than 108 inches and less than or equal to 130 inches.

49. Write a compound inequality to describe this situation. $108 \leq L + D \leq 130$

50. If the distance around the thickest part of a package you want to mail is 24 inches, describe the range of lengths that would classify your package as oversized. $84 \leq L \leq 106$

GEOMETRY For Exercises 51 and 52, use the following information. The Triangle Inequality Theorem states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.

51. Write three inequalities to express the relationships among the sides of $\triangle ABC$.
   1. $a + b > c$, $a + c > b$, $b + c > a$

52. Write a compound inequality to describe the range of possible measures for side $c$ in terms of $a$ and $b$. Assume that $a > b > c$. (Hint: Solve each inequality you wrote in Exercise 51 for $c$.) $a - b < c < a + b$

53. CRITICAL THINKING Graph each set on a number line. a-d. See margin.
   a. $-2 < x < 4$
   b. $x < -1$ or $x > 3$
   c. $-2 < x < 4$ and $(x < -1$ or $x > 3)$ (Hint: This is the intersection of the graphs in part a and part b.)
   d. Solve $3 < |x + 2| \leq 8$. Explain your reasoning and graph the solution set.

54. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See pp. 53A-53B.

How are compound inequalities used in medicine? Include the following in your answer:
- an explanation as to when to use and when to use or when writing a compound inequality,
- an alternative way to write $h \equiv 10$ and $h \leq 16$, and
- an example of an acceptable number of hours for this fasting state and a graph to support your answer.

www.algebra2.com/self_check_quiz

Lesson 1-6 Solving Compound and Absolute Value Inequalities

The union of the graph of $x > 1$ or $x < -5$ and the graph of $-10 \leq x \leq 6$ is shown below. From this we can see that the solution can be rewritten as $(-10 \leq x < -5)$ or $(1 < x \leq 6)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-12</th>
<th>-8</th>
<th>-4</th>
<th>0</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
</table>

Enrichment, p. 36

Conjunctions and Disjunctions
An absolute value inequality may be solved as a compound sentence.

**Compound**
- **Disjunction** $|x| < 10$
- $-10 > x > 10$
- Every solution for $|x| < 10$ is a replacement for $x$ that makes both $x < 10$ and $x > -10$ true. Choose a compound sentence that contains two statements by the word or in its conjunction.

**Example**
- $|x - 3| = 7$ has solutions $\{x = 10\}$ or $\{x = -4\}$.
55. SHORT RESPONSE  Solve $|2x + 11| > 1$ for $x$.  $x > -5$ or $x < -6$

56. If $5 < a < 7 < b < 14$, then which of the following best defines $\frac{a}{b}$?

A. $\frac{5}{7} < \frac{a}{b} < \frac{1}{2}$
B. $\frac{5}{14} < \frac{a}{b} < \frac{1}{2}$
C. $\frac{5}{7} < \frac{a}{b} < 1$
D. $\frac{5}{14} < \frac{a}{b} < 1$

LOGIC MENU  For Exercises 57–60, use the following information.

You can use the operators in the LOGIC menu on the TI-83 Plus to graph compound and absolute value inequalities. To display the LOGIC menu, press $\text{2nd} \rightarrow \text{MATH NUM}$.

57. Clear the Y= list. Enter $(5x + 2 > 12)$ and $(3x - 8 < 1)$ as Y1. With your calculator in DOT mode and using the standard viewing window, press $\text{TRACE}$. Make a sketch of the graph displayed. See margin for sketch.

58. Using the TRACE function, investigate the graph. Based on your investigation, what inequality is graphed? $2 < x < 3$

59. Write the expression you would enter for Y1 to find the solution set of the compound inequality $5x + 2 \geq 3$ or $5x + 2 \leq -3$. Then use the graphing calculator to find the solution set.

