Lesson Objectives

3-1 Solving Systems of Equations by Graphing (pp. 110–115)
- Solve systems of linear equations by graphing.
- Determine whether a system of linear equations is consistent and independent, consistent and dependent, or inconsistent.

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<th>Block</th>
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<td>Advanced</td>
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| 3-2 Solving Systems of Equations Algebraically (pp. 116–122)
- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

3-3 Solving Systems of Inequalities by Graphing (pp. 123–128)
- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.
  Follow-Up: Systems of Linear Inequalities

3-4 Linear Programming (pp. 129–135)
- Find the maximum and minimum values of a function over a region.
- Solve real-world problems using linear programming.

3-5 Solving Systems of Equations in Three Variables (pp. 136–144)
  Preview: Graphing Equations in Three Variables
- Solve systems of linear equations in three variables.
- Solve real-world problems using systems of linear equations in three variables.

Study Guide and Practice Test (pp. 145–149)
Standardized Test Practice (pp. 150–151)

Chapter Assessment

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<td>TOTAL</td>
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<td>5</td>
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Pacing suggestions for the entire year can be found on pages T20–T21.
# Chapter Resource Manager

## CHAPTER 3 RESOURCE MASTERS

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*Key to Abbreviations:* GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual


3.1 Solving Systems of Equations by Graphing

This lesson extends the relationship between linear equations and graphed lines by dealing with systems of two linear equations. One main idea of the lesson is that the intersection of the two lines represents a point that satisfies both equations, so the coordinates of that point are the solution to the system. In the lesson, students draw the graph for each equation, find the coordinates of the intersection, and confirm that those coordinates satisfy both equations. A second idea in the lesson uses the fact that, for any two lines in a plane, the lines must be intersecting, parallel, or coincident. If the lines intersect, then the system has exactly one solution and is called consistent and independent. If the lines are parallel, then the system has no solution and is called inconsistent. If the lines coincide, then the system has infinitely many solutions and is called consistent and dependent.

3.2 Solving Systems of Equations Algebraically

Students explore three algebraic properties as the basis for solving systems of two linear equations (all three properties were discussed in Lesson 1-3). One property is substitution. Students solve one of two equations for one of the two variables and then substitute the resulting expression into the other equation. That gives them a single equation in one variable which they solve and then use that solution in either of the two-variable equations to solve for the other variable.

A second property is addition. Given two equations in standard form such that one variable has coefficients that are additive inverses, students add the two equations. The resulting equation is in one variable. Students solve that equation and then solve for the other variable.

A third property is multiplication. Given a system of two linear equations in standard form, students can select two numbers so that, after multiplying each equation by one of the numbers, the result is another system of two equations in which one variable has coefficients that are additive inverses. This new system can then be solved by addition.

The method of solution based on addition or multiplication-and-addition is called solving by elimination.
Solving Systems of Inequalities by Graphing

This lesson combines two skills, modeling inequalities as regions of the coordinate plane and solving systems of linear inequalities. The region modeled by an inequality has a boundary line. Rewriting the inequality as an equality describes that boundary line algebraically. For a system of two inequalities, students identify the region that is common to the two inequalities. Also, given a system with at least three inequalities, students identify the region bounded by the system. Then, using pairs of boundary-line equations as a system of equations, they find the coordinates of the vertices of the region.

Two cases are explored for a system of two inequalities where the boundary lines are parallel. In one case, the region common to the two inequalities is between the two parallel boundary lines. In the other case, there is no region common to the two inequalities, so the solution to the system is \( \emptyset \).

Linear Programming

Solving a linear programming problem adds two steps to the skills of the previous lesson, evaluating a given function using the vertex coordinates and then comparing the results to identify the point that maximizes or minimizes the function. In a linear programming problem, each of the inequalities in the system is called a constraint. The region that represents the system's solution is called the feasible region, and the intersections of pairs of boundary lines are referred to as the vertices of the feasible region. The function to be evaluated usually describes net income or cost. A bounded feasible region is one whose outline is a polygon, and an unbounded feasible region is one whose boundary is not closed. Students are given the fact that for a bounded region, the maximum and minimum values of the income or cost function always occur at a vertex. For an unbounded feasible region, the income or cost function may have no maximum or minimum value.

Solving Systems of Equations in Three Variables

This lesson stresses graphical models to explore four possibilities for the solution of a system as the intersection of three planes. (1) The three planes can intersect in a point, and the ordered triple for that point is the single solution to the system. (2) The three planes can intersect in a line, and the system has an infinite number of solutions, any point on that line. (3) Two equations represent one plane, which intersects the third plane; the solution is any point on the line of intersection. (4) The three planes have no point in common, either because the planes are parallel or the lines of intersection of pairs of planes are parallel, and the solution is the empty set. (Another possibility, not mentioned, is that all three equations represent the same plane and the solution is any point on that plane.

The lesson stresses algebraic manipulation for solving a system of three equations in three variables; selecting two of the equations and using elimination to produce a system of two equations in two variables; solving that two-equation, two-variable system and finding the values of the two variables; substituting the values of those two variables into any of the three-variable equations and finding the value of the third variable.

During the algebraic process of solving for the variables, finding always-false statements such as \( 0 = 1 \) means that at least two of the planes are parallel; that system can have no solution. Finding an always-true statement such as \( 0 = 0 \) means at least two of the planes coincide. If the third plane intersects or coincides with that plane, then the number of solutions is infinite; if the third plane is parallel to the first plane, then the system has no solution.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe’s Algebra 2 Key Concepts: Mathematical Background and Teaching Notes, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Graphing Systems of Equations (Lesson 18)
- Substitution (Lesson 19)
- Elimination Using Addition and Subtraction (Lesson 20)
- Elimination Using Multiplication (Lesson 21)
### Additional Intervention Resources

- The Princeton Review’s *Cracking the SAT & PSAT*
- The Princeton Review’s *Cracking the ACT*
- ALEKS

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### TestCheck and Worksheet Builder

This networkable software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.
Chapter 3
Systems of Equations and Inequalities

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

<table>
<thead>
<tr>
<th>Algebra 2 Lesson</th>
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ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student’s needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 109
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 112, 119, 125, 132, 142, 145)
- Writing in Math questions in every lesson, pp. 114, 121, 127, 134, 144
- Reading Study Tip, pp. 124, 129
- WebQuest, p. 120

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 109, 145
- Study Notebook suggestions, pp. 113, 119, 125, 132, 136, 142
- Modeling activities, pp. 115, 127
- Speaking activities, pp. 122, 144
- Writing activities, p. 135
- Differentiated Instruction, (Verbal/Linguistic), pp. 125, 141
- ELL Resources, pp. 108, 114, 121, 125, 126, 134, 141, 143, 145

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (Chapter 3 Resource Masters, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (Chapter 3 Resource Masters, pp. 123, 129, 135, 141, 147)
- Vocabulary PuzzleMaker software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- Teaching Mathematics with Foldables provides suggestions for promoting cognition and language.
- Reading and Writing in the Mathematics Classroom
- WebQuest and Project Resources

For more information on Reading and Writing in Mathematics, see pp. T6–T7.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Systems of linear equations and inequalities can be used to model real-world situations in which many conditions must be met. For example, hurricanes are classified using inequalities that involve wind speed and storm surge. Weather satellites provide images of hurricanes, which are rated on a scale of 1 to 5. You will learn how to classify the strength of a hurricane in Lesson 3-3.