60. A graphing calculator can also be used to solve absolute value inequalities. Write the expression you would enter for Y1 to find the solution set of the inequality $|2x - 6| > 10$. Then use the graphing calculator to find the solution set. (Hint: The absolute value operator is item 1 on the MATH NUM menu.)

abs$(2x - 6) > 10$; $(x | x < -2$ or $x > 8)$

Maintain Your Skills

Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line. (Lesson 1-5)

61. $2d + 15 \geq 3$
62. $7x + 11 > 9x + 3$
63. $3n + 4(n + 3) < 5(n + 2)$
   \[d \geq -6$ or $(-\infty, +\infty)$
   \[x < 4$ or $(-\infty, 4)$
   \[n < -1$ or $(-\infty, -1)$

64. CONTESTS  To get a chance to win a car, you must guess the number of keys in a jar to within 5 of the actual number. Those who are within this range are given a key to try in the ignition of the car. Suppose there are 587 keys in the jar. Write and solve an equation to determine the highest and lowest guesses that will give contestants a chance to win the car. (Lesson 1-4) \[|x - 587| = 5; \text{highest: 592 keys, lowest: 582 keys}$

Solve each equation. Check your solutions.

65. $5| x - 3 | = 15$ \[10, 16]$66. |2x + 7| = 15$ \[ -11, 4]$67. |8x + 7| = -4$ \[ \emptyset$

Name the property illustrated by each statement. (Lesson 1-3)

68. If $3x = 10$, then $3x + 7 = 10 + 7$.  Addition ($=$)
69. If $-5 = 4y - 8$, then $4y - 8 = -5$.  Symmetric ($=$)
70. If $-2x - 5 = 9$ and $9 = 6x + 1$, then $-2x - 5 = 6x + 1$.  Transitive ($=$)

Simplify each expression. (Lesson 1-2)

71. $6a - 2b - 3a + 9b$ \[3a + 7b$
72. $-2(m - 4n) - 3(5n + 6)$ \[-2m - 7n - 18$

Find the value of each expression. (Lesson 1-1)

73. $6(5 - 8) \div 9 + 4$ \[2$74. $(3 + 7)^2 - 16 \div 2$ \[92$75. $\frac{7(1 - 4)}{8 - 5}$ \[-7$
Vocabulary and Concept Check

Choose the term from the list above that best matches each example.

1. \( y > 3 \) or \( y < -2 \)  
   - **compound inequality**  

2. \( 0 + (-4b) = -4b \)  
   - **Identity Property**  

3. \( (m - 1)(-2) = -2(m - 1) \)  
   - **Commutative Property**  

4. \( 35x + 56 = 7(5x + 8) \)  
   - **Distributive Property**  

5. \( ab + 1 = ab + 1 \)  
   - **Reflexive Property**  

6. If \( 2x = 3y - 4, 3y - 4 = 7 \), then \( 2x = 7 \).  
   - **Transitive Property**  

7. \( 4(0.25) = 1 \)  
   - **Multiplication Property**  

8. \( 2p + (4 + 9r) = (2p + 4) + 9r \)  
   - **Associative Property**  

9. \( |5n| \)  
   - **absolutely value**  

10. \( 6y + 5z - 2(x + y) \)  
    - **algebraic expression**

Lesson-by-Lesson Review

**Expressions and Formulas**

**Concept Summary**

- **Order of Operations**

  **Step 1** Simplify the expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars.

  **Step 2** Evaluate all powers.

  **Step 3** Do all multiplications and/or divisions from left to right.

  **Step 4** Do all additions and/or subtractions from left to right.

**Example**

Evaluate \( \frac{y^3}{3ab + 2} \) if \( y = 4, a = -2, \) and \( b = -5 \).

\[
\frac{y^3}{3ab + 2} = \frac{4^3}{3(-2)(-5) + 2} \quad y = 4, a = -2, \text{ and } b = -5
\]

\[
= \frac{64}{3(10) + 2} \quad \text{Evaluate the numerator and denominator separately.}
\]

\[
= \frac{64}{32} \quad \text{or } 2
\]

www.algebra2.com/vocabulary_review

**MindJogger Videoquizzes**

**ELL** MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

- **Round 1** Concepts (5 questions)
- **Round 2** Skills (4 questions)
- **Round 3** Problem Solving (4 questions)

**Foldables Study Organizer**

For more information about Foldables, see Teaching Mathematics with Foldables.