**Key Vocabulary**
- system of equations (p. 110)
- substitution method (p. 116)
- elimination method (p. 118)
- linear programming (p. 130)
- ordered triple (p. 136)

**Vocabulary Builder**
The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 3 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 3 test.
Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 3.

For Lesson 3-1

Graph Linear Equations

1. Graph each equation. (For review, see Lesson 2-2.)
   1. 2y = x
   2. y = x - 4
   3. y = 2x - 3
   4. x + 3y = 6
   5. 2x + 3y = -12
   6. 4y - 5x = 10

For Lesson 3-2

Solve each equation for y. (For review, see Lesson 1-3.)

7. 2x + y = 0  y = -2x
8. x - y = -4  y = x + 4
9. 6x + 2y = 12  y = 6 - 3x
10. 8 - 4y = 5x  y = 2 - \frac{5}{4}x
11. \frac{1}{2}y + 3x = 1  y = 2 - 6x
12. \frac{1}{3}x - 2y = 8  y = \frac{1}{6}x - 4

For Lessons 3-3 and 3-4

Graph Inequalities

13. y \geq -2
14. x + y \leq 0
15. y < 2x - 2
16. x + 4y < 3
17. 2x - y \geq 6
18. 3x - 4y < 10

For Lesson 3-5

Evaluate Expressions

19. 3x + 2y - z = -9
20. 3y - 8z = -13
21. x - 5y + 4z = 0
22. 2x + 9y + 4z = 11
23. 2x - 6y - 5z = -22
24. 7x - 3y + 2z = -20

Make this Foldable to record information about systems of linear equations and inequalities. Begin with one sheet of 11" x 17" paper and four sheets of grid paper.

Step 1 Fold and Cut

Fold the short sides of the 11" x 17" paper to meet in the middle. Cut each tab in half as shown.

Step 2 Staple and Label

Insert 2 folded half-sheets of grid paper in each tab. Staple at edges. Label each tab as shown.

Reading and Writing As you read and study the chapter, fill the tabs with notes, diagrams, and examples for each topic.

Paraphrasing or Summarizing After students make their Foldable, have them label the tabs to correspond to the lessons in this chapter, combining Lessons 3-1 and 3-2 on the first tab. Students use their Foldable to take notes, define terms, record concepts, and write examples. At the end of each lesson, ask students to paraphrase or write a summary of the main ideas and supporting details presented in the lesson. Summaries are useful for condensing data and realizing what is important.
5-Minute Check Transparency 3-1 Use as a quiz or review of Chapter 2.

Mathematical Background notes are available for this lesson on p. 108C.

Building on Prior Knowledge
In Chapter 2, students solved and graphed equations. In this lesson, students use similar procedures to solve and graph systems of equations.

How can a system of equations be used to predict sales?
Ask students:
• Are in-store sales or online sales growing at a faster rate?
  
• How can you tell which type of sales are growing at a faster rate?
  The slope, 7.5, of the online sales graph is greater than the other slope of 4.2, which means the line for online sales is steeper.

GRAPH SYSTEMS OF EQUATIONS A system of equations is two or more equations with the same variables. To solve a system of equations, find the ordered pair that satisfies all of the equations. One way to do this is to graph the equations on the same coordinate plane. The point of intersection represents the solution.

Example 1 Solve by Graphing
Solve the system of equations by graphing.

\[2x + y = 5\]
\[x - y = 1\]

Write each equation in slope-intercept form.

\[2x + y = 5 \rightarrow y = -2x + 5\]
\[x - y = 1 \rightarrow y = x - 1\]

The graphs appear to intersect at (2, 1).

CHECK Substitute the coordinates into each equation.

\[2(2) + 1 \neq 5 \quad x - 1 \neq 1\]
\[4 + 1 \neq 5 \quad 1 \neq 1\]

The solution of the system is (2, 1).

Systems of equations are used in businesses to determine the break-even point. The break-even point is the point at which the income equals the cost. If a business is operating at the break-even point, it is neither making nor losing money.
**Example 2** Break-Even Point Analysis

Travis and his band are planning to record their first CD. The initial start-up cost is $1500, and each CD will cost $4 to produce. They plan to sell their CDs for $10 each. How many CDs must the band sell before they make a profit?

Let \( x \) = the number of CDs, and let \( y \) = the number of dollars.

\[
\begin{align*}
\text{Cost of } x \text{ CDs} & = \text{cost per CD} \times x + \text{startup cost}, \\
\text{Income from } x \text{ CDs} & = \text{price per CD} \times x \times \text{number of CDs}.
\end{align*}
\]

The graphs intersect at (250, 2500). This is the break-even point. If the band sells fewer than 250 CDs, they will lose money. If the band sells more than 250 CDs, they will make a profit.

**CLASSIFY SYSTEMS OF EQUATIONS** Graphs of systems of linear equations may be intersecting lines, parallel lines, or the same line. A system of equations is **consistent** if it has at least one solution and **inconsistent** if it has no solutions. A consistent system is **independent** if it has exactly one solution or **dependent** if it has an infinite number of solutions.

**Example 3** Intersecting Lines

Graph the system of equations and describe it as **consistent and independent**, **consistent and dependent**, or **inconsistent**.

\[
x + \frac{1}{2}y = 5 \\
3y - 2x = 6
\]

Write each equation in slope-intercept form.

\[
x + \frac{1}{2}y = 5 \rightarrow y = -2x + 10 \\
3y - 2x = 6 \rightarrow y = \frac{2}{3}x + 2
\]

The graphs intersect at (3, 4). Since there is one solution, this system is consistent and independent.

**Example 4** Same Line

Graph the system of equations and describe it as **consistent and independent**, **consistent and dependent**, or **inconsistent**.

\[
9x - 6y = 24 \\
6x - 4y = 16
\]

\[
9x - 6y = 24 \rightarrow y = \frac{3}{2}x - 4 \\
6x - 4y = 16 \rightarrow y = \frac{3}{2}x - 4
\]

Since the equations are equivalent, their graphs are the same line. Any ordered pair representing a point on that line will satisfy both equations.

So, there are infinitely many solutions to this system. This system is consistent and dependent.

www.algebra2.com/extra_examples

Lesson 3-1 Solving Systems of Equations by Graphing

2 Teach

**GRAPH SYSTEMS OF EQUATIONS**

**In-Class Examples**

1. Solve the system of equations by graphing:

\[
x - 2y = 0 \\
x + y = 6 (4, 2)
\]

2. A service club is selling copies of their holiday cookbook to raise funds for a project. The printer’s set-up charge is $200, and each book costs $2 to print. The cookbooks will sell for $6 each. How many cookbooks must the members sell before they make a profit?

\[
\begin{align*}
\text{Number of Cookbooks} & = \text{Number of CDs} \\
\text{Dollars} & = \text{Income} - \text{Cost}
\end{align*}
\]

3. Graph the system of equations and describe it as **consistent and independent**, **consistent and dependent**, or **inconsistent**.

\[
x - y = 5 \quad \text{consistent and independent} \\
x + 2y = -4
\]

**CLASSIFY SYSTEMS OF EQUATIONS**

**In-Class Example**

Interpersonal Have students work in pairs to write three systems of equations, one that is **consistent and independent**, one that is **consistent and dependent**, and another that is **inconsistent**. To simplify the activity somewhat, you might require that each system include the equation \( 2x + y = 1 \). Pairs can exchange their systems with another pair of students to have their work checked.
Graph the system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent.

4. \(3x + 4y = 12\)
   \(6x + 8y = -16\)

5. \(3x + 4y = 12 \rightarrow y = -\frac{3}{4}x + 3\)
   \(6x + 8y = -16 \rightarrow y = -\frac{3}{4}x - 2\)

The lines do not intersect. Their graphs are parallel lines. So, there are no solutions that satisfy both equations. This system is inconsistent.

The relationship between the graph of a system of equations and the number of its solutions is summarized below.

**Concept Summary**

- **Consistent and Independent**
  - Intersection of lines: one solution

- **Consistent and Dependent**
  - Same line: infinitely many solutions

- **Inconsistent**
  - Parallel lines: no solution

**Check for Understanding**

1. Explain why a system of linear equations cannot have exactly two solutions.

2. **OPEN ENDED** Give an example of a system of equations that is consistent and independent. Sample answer: \(x + y = 4, x - y = 2\)

3. Explain why it is important to check a solution found by graphing in both of the original equations. A graph is used to estimate the solution. To determine that the point lies on both lines, you must check that it satisfies both equations.

4. Solve each system of equations by graphing. 4–6. See margin for graphs.
   
   4. \(y = 2x + 9\)
      \(y = -x + 3\)
      \((-2, 5)\)
   
   5. \(3x + 2y = 10\)
      \(2x + 3y = 10\)
      \((2, 2)\)
   
   6. \(4x - 2y = 22\)
      \(6x + 9y = -3\)
      \((4, -3)\)

   Graph each system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent. 7–9. See margin for graphs.

   7. \(y = 6 - x\)\(\text{ cons. and ind.}\)
   \(y = x + 4\)

   8. \(x + 2y = 2\)\(\text{ incon.}\)
   \(2x + 4y = 8\)

   9. \(x - 2y = 8\)\(\text{ cons. and}\)
   \(\frac{1}{2}x - y = 4\)\(\text{ dep.}\)
PHOTOS For Exercises 10–12, use the graphic at the right.

10. Write equations that represent the cost of developing a roll of film at each lab.

11. Under what conditions is the cost to develop a roll of film the same for either store?

12. When is it best to use The Photo Lab and when is it best to use Specialty Photos? You should use Specialty Photos if you are developing less than 30 prints, and you should use The Photo Lab if you are developing more than 30 prints.

★ indicates increased difficulty

Practice and Apply


13. \( y = 2x - 4 \)
   \( y = -3x + 1 \) (1, -2)

14. \( y = 3x - 8 \)
   \( y = -x - 8 \) (0, -8)

15. \( x + 2y = 6 \)
   \( 2x + y = 9 \) (4, 1)

16. \( 2x + 3y = 12 \)
   \( 2x - y = 4 \) (3, 2)

17. \( 3x - 7y = -6 \)
   \( x + 2y = 11 \) (5, 3)

18. \( 5x - 11 = 4y \)
   \( 7x - 1 = 8y \) (7, 6)

19. \( 2x + 3y = 7 \)
   \( 2x - 3y = 7 \) (3.5, 0)

20. \( 8x - 3y = -3 \)
   \( 4x - 2y = -4 \) (1.5, 5)

21. \( \frac{1}{4}x + 2y = 5 \)
   \( 2x - y = 6 \) (4, 2)

22. \( \frac{2}{3}x + y = -3 \)

23. \( \frac{1}{2}x - y = 0 \)

24. \( \frac{4}{3}x + \frac{1}{3}y = 3 \)

Graph each system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent. 25–36. See pp. 151A–151F for graphs.

25. \( y = x + 4 \) (incon.)
   \( y = x - 4 \)

26. \( y = x + 3 \) (cons. and ind.)
   \( y = 2x + 6 \)

27. \( x + y = 4 \) (cons.)
   \( -x + y = 9 \) (and ind.)

28. \( 3x + y = 3 \) (cons.)
   \( 6x + 2y = 6 \) (and dep.)

29. \( y = x - 5 \) (incon.)
   \( 2y = 2x - 8 \)

30. \( 4x - 2y = 6 \) (cons.)
   \( 6x - 3y = 9 \) (and dep.)

31. \( 2y = x \) (cons. and ind.)
   \( 8y = 2x + 1 \)

32. \( 2y = 5 - x \) (incon.)
   \( 6y = 7 - 3x \)

33. \( 0.8x - 1.5y = 10 \) (cons.)
   \( 1.2x + 2.5y = 4 \) (and ind.)

34. \( 1.6y = 0.4x + 1 \) (cons.)
   \( 0.4y = 0.1x + 0.25 \) (and dep.)

35. \( 3y - x = -2 \) (incon.)
   \( y - \frac{1}{3}x = 2 \)

36. \( 2y - 4x = 3 \) (cons. and ind.)
   \( \frac{4}{3}x - y = -2 \)

37. GEOMETRY The sides of an angle are parts of two lines whose equations are \( 2y + 3x = -7 \) and \( 3y - 2x = 9 \). The angle’s vertex is the point where the two sides meet. Find the coordinates of the vertex of the angle. (-3, 1)

38. GEOMETRY The graphs of \( y - 2x = 1 \), \( 4x + y = 7 \), and \( 2y - x = -4 \) contain the sides of a triangle. Find the coordinates of the vertices of the triangle.