Since this is your students’ first use of the Foldables, you may want to show some good examples, and ask volunteers to name the main ideas and procedures that they included. Then have everyone add any information they may have overlooked.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.
Study Guide and Review

Chapter 1 Study Guide and Review

Exercises Find the value of each expression. See Example 1 on page 6.

11. \(10 + 16 \div 4 + 8\) \hspace{1cm} 12. \([21 - (9 - 2)] \div 2\) \hspace{1cm} 13. \(\frac{14(8 - 15)}{2}\)

Evaluate each expression if \(a = 12\), \(b = 0.5\), \(c = -3\), and \(d = \frac{1}{3}\).

See Examples 2 and 3 on page 7.

14. \(6b - 5c\) \hspace{1cm} 15. \(c^3 + ad\) \hspace{1cm} 16. \(\frac{9c + ab}{c}\) \hspace{1cm} 17. \(a(b^2 + a)\)

1-2 Properties of Real Numbers

Concept Summary

- Real numbers (R) can be classified as rational (Q) or irrational (I).
- Rational numbers can be classified as natural numbers (N), whole numbers (W), and/or integers (Z).
- Use the properties of real numbers to simplify algebraic expressions.

Example Simplify \(4(2b + 6c) + 3b - c\).

\[
4(2b + 6c) + 3b - c = 8b + 24c + 3b - c \\
= 8b + 3b + 24c - c \\
= (8 + 3)b + (24 - 1)c \\
= 11b + 23c
\]

Exercises Name the sets of numbers to which each value belongs.

See Example 1 on page 12.

18. \(-\sqrt{9}\) \hspace{1cm} 19. \(1.6\) \hspace{1cm} 20. \(\frac{35}{7}\) \hspace{1cm} 21. \(\sqrt{18}\)

1-3 Solving Equations

Concept Summary

- Verbal expressions can be translated into algebraic expressions using the language of algebra, using variables to represent the unknown quantities.
- Use the properties of equality to solve equations.

Example Solve \(4(a + 5) - 2(a + 6) = 3\).

\[
4(a + 5) - 2(a + 6) = 3 \hspace{1cm} \text{Original equation} \\
4a + 20 - 2a - 12 = 3 \hspace{1cm} \text{Distributive Property} \\
2a + 8 = 3 \hspace{1cm} \text{Commutative, Distributive, and Substitution Properties} \\
2a = -5 \hspace{1cm} \text{Subtraction Property (\(-\))} \\
a = -2.5 \hspace{1cm} \text{Division Property (\(-\))}
\]
Chapter 1  Study Guide and Review

1-4

Solving Absolute Value Equations

Concept Summary
• For any real numbers \( a \) and \( b \), where \( b \geq 0 \), if \( |a| = b \), then \( a = b \) or \( a = -b \).

Example
Solve \(|2x + 9| = 11\).

Case 1 \( a = b \) or Case 2 \( a = -b \)

\[
\begin{align*}
\text{Case 1} & \quad 2x + 9 = 11 \\
& \quad 2x = 2 \\
& \quad x = 1 \\
\text{Case 2} & \quad 2x + 9 = -11 \\
& \quad 2x = -20 \\
& \quad x = -10 \\
\end{align*}
\]

The solution set is \{1, -10\}. Check these solutions in the original equation.

Exercises  Solve each equation. Check your solutions.
See Examples 1–4 on pages 28–30.

34. \(|x + 11| = 42\) \{31, -53\}  35. \(|x + 6| = 36\) \{6, -18\}  36. \(|4x - 5| = -25\) \{\}
37. \(|x + 7| = 3x - 5\) \{6\}  38. \(|y - 5| = -2 = 10\) \{\}  39. \(|3x + 4| = 4x + 8\) \{\-

1-5

Solving Inequalities

Concept Summary
• Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.
• When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be reversed.

Example
Solve \(5 - 4a > 8\). Graph the solution set on a number line.

\[
\begin{align*}
5 - 4a & > 8 & \text{Original inequality} \\
-4a & > 3 & \text{Subtract 5 from each side.} \\
a & < \frac{-3}{4} & \text{Divide each side by -4, reversing the inequality symbol.} \\
\end{align*}
\]

The solution set is \(a < \frac{-3}{4}\).

The graph of the solution set is shown at the right.
Study Guide and Review

Answers

40. \{w \mid w < -4\} or \((-\infty, -4)\)
   \[-8 -6 -4 -2 0 2\]

41. \{x \mid x \geq 5\} or \([5, +\infty)\)
   \[-1 0 1 2 3 4 5 6 7 8 9 10\]

42. \{n \mid n \leq 24\} or \((-\infty, 24]\)
   \[18 20 22 24 26 28\]

43. \{|x| > 2\} or \((2, +\infty)\)
   \[-2 0 2 4 6 8\]

44. \{|z| \geq 6\} or \([6, +\infty)\)
   \[1 2 3 4 5 6 7 8 9 10 11 12\]

45. \{|x| > -1.8\} or \((-1.8, +\infty)\)
   \[-2.2 -2.0 -1.8 -1.6 -1.4 -1.2\]

46. \{-1 < a < 4\}
   \[-4 -2 0 2 4 6\]

47. \(\left\{y \mid \frac{5}{3} < y \leq 5\right\}\)
   \[1 2 3 4 5 6\]

48. \(|x| < -11\) or \(|x| > 11\)
   \[-16 -8 0 8 16 24\]

49. \(|y| - 9 \leq y \leq 18\)
   \[-12 -6 0 6 12 18\]

50. All real numbers
   \[-6 -4 -2 0 2 4\]

51. \(\left\{b \mid b < -4 \text{ or } b > -\frac{10}{3}\right\}\)
   \[-4 -3 -2 -1\]

Answers \(p.\) 51

25. \((-\infty, 3)\)
   \[-2 0 2 4 6 8\]

26. \([2, +\infty)\)
   \[-2 0 2 4 6 8\]

27. \((\infty, 3)\)
   \[-2 0 2 4 6 8\]

28. \([-13, 3]\)
   \[-16 -12 -8 -4 0 4\]

29. \((-1, 2]\)
   \[-2 -1 0 1 2\]

30. \(|y| < -\frac{4}{3} \text{ or } y > 2\)
   \[-2 -1 0 1 2 3\]

Exercises

Solve each inequality. Describe the solution set using set builder notation or interval notation. Then graph the solution set on a number line.

See Examples 1–3 on pages 34–35. 40–45. See margin.

40. \(-7w > 28\)
41. \(3x + 4 \geq 19\)
42. \(\frac{N}{12} + 5 \leq 7\)
43. \(3(6 - 5a) < 12a - 36\)
44. \(2 - 3z \geq 7(8 - 2z) + 12\)
45. \(8(2x - 1) > 11x - 17\)

1–6

Solving Compound and Absolute Value Inequalities

Concept Summary

- The graph of an and compound inequality is the intersection of the solution sets of the two inequalities.
- The graph of an or compound inequality is the union of the solution sets of the two inequalities.
- For all real numbers \(a\) and \(b\), \(b > 0\), the following statements are true.
  1. If \(|a| < b\) then \(-b < a < b\).
  2. If \(|a| > b\) then \(a > b\) or \(a < -b\).

Examples

Solve each inequality. Graph the solution set on a number line.

1. \(-19 < 4d - 7 \leq 13\)
   \(-19 < 4d - 7 \leq 13\) Original inequality
   \(-12 < 4d \leq 20\) Add 7 to each part.
   \(-3 < d \leq 5\) Divide each part by 4.
   The solution set is \(|x| - 3 < d \leq 5\).

2. \(2x + 4 \geq 12\)
   \(2x + 4 \geq 12\) is equivalent to \(2x + 4 \geq 12\) or \(2x + 4 \leq -12\).
   