39. \( y = 52 + 0.23x \), \( y = 80 \)

40. See margin for graph (120, 80)

41. Deluxe Plan

www.algebra2.com/self_check_quiz

Lesson 3-1 Solving Systems of Equations by Graphing 113

Answers

8. [Graph image]

9. [Graph image]

38. [Graph image]

40. [Graph image]
42. Supply, 200,000; demand, 300,000; prices will tend to rise.

43. Supply, 300,000; demand, 200,000; prices will tend to fall.

44. At what quantity will the prices stabilize? What is the equilibrium price for this product? $250,000; $10

ECONOMICS For Exercises 42–44, use the graph below that shows the supply and demand curves for a new multivitamin.
In Economics, the point at which the supply equals the demand is the equilibrium price. If the supply of a product is greater than the demand, there is a surplus and prices fall. If the supply is less than the demand, there is a shortage and prices rise.

42. If the price for vitamins is $8 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?
43. If the price for vitamins is $12 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?
44. At what quantity will the prices stabilize? What is the equilibrium price for this product? $250,000; $10

POPULATION For Exercises 45–47, use the graph that shows 2000 state populations.

45. Write equations that represent the populations of Florida and New York x years after 2000. Assume that both states continue to gain the same number of residents every year. Let y equal the population in thousands.

46. Graph both equations for the years 2000 to 2020. Estimate when the populations of both states will be equal. See margin for graph; 2015.
47. Do you think Florida will overtake New York as the third most populous state by 2010? by 2020? Explain your reasoning.

48. CRITICAL THINKING State the conditions for which the system below is:
   (a) consistent and dependent, (b) consistent and independent, (c) inconsistent.
\[ 3x + 4y = 2 \]
\[ 6x + 8y = 4 \]
49. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

**How can a system of equations be used to predict sales?**
Include the following in your answer:
- an explanation of the real-world meaning of the solution of the system of equations in the application at the beginning of the lesson, and
- a description of what a business owner would learn if the system of equations representing the in-store and online sales is inconsistent.

Enrichment, p. 124

Investments
The following graph shows the value of two different investments over time. Line A represents an initial investment of $50,000 with a 5% annual rate of increase and a 5% annual rate of decrease. Line B represents an initial investment of $50,000 in a certificate of deposit with a 5% annual rate of increase and a 5% annual rate of decrease. Line C represents an initial investment of $50,000 in a passbook savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line D represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line E represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line F represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line G represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line H represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line I represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line J represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line K represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line L represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line M represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line N represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line O represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line P represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line Q represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line R represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line S represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line T represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line U represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line V represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line W represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line X represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line Y represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line Z represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line AA represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line BB represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line CC represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line DD represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line EE represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line FF represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line GG represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line HH represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line II represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line JJ represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line KK represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line LL represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line MM represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line NN represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line OO represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line PP represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line QQ represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line RR represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line SS represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line TT represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line UU represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line VV represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line WW represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line XX represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line YY represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease. Line ZZ represents an initial investment of $50,000 in a savings account with a 5% annual rate of increase and a 5% annual rate of decrease.
50. What are the coordinates \((x, y)\) at which the graphs of \(2x + 3y = 12\) and 
\(2x - y = 4\) intersect?  
\(\text{A} \quad (3, 2) \quad \text{B} \quad (2, 3) \quad \text{C} \quad (1, -2) \quad \text{D} \quad (-3, 6)\)

51. Which equation has the same graph as \(4x + 8y = 12?\)  
\(\text{A} \quad x + y = 3 \quad \text{B} \quad 2x + y = 3 \quad \text{C} \quad x + 2y = 3 \quad \text{D} \quad 2x + 2y = 6\)

**Getting Ready for the Next Lesson**

**Standardized Test Practice**

**Graphing Calculator**

**INTERSECT FEATURE** To use a TI-83 Plus to solve a system of equations, graph both equations on the same screen. Then, select Intersect, which is option 5 under the Calc menu, to find the coordinates of the point of intersection. Solve each system of equations to the nearest hundredth.

- **54.** \((4, 3.42)\)
- **55.** \((3.005, 4)\)
- **56.** \((-9, 3.75)\)
- **57.** \((2.64, 42.43)\)

**Maintain Your Skills**

**Mixed Review**

Graph each inequality. *(Lesson 2-7)* 58–60. See margin.

- **58.** \(y \geq 5 + 3x\)
- **59.** \(2x + y > -4\)
- **60.** \(2y - 1 \leq x\)

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise. *(Lesson 2-6)*

- **61.**
- **62.**
- **63.**

Solve each equation. Check your solutions. *(Lesson 1-4)*

- **64.** \(|x| - 5 = 8 \quad (-13, 13)\)
- **65.** \(|w + 3| = 12 \quad (-15, 9)\)
- **66.** \(|6a - 4| = -2 \quad \emptyset\)
- **67.** \(3|2t - 1| = 15 \quad (-2, 3)\)
- **68.** \(|4r + 7| - 7 = 10 \quad \{-5, \frac{7}{2}\}\)
- **69.** \(|k + 7| = 3k - 11 \quad \{9\}\)

Write an algebraic expression to represent each verbal expression. *(Lesson 1-3)*

- **70.** the sum of 8 and 2 times a number \(8 + 2n\)
- **71.** six less than the square of a number \(x^2 - 6\)
- **72.** four times the sum of a number and 5 \(4(a + 5)\)
- **73.** the quotient of a number and 3 increased by \(\frac{2}{3} + 1\)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Simplify each expression. *(To review simplifying expressions, see Lesson 1-2)*

- **74.** \((3x + 5) - (2x + 3)\) \(x + 2\)
- **75.** \((3y - 11) + (6y + 12)\) \(9y + 1\)
- **76.** \((5x - y) + (-8x + 7y)\) \(-3x + 6y\)
- **77.** \((6x + 3y - 1)\) \(12x + 18y - 6\)
- **78.** \((4x + 2y - x + 2)\) \(15x + 10y + 10\)
- **79.** \((3x + 4y) - 2(x + 4y)\) \(x + 4y\)

**Answer**

49. You can use a system of equations to track sales and make predictions about future growth based on past performance and trends in the graphs. Answers should include the following.

- The coordinates \((6, 54)\) represent that 6 years after 1999 both the in-store sales and online sales will be $54,000.

- The in-store sales and the online sales will never be equal and in-store sales will continue to be higher than online sales.
Solving Systems of Equations Algebraically

What You'll Learn

- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

How can systems of equations be used to make consumer decisions?

In January, Yolanda’s long-distance bill was $5.50 for 25 minutes of calls. The bill was $6.54 in February, when Yolanda made 38 minutes of calls. What are the rate per minute and flat fee the company charges? Let $x$ equal the rate per minute, and let $y$ equal the monthly fee.