   \(2x + 4 \geq 12\) or \(2x + 4 \leq -12\) Original inequality
   \(2x \geq 8\)
   \(2x \leq -16\)
   \(x \geq 4\)
   \(x \leq -8\)
   Divide each side by 2.
   The solution set is \(|x| x \geq 4 \text{ or } x \leq -8\).

Exercises

Solve each inequality. Graph the solution set on a number line.

See Examples 1–5 on pages 40–42. 46–51. See margin.

46. \(-1 < 3a + 2 < 14\)
47. \(-1 < 3(y - 2) < 9\)
48. \(|x| + 1 > 12\)
49. \(2y - 9 \leq 27\)
50. \(|5n - 8| > -4\)
51. \(|3b + 11| > 1\)

50 Chapter 1 Solving Equations and Inequalities
25. Solve each equation. Check your solution(s).
21. all reals
22. \[ |8w + 2| + 2 = 0 \] 
23. \[ 12 \cdot \frac{1}{3}y + 3 = 6 - 7, \quad -5 \] 
24. \[ 2|2y - 6| + 4 = 8 \quad 2, \quad 4 \]

26. For Exercises 31 and 32, define a variable, write an equation or inequality, and solve the problem. 31. \( m = \text{miles traveled}; \quad 19.50 + 0.18m = 33; \quad 75 \text{ mi} \)
32. \( s = \text{score on last test}; \quad s + 87 + 89 + 76 + 77 \geq 80; \quad \) at least 71

33. \( \text{STANDARDIZED TEST PRACTICE} \)
If \( \frac{a}{b} = 8 \) and \( ac - 5 = 11 \), then \( bc = \) \[ \frac{5}{8}; \quad \text{or} \] cannot be determined.

34. Portfolio Suggestion
Introduction Translating words into algebraic expressions involves reading the words, deciding what they mean mathematically, and using the correct notation to write the translation. One way to build the skills involved is to go in the opposite direction, translating algebraic expressions into words.

Ask Students Write an expression or equation and create a word problem about it. Exchange your problem with a partner and translate what you receive into an expression or equation. Place your problem in your portfolio.
These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the Chapter 1 Resource Masters.

**Standardized Test Practice Student Recording Sheet, p. A1**

**Part 1: Multiple Choice**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In the square at the right, what is the value of $x$?   
   
   |   |   |
   |---|---|---|
   | 1 | 2 |
   | 3 | 4 |
   | 5 |
   
   A) 1   B) 2
   C) 3   D) 4

2. On a college math test, 18 students earned an A. This number is exactly 30% of the total number of students in the class. How many students are in the class?   
   
   |   |   |
   |---|---|---|
   | 5 | 23 |
   | 48 | 60 |
   | |
   
   A) 5   B) 23
   C) 48   D) 60

3. A student computed the average of her 7 test scores by adding the scores together and dividing this total by the number of tests. The average was 87. On her next test, she scored a 79. What is her new test average?   
   
   |   |   |
   |---|---|---|
   | 83 | 84 |
   | 85 | 86 |
   | |
   
   A) 83   B) 84
   C) 85   D) 86

4. If the perimeter of $\triangle PQR$ is 3 times the length of $PQ$, then $PR =$   
   
   |   |   |
   |---|---|---|
   | 4 | 6 |
   | 7 | 8 |
   | |
   
   A) 4   B) 6
   C) 7   D) 8

5. If a different number is selected from each of the three sets shown below, what is the greatest sum these 3 numbers could have?   
   
   |   |   |
   |---|---|---|
   | 13 | 14 |
   | 17 | 21 |
   | |
   
   A) 13   B) 14
   C) 17   D) 21

6. A pitcher contains $a$ ounces of orange juice. If $b$ ounces of juice are poured from the pitcher into each of $c$ glasses, which expression represents the amount of juice remaining in the pitcher?   
   
   |   |   |
   |---|---|---|
   | $\frac{a}{b} + c$ | $ab - c$ |
   | $a - bc$ | $\frac{a}{bc}$ |
   | |
   