- January bill: $25x + y = 5.50$
- February bill: $38x + y = 6.54$

It is difficult to determine the exact coordinates of the point where the lines intersect from the graph. For systems of equations like this one, it may be easier to solve the system by using algebraic methods.

In-Class Example

1. Use substitution to solve the system of equations.

   \[ x + 2y = 8 \]
   \[ \frac{1}{2}x - y = 18 \]

   Solve the first equation for $x$ in terms of $y$.

   \[ x + 2y = 8 \quad \text{First equation} \]
   \[ x = 8 - 2y \quad \text{Subtract } 2y \text{ from each side.} \]

   Substitute $8 - 2y$ for $x$ in the second equation and solve for $y$.

   \[ \frac{1}{2}(8 - 2y) - y = 18 \quad \text{Second equation} \]
   \[ 4 - y - y = 18 \quad \text{Distributive Property} \]
   \[ -2y = 14 \quad \text{Subtract } 4 \text{ from each side.} \]
   \[ y = -7 \quad \text{Divide each side by } -2. \]

   Now, substitute the value for $y$ in either original equation and solve for $x$.

   \[ x + 2y = 8 \quad \text{First equation} \]
   \[ x + 2(-7) = 8 \quad \text{Replace } y \text{ with } -7. \]
   \[ x = 22 \quad \text{The solution of the system is } (22, -7). \]
**Quantitative Comparison Test Item**

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- **A** the quantity in Column A is greater,
- **B** the quantity in Column B is greater,
- **C** the two quantities are equal, or
- **D** the relationship cannot be determined from the information given.

\[
\begin{align*}
2x + y &= 11 \\
x + 3y &= 13
\end{align*}
\]

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
</tbody>
</table>

**Read the Test Item**

You are asked to compare the values of \(x\) and \(y\). Since this is a system of equations, you may be able to find the exact values for each variable.

**Solve the Test Item**

**Step 1**  
Solve the first equation for \(y\) in terms of \(x\) since the coefficient of \(y\) is 1.

\[
\begin{align*}
2x + y &= 11 \\
y &= 11 - 2x & \text{Subtract } 2x \text{ from each side.}
\end{align*}
\]

**Step 2**  
Substitute \(11 - 2x\) for \(y\) in the second equation.

\[
\begin{align*}
x + 3y &= 13 & \text{Second equation} \\
x - 3(11 - 2x) &= 13 & \text{Substitute } 11 - 2x \text{ for } y, \\
x + 33 - 6x &= 13 & \text{Distributive Property} \\
-5x &= -20 & \text{Simplify.} \\
x &= 4 & \text{Divide each side by } -5.
\end{align*}
\]

**Step 3**  
Now replace \(x\) with 4 in either equation to find the value of \(y\).

\[
\begin{align*}
2x + y &= 11 & \text{First equation} \\
2(4) + y &= 11 & \text{Substitute 4 for } x. \\
8 + y &= 11 & \text{Multiply.} \\
y &= 3 & \text{Subtract } 8 \text{ from each side.}
\end{align*}
\]

**Step 4**  
Check the solution.

\[
\begin{align*}
2x + y &= 11 & \text{Original equation} \\
2(4) + 3 &= 11 & \text{Replace } x \text{ with 4 and } y \text{ with 3.} \\
8 + 3 &= 11 & \text{Simplify.} \\
11 &= 11 & \checkmark
\end{align*}
\]

**Step 5**  
Compare the values of \(x\) and \(y\) to answer the original problem.

\[x = 4 \text{ and } y = 3\]

\[4 > 3\]

So, \(x > y\).

The answer is **A**.

---

**Test-Taking Tip**

Memorize the choices for A, B, C, and D in the quantitative comparison questions. You will save time by not having to refer to them for every question.
3 Use the elimination method to solve the system of equations.
\[ x + 2y = 10 \]
\[ x + y = 6 \quad (2, 4) \]

4 Use the elimination method to solve the system of equations.
\[ 2x + 3y = 12 \]
\[ 5x - 2y = 11 \quad (3, 2) \]

**Teaching Tip** Remind students to take time to plan their solution strategy before they start their calculations. Stress that a little forethought may show that it is easier to eliminate one of the variables rather than the other.

**Concept Check**
Ask students what they might look for to help them plan their strategy for solving a system of equations by elimination. Sample answer: Look to see if the coefficients for one of the variables are already the same or if they are opposites. If so, that variable can be eliminated by subtracting or adding the given equations.

**In-Class Examples**

**Example 3** Solve by Using Elimination

Use the elimination method to solve the system of equations.
\[ 4a + 2b = 15 \]
\[ 2a + 2b = 7 \]

In each equation, the coefficient of \( b \) is 2. If one equation is subtracted from the other, the variable \( b \) will be eliminated.

\[ 4a + 2b = 15 \]
\[ (-) 2a + 2b = 7 \]

\[ 2a = 8 \quad \text{Subtract the equations.} \]
\[ a = 4 \quad \text{Divide each side by 2.} \]

Now find \( b \) by substituting 4 for \( a \) in either original equation.

\[ 2(4) + 2b = 7 \quad \text{Replace} \ a \ \text{with} \ 4. \]
\[ 8 + 2b = 7 \quad \text{Multiply.} \]
\[ 2b = -1 \quad \text{Subtract 8 from each side.} \]
\[ b = -\frac{1}{2} \quad \text{Divide each side by 2.} \]

The solution is \((4, -\frac{1}{2})\).

**Example 4** Multiply, Then Use Elimination

Use the elimination method to solve the system of equations.
\[ 3x - 7y = -14 \]
\[ 5x + 2y = 45 \]

Multiply the first equation by 2 and the second equation by 7. Then add the equations to eliminate the \( y \) variable.

\[ 3x - 7y = -14 \quad \text{Multiply by 2.} \]
\[ 6x - 14y = -28 \]
\[ 5x + 2y = 45 \quad \text{Multiply by 7.} \]
\[ 35x + 14y = 315 \]
\[ 41x = 287 \quad \text{Add the equations.} \]
\[ x = 7 \quad \text{Divide each side by 41.} \]

Replace \( x \) with 7 and solve for \( y \).

\[ 3x - 7y = -14 \quad \text{First equation} \]
\[ 3(7) - 7y = -14 \quad \text{Replace} \ x \ \text{with} \ 7. \]
\[ 21 - 7y = -14 \quad \text{Multiply.} \]
\[ -7y = -35 \quad \text{Subtract 21 from each side.} \]
\[ y = 5 \quad \text{Divide each side by} \ -7. \]

The solution is \((7, 5)\).