   A) $\frac{a}{b} + c$   B) $ab - c$
   C) $a - bc$   D) $\frac{a}{bc}$

7. The sum of three consecutive integers is 135. What is the greatest of the three integers?   
   
   |   |   |
   |---|---|---|
   | 43 | 44 |
   | 45 | 46 |
   | |
   
   A) 43   B) 44
   C) 45   D) 46

8. The ratio of girls to boys in a class is 5 to 4. If there are a total of 27 students in the class, how many are girls?   
   
   |   |   |
   |---|---|---|
   | 15 | 12 |
   | 9 | 5 |
   | |
   
   A) 15   B) 12
   C) 9   D) 5

9. For which of the following ordered pairs $(x, y)$ is $x + y > 3$ and $x - y < -2$?   
   
   |   |   |
   |---|---|---|
   | (0, 3) | (3, 4) |
   | (5, 3) | (2, 5) |
   | |
   
   A) (0, 3)   B) (3, 4)
   C) (5, 3)   D) (2, 5)

10. If the area of $\triangle ABD$ is 280, what is the area of the polygon $ABCD$?   
   
   |   |   |
   |---|---|---|
   | 560 | 630 |
   | 700 | 840 |
   | |
   
   A) 560   B) 630
   C) 700   D) 840

**Additional Practice**

See pp. 55–56 in the Chapter 1 Resource Masters for additional standardized test practice.

**Test-Taking Tip**

To solve equations or inequalities, you can replace the variables in the question with the values given in each answer choice. The answer choice that results in true statements is the correct answer choice.

TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.
11. In the triangle below, \(x\) and \(y\) are integers. If \(25 < y < 30\), what is one possible value of \(x\)? 
\(122, 124, 126,\) or \(128\)

12. If \(n\) and \(p\) are each different positive integers and \(n + p = 4\), what is one possible value of \(3n + 4p\)? \(13\) or \(15\)

13. In the figure at the right, what is the value of \(x\)? \(55\)

14. One half quart of lemonade concentrate is mixed with \(1\frac{1}{2}\) quarts of water to make lemonade for 6 people. If you use the same proportions of concentrate and water, how many quarts of lemonade concentrate are needed to make lemonade for 21 people? \(1.75\) or \(7/4\)

15. If 25 percent of 300 is equal to 500 percent of \(t\), then \(t\) is equal to what number? \(15\)

16. In the figure below, what is the area of the shaded square in square units? \(13\)

17. There are 140 students in the school band. One of these students will be selected at random to be the student representative. If the probability that a brass player is selected is \(\frac{2}{5}\), how many brass players are in the band? \(56\)

18. A shelf holds fewer than 50 cans. If all of the cans on this shelf were put into stacks of five cans each, no cans would remain. If the same cans were put into stacks of three cans each, one can would remain. What is the greatest number of cans that could be on the shelf? \(40\)

19. \[\frac{3}{4} \quad \frac{4}{3}\]

20. \(+13 \quad +14\)

21. \[0 < s < \frac{3}{4} \quad \frac{1}{4} \quad 3\]

22. \[\ell \parallel m \quad \ell \quad m \quad 120 \quad 2\]

23. The average (arithmetic mean) of \(s\) and \(t\) is greater than the average of \(s\) and \(w\).
Page 13, Algebra Activity

1. 

2. 

3. 

4. 

Page 19, Follow-Up of Lesson 1-2

Algebra Activity

2. 

\[ 0 + 2 = 2 \quad 2 + 3 = 5 \quad 5 + 4 = 9 \]

8. 

\[ 10 \quad 10 - 3 \div 2 = 35 \]

9. 

\[ n \quad n \quad n - 3 \quad n - 3 \quad 2 \]

10. 

Page 27, Lesson 1-3

76. 

To find the most effective level of intensity for your workout, you need to use your age and 10-second pulse count. You must also be able to solve the formula given for \( A \). Answers should include the following.

- Substitute 0.80 for \( I \) and 27 for \( P \) in the formula \( I = \frac{60 \times P}{220 - A} \) and solve for \( A \). To solve this equation, divide the product of 6 and 28 by 0.8. Then subtract 220 and divide by 1. The result is 17.5. This means that this person is 17 years old.

- To find the intensity level for different values of \( A \) and \( P \) would require solving a new equation but using the same steps as described above. Solving for \( A \) would mean that for future calculations of \( A \) you would only need to simplify an expression, 220 - \( \frac{6P}{I} \), rather than solve an equation.

Pages 37–38, Lesson 1-5

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

13. 

\[ x \quad x - 1 \]

\[ y = x \times 1 \div 2 \]

\[ y = x \times 3 \div 2 + x = 0.5x^2 - 1.5x + x = 0.5x^2 - 0.5x \]

\[ y = 0.5x^2 - 0.5x \]
$55. To find where Plan 2 would cost less than Plan 1, we need to find where the cost for Plan 2 for 400 minutes or less would be less than the cost for Plan 1 for 400 minutes or less.

The cost for Plan 2 for any number of minutes can be represented by the expression $55 + 0.4n$, where $n$ is the number of minutes used. The cost for Plan 1 for any number of minutes can be represented by the expression $35 + 0.4n - 150/n$, where $n$ is the number of minutes used.

To find where Plan 2 is cheaper than Plan 1 for 400 minutes or less, we set the cost for Plan 2 equal to the cost for Plan 1 and solve for $n$.

$55 + 0.4n = 35 + 0.4n - 150/n$

Simplifying, we get:

$200 = 150/n$

Solving for $n$, we get:

$n = 150/200 = 0.75$

Therefore, Plan 2 is cheaper than Plan 1 for 400 minutes or less if the number of minutes used is greater than 0.75.

20. \[ -6 -4 -2 0 2 4 \]

21. \[ -7 -6 -5 -4 -3 -2 \]

22. \[ -4 -2 0 2 4 6 \]

23. \[ -6 -4 -2 0 2 4 \]

24. \[ -1 0 1 2 \]

25. \[ -4 -2 0 2 4 6 \]

26. \[ -2 0 2 4 6 \]

27. \[ 0 0.5 1 1.5 2 2.5 \]


29. \[ -286 -284 -282 -280 -278 -276 \]

30. \[ -20 -18 -16 -14 -12 -10 \]

31. \[ -8 -6 -4 -2 0 2 \]

32. \[ 2.0 2.2 2.4 2.6 2.8 3.0 \]

33. \[ -6 -4 -2 0 2 4 \]

34. \[ 0 \[ 1/7 2/7 3/7 4/7 5/7 6/7 7/7 \]

35. \[ -1 -3/5 -1 3/5 1 \]

36. \[ -6 -4 -2 0 2 4 6 \]

37. \[ -6 -4 -2 0 2 4 \]

38. \[ -4 -3 -2 -1 0 1 \]

39. \[ -2 -1 0 1 \]

40. \[ -4 -2 0 2 4 6 \]

41. \[ -4 -2 0 2 4 6 \]

42. \[ -4 -2 0 2 4 6 \]

43. \[ 0 1 2 3 4 5 \]

44. \[ -2 -1 0 1 2 3 \]

53. \[ 150 < 400 \]

\[ n \]

\[ 2 \ n \]

\[ 1 \]

\[ 35 \]

\[ 40 \]

\[ 150 \]

\[ 35 + 0.4n - 150 \]

\[ 2 \]

\[ 400 \]

\[ 55 \]

\[ 2 \]

\[ 1 \]

\[ 55 < 35 + 0.4n - 150 \]

\[ n \]

\[ n \]

\[ n \]

\[ n \]

\[ 200 \]

\[ 2 \]

\[ n \]

\[ 2 \]

\[ 1 \]

\[ 55 \]

\[ > 200 \]

\[ 2 \]

\[ and \]

\[ or \]

\[ 10 \leq h \leq 16 \]

\[ 12 \]

\[ 10 \leq h \leq 16 \]

\[ 8 \]

\[ 9 \]

\[ 10 \]

\[ 11 \]

\[ 12 \]

\[ 13 \]

\[ 14 \]

\[ 15 \]

\[ 16 \]

\[ 17 \]

\[ 18 \]

\[ 19 \]