**Teacher to Teacher**

Vickie McGlohon

DH Conley H.S., Greenville, NC

“I have students make up 10 word problems that can be solved using a system of two or three equations. All problems must be related to a theme, such as sports. The problems must be illustrated and bound for presentation.”
Guided Practice

1. OPEN ENDED Give an example of a system of equations that is more easily solved by substitution and a system that is more easily solved by elimination.

2. Make a conjecture about the solution of a system of equations if the result of subtracting one equation from the other is 0 = 0.

3. FIND THE ERROR Juanita and Vincent are solving the system

\[ \begin{align*}
2x - y &= 6 \\
32x + y &= 10
\end{align*} \]

Juanita

\[ \begin{align*}
2x - y &= 6 \\
(-)32x + y &= 10 \\
0 &= -4
\end{align*} \]

The statement 0 = -4 is never true, so there is no solution.

Vincent

\[ \begin{align*}
2x - y &= 6 \\
(-)2x + y &= 10 \\
4x &= 16 \\
4 &= x \\
- y &= 2 \\
y &= 2
\end{align*} \]

The solution is \( (4,2) \).

Who is correct? Explain your reasoning.

Solve each system of equations by using substitution.

4. \[ \begin{align*}
y &= 3x - 4 \\
y &= 4 + x
\end{align*} \] \( (4,8) \)

5. \[ \begin{align*}
4c + 2d &= 10 \\
c + 3d &= 10
\end{align*} \] \( (1,3) \)

Solve each system of equations by using elimination.

6. \[ \begin{align*}
2x - 3y &= 11 \\
2x + 2y &= 6
\end{align*} \] \( (4,-1) \)

7. \[ \begin{align*}
2p + 4q &= 18 \\
3p - 6q &= 3
\end{align*} \] \( (5,2) \)

Solve each system of equations by using either substitution or elimination.

8. \[ \begin{align*}
a - b &= 2 \\
-2a + 3b &= 3
\end{align*} \] \( (9,7) \)

9. \[ \begin{align*}
5m + n &= 10 \\
4m + n &= 4
\end{align*} \] \( (6,-20) \)

10. \[ \begin{align*}
3x - 2y &= -1 \\
8x = 5 + 12y
\end{align*} \] no solution

Logical Have students summarize the various algebraic methods for solving a system of equations using if-then statements and examples. Sample: “If one of the equations has a variable with a coefficient of 1 (such as \( x + 3y = 9 \) or \( 5x - y = 13 \)), consider the substitution method.”
12. **QUANTITATIVE COMPARISON** Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- (A) the quantity in Column A is greater,
- (B) the quantity in Column B is greater,
- (C) the two quantities are equal, or
- (D) the relationship cannot be determined from the information given.

\[
\begin{align*}
4x + 3y & = 7 \\
2x + y & = 1
\end{align*}
\]

<table>
<thead>
<tr>
<th>Column A</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2x + 2y</td>
<td>6</td>
</tr>
</tbody>
</table>

**Practice and Apply**

Solve each system of equations by using substitution.

13. \(2y - 3k = 3\) \(j + k = 14\) \((9, 5)\)  
14. \(2r + s = 11\) \(6r - 2s = -2\) \((2, 7)\)  
15. \(5a - b = 17\) \(3a + 2b = 5\) \((3, -2)\)  

Solve each system of equations by using elimination.

16. \(-w - z = -2\) \(4w + 5z = 16\) \((6, 8)\) no solution  
17. \(6c + 3d = 12\) \(2c = 8 - d\)  
18. \(2x + 4y = 6\) \(7x = 4 + 3y\) \((1, 1)\)  

Solve each system of equations by using either substitution or elimination.

19. \(u + v = 7\) \(2u + v = 11\) \((4, 3)\)  
20. \(m - n = -9\) \(7m + 2n = 9\) \((-1, 8)\)  
21. \(3p - 5q = 6\) \(2p - 4q = 4\) \((2, 0)\)  
22. \(4x - 5y = 17\) \(3x + 4y = 5\) \((3, -1)\)  
23. \(2c + 6d = 14\) \(c - 3d = 8\) \((-7, 9)\)  
24. \(3s + 2t = -3\) \((10, -1)\)  
25. \(r + 4s = -8\) \((4, -3)\)  
26. \(10m - 9n = 15\) \((6, 5)\)  
27. \(3c - 7d = -3\) \((2, 6)\)  
28. \(6x - 8y = 50\) \((7, -1)\)  
29. \(2p = 7 + q\) \(6p - 3q = 24\) no solution  
30. \(3x = -31 + 2y\) \((5, 8)\)  
31. \(3u + 5v = 6\) \(2u - 4v = -7\) \((- \frac{1}{2}, 2)\)  
32. \(3a - 2b = -3\) \((3, 2)\)  
33. \(s + 3t = 27\) \((\frac{1}{2}, 2)\)  
34. \(f - 2g = \frac{1}{6}f + \frac{1}{3}g = 1\) infinitely many  
35. \(0.25x + 1.75y = 1.25\) \(0.5x + 2.5y = 2\) \((1.5, 0.5)\)  
36. \(0.4m + 1.8n = 8\) \(1.2m + 3.4n = 16\) \((2, 4)\)  

**SKIING** For Exercises 39 and 40, use the following information.

All 28 members in Crestview High School’s Ski Club went on a one-day ski trip. Members can rent skis for $16.00 per day or snowboards for $19.00 per day. The club paid a total of $478 for rental equipment.

39. Write a system of equations that represents the number of members who rented the two types of equipment. \(x + y = 28\) \(16x + 19y = 478\)

40. How many members rented skis and how many rented snowboards?

18 members rented skis and 10 members rented snowboards.

**Answer**

51. You can use a system of equations to find the monthly fee and rate per minute charged during the months of January and February. Answers should include the following.

- The coordinates of the point of intersection are \((0.08, 3.5)\).
- Currently, Yolanda is paying a monthly fee of $3.50 and an additional 8¢ per minute. If she graphs \(y = 0.08x + 3.5\) (to represent what she is paying currently) and \(y = 0.10x + 3\) (to represent the other long-distance plan) and finds the intersection, she can identify which plan would be better for a person with her level of usage.

120 Chapter 3 Systems of Equations and Inequalities
41. **HOUSING** Campus Rentals rents 2- and 3-bedroom apartments for $700 and $900 per month, respectively. Last month they had six vacant apartments and reported $4600 in lost rent. How many of each type of apartment were vacant?

**Solution:** Let x be the number of 2-bedroom apartments and y be the number of 3-bedroom apartments. We have two equations: 2x + 3y = 4600 (total lost rent) and x + y = 6 (total number of vacancies). Solving the system algebraically, we get x = 4 and y = 2. Therefore, there were 4 2-bedroom apartments and 2 3-bedroom apartments vacant last month.

**42. GEOMETRY** Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines whose equations are 2x + y = −12, 2x − y = −8, 2x − y = −4, and 4x + 2y = 24. (−5, −2), (4, 4), (−2, −8), (1, 10)

**INVENTORY** For Exercises 43 and 44, use the following information.

Heung-Soo is responsible for checking a shipment of technology equipment that contains laser printers that cost $700 each and color monitors that cost $200 each. He counts 30 boxes on the loading dock. The invoice states that the order totals $15,000.

43. Write a system of two equations that represents the number of each item.

44. How many laser printers and how many color monitors were delivered?

16 printers, 12 monitors

**45.** Write a system of equations that represents the number of each type of question. 2x + 4y = 100, y = 2x

46. How many true/false questions and multiple-choice questions will be on the test? 10 true/false, 20 multiple-choice

47. If most of his students can answer true/false questions within 1 minute and multiple-choice questions within 2 minutes, will they have enough time to finish the test in 45 minutes? Yes; they should finish the test within 40 minutes.

**EXERCISE** For Exercises 48 and 49, use the following information.

Megan exercises every morning for 40 minutes. She does a combination of step aerobics, which burns about 11 Calories per minute, and stretching, which burns about 4 Calories per minute. Her goal is to burn 335 Calories during her routine.

48. Write a system of equations that represents Megan’s morning workout.

49. How long should she participate in each activity in order to burn 335 Calories? 25 min of step aerobics, 15 min of stretching

50. **CRITICAL THINKING** Solve the system of equations. (4, 6)

\[
\begin{align*}
\frac{1}{3}x + \frac{3}{4}y &= \frac{3}{4} \\
\frac{3}{2} - \frac{2}{12} &\quad \text{Hint: Let } m = \frac{1}{2} \\
x &= \frac{1}{2} y
\end{align*}
\]

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See margin.

How can a system of equations be used to make consumer decisions?

Include the following in your answer:

• a solution of the system of equations in the application at the beginning of the lesson,

• an explanation of how Yolanda can use a graph to decide whether she should change to a long-distance plan that charges $0.10 per minute and a flat fee of $3.00 per month.

www.algebra2.com/self_check_quiz

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**Lesson 3-2 Solving Systems of Equations Algebraically**

**Enrichment, p. 130**

**Using Coordinates**

From one observation point, the line of sight to a distant plane is given by \(x + y = 3\). From another observation point, the line of sight to the plane is given by \(x + 2y = 10\). What are the coordinates of the point at which the rays intersect?

Solve the system of equations:

\[
\begin{align*}
\frac{1}{3}x + \frac{3}{4}y &= \frac{3}{4} \\
\frac{3}{2} - \frac{2}{12} &\quad \text{Hint: Let } m = \frac{1}{2} \\
x &= \frac{1}{2} y
\end{align*}
\]

**Study Guide and Intervention, p. 125 (shown) and p. 126**

**Lesson 3-2 Solving Systems of Equations Algebraically**
Determine whether the given point satisfies each inequality. (Lesson 2-7)

57. \(x + y \leq 3\)
58. \(5y - 4x < -20\)
59. \(3x + 9y \geq -15\)

Write each equation in standard form. Identify \(A, B,\) and \(C.\) (Lesson 2-2)

60. \(y = 7x + 4\)
61. \(x = y\)
62. \(3x = 2 - 5y\)
63. \(6x = 3y - 9\)
64. \(y = \frac{1}{2}x - 3\)
65. \(\frac{2}{3}y - 6 = 1 - x\)

66. **ELECTRICITY** Use the formula \(I = \frac{E}{R + r}\) to find the amount of current \(I\) (in amperes) produced if the electromotive force \(E\) is 1.5 volts, the circuit resistance \(R\) is 2.35 ohms, and the resistance \(r\) within a battery is 0.15 ohms. (Lesson 1-1)

\(0.6\) ampere

---

**Practice Quiz 1**

Solve each system of equations by graphing. (Lesson 3-1)

1. \(y = 3x + 10\)
   \(y = -x + 6\)
   \((-1, 7)\)

2. \(2x + 3y = 12\)
   \(2x - y = 4\)
   \((3, 2)\)

Solve each system of equations by using either substitution or elimination. (Lesson 3-2)

3. \(y = x + 5\)
   \(x + y = 9\)
   \((2, 7)\)

4. \(2x + 6y = 2\)
   \(3x + 2y = 10\)
   \((4, -1)\)

5. **AIRPORTS** According to the Airports Council International, the busiest airport in the world is Atlanta’s Hartsfield International Airport, and the second busiest is Chicago’s O’Hare Airport. Together they handled 150.5 million passengers in the first six months of 1999. If Hartsfield handled 5.5 million more passengers than O’Hare, how many were handled by each airport? (Lesson 3-2)

Hartsfield, 78 million; O’Hare, 72.5 million

---

**Answers (Practice Quiz 1)**

1.

2.

54.

55.

56.
**Solving Systems of Inequalities by Graphing**

**What You’ll Learn**
- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

**Vocabulary**
- system of inequalities

**How can you determine whether your blood pressure is in a normal range?**

During one heartbeat, blood pressure reaches a maximum pressure (systolic) and a minimum pressure (diastolic), which are measured in millimeters of mercury (mm Hg). Blood pressure is expressed as the maximum pressure over the minimum pressure—for example, 120/80. Normal blood pressure for people under 40 ranges from 100 to 140 mm Hg for the maximum and from 60 to 90 mm Hg for the minimum. This information can be represented by a system of inequalities.

**GRAPH SYSTEMS OF INEQUALITIES**

To solve a system of inequalities, we need to find the ordered pairs that satisfy all of the inequalities in the system. One way to solve a system of inequalities is to graph the inequalities on the same coordinate plane. The solution set is represented by the intersection of the graph.

**Example 1**

**Intersecting Regions**

Solve each system of inequalities by graphing.

**a.**  
\[ y > -2x + 4 \]  
\[ y \leq x - 2 \]

solution of \( y > -2x + 4 \) \( \rightarrow \) Regions 1 and 2

solution of \( y \leq x - 2 \) \( \rightarrow \) Regions 2 and 3

The intersection of these regions is Region 2, which is the solution of the system of inequalities. Notice that the solution is a region containing an infinite number of ordered pairs.

**b.**  
\[ y > x + 1 \]
\[ |y| \leq 3 \]

The inequality \( |y| \leq 3 \) can be written as \( y \leq 3 \)
and \( y \geq -3 \).

Graph all of the inequalities on the same coordinate plane and shade the region or regions that are common to all.

**5-Minute Check Transparency 3-3**

Use as a quiz or review of Lesson 3-2.

**Mathematical Background**

notes are available for this lesson on p. 108D.

**Ask students:**
- What do the letters Hg represent? Hg is the symbol for mercury.
- Do you think the blood pressure ranges for people over 40 are higher or lower than those for people under 40? higher

**GRAPH SYSTEMS OF INEQUALITIES**

Solve each system of inequalities by graphing.

\[ y \geq 2x - 3 \]
\[ y < -x + 2 \]  
(continued on the next page)
In-Class Examples

1. (continued)

b. \( y \leq -x + 1 \)
\[ |x + 1| < 3 \]

\[ y = -x + 1 \]
\[ y = -x - 2 \]
\[ x = -4 \]
\[ x = 2 \]

2. Solve the system of inequalities by graphing.
\[ y \geq \frac{-3}{4}x + 1 \]
\[ y \leq \frac{-3}{4}x - 2 \]
\[ \emptyset \]

3. **MEDICINE** Medical professionals recommend that patients have a cholesterol level below 200 milligrams per deciliter (mg/dL) of blood and a triglyceride level below 150 mg/dL. Write and graph a system of inequalities that represents the range of cholesterol levels and triglyceride levels for patients. Let \( c \) represent the cholesterol levels and \( t \) represent the triglyceride levels. **Source:** American Heart Association

**0 \leq c < 200, 0 \leq t < 150**

It is possible that two regions do not intersect. In such cases, we say the solution is the empty set \( \emptyset \) and no solution exists.

**Example 2** Separate Regions

Solve the system of inequalities by graphing.
\[ y > \frac{1}{2}x + 1 \]
\[ y \leq \frac{1}{2}x - 3 \]

Graph both inequalities. The graphs do not overlap, so the solutions have no points in common.

The solution set is \( \emptyset \).

**Example 3** Write and Use a System of Inequalities

**SPACE EXPLORATION** When NASA chose the first astronauts in 1959, size was important since the space available inside the Mercury capsule was very limited. NASA wanted men who were at least 5 feet 4 inches, but no more than 5 feet 11 inches tall, and who were between 21 and 40 years of age. Write and graph a system of inequalities that represents the range of heights and ages for qualifying astronauts.

Let \( h \) represent the height of an astronaut in inches. The acceptable heights are at least 5 feet 4 inches (64 inches) and no more than 5 feet 11 inches (71 inches). We can write this information as two inequalities.

\[ 64 \leq h \leq 71 \]

Let \( a \) represent the age of an astronaut. The acceptable ages can also be written as two inequalities.

\[ 21 < a < 40 \]

Graph all of the inequalities. Any ordered pair in the intersection of the graphs is a solution of the system.

**FIND VERTICES OF A POLYGONAL REGION** Sometimes, the graph of a system of inequalities forms a polygonal region. You can find the vertices of the region by determining the coordinates of the points at which the boundary lines intersect.

**Example 4** Find Vertices

Find the coordinates of the vertices of the figure formed by \( x + y \geq -1 \), \( x - y \leq 6 \), and \( 12y + x \leq 32 \).

Graph each inequality. The intersection of the graphs forms a triangle.

The coordinates \((-4, 3)\) and \((8, 2)\) can be determined from the graph. To find the coordinates of the third vertex, solve the system of equations \( x + y = -1 \) and \( x - y = 6 \).
Add the equations to eliminate $y$.

\[
\begin{align*}
 x + y & = -1 \\
 (+) x - y & = 6 \\
 2x & = 5 \quad \text{Add the equations.} \\
 x & = \frac{5}{2} \quad \text{Divide each side by 2.}
\end{align*}
\]

Now find $y$ by substituting $\frac{5}{2}$ for $x$ in the first equation.

\[
\begin{align*}
 x + y & = -1 \quad \text{First equation} \\
 \frac{5}{2} + y & = -1 \quad \text{Replace } x \text{ with } \frac{5}{2}. \\
 y & = -\frac{7}{2} \quad \text{Subtract } \frac{5}{2} \text{ from each side.}
\end{align*}
\]

The vertices of the triangle are at $(-4, 3), (8, 2), \text{ and } (\frac{5}{2}, \frac{-7}{2})$.

---

### Check for Understanding

**Concept Check**

**1. Sample answer:**

$y > x + 3, y < x - 2$

**2.** Tell whether the following statement is true or false. If false, give a counterexample. A system of two linear inequalities has either no points or infinitely many points in its solution. **True**

**3.** State which region is the solution of the following systems of inequalities.

<table>
<thead>
<tr>
<th>Inequality 1</th>
<th>Inequality 2</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y \geq x$</td>
<td>$y \geq -x$</td>
<td>Region 1</td>
</tr>
<tr>
<td>$y \leq -x$</td>
<td>$y \leq x$</td>
<td>Region 2</td>
</tr>
<tr>
<td>$y = x$</td>
<td>$y = -x$</td>
<td>Region 3</td>
</tr>
</tbody>
</table>

---

### Guided Practice

1. Solve each system of inequalities by graphing. 4–7. See pp. 151A–151F.

**4.** $x \leq 4, y > 2$

**5.** $y \geq x - 2, y \leq -2x + 4$

**6.** $|x - 1| \leq 2, x + y > 2$

**7.** $x \leq 1, y < 2x + 1, x + 2y \geq -3$

Find the coordinates of the vertices of the figure formed by each system of inequalities.

**8.** $y \leq x \quad (-3, -3), (2, 2), (5, -3), \quad y \geq -3$

**9.** $y \geq x - 3 \quad (-4, 3), (1, -2), (2, 9), \quad y \leq x + 7 \quad (7, 4)$

10. **SHOPPING** For Exercises 10 and 11, use the following information.

Willis has been sent to the grocery store to purchase bagels and muffins for the members of the track team. He can spend at most $28. A package of bagels costs $2.50 and contains 6 bagels. A package of muffins costs $3.50 and contains 8 muffins. He needs to buy at least 12 bagels and 24 muffins.

**10.** Graph the region that shows how many packages of each item he can purchase. **See pp. 151A–151F.**

**11.** Give an example of three different purchases he can make.

---

### Application

**11. Sample answer:**

- 3 pkgs. of bagels, 4 pkgs. of muffins; 4 pkgs. of bagels, 4 pkgs. of muffins; 3 pkgs. of bagels, 5 pkgs. of muffins.

---

### Differentiated Instruction

**Verbal/Linguistic** Have students write a list of tips to help someone draw the graphs of systems of inequalities and find the vertices easily and efficiently.

---

### Find Vertices of a Polygonal Region

**In-Class Example**

4. Find the coordinates of the vertices of the figure formed by $2x - y \geq -1, x + y \leq 4,$ and $x + 4y \geq 4$.

**Exercise:**

- **Region 1:** \((0, 1), (1, 3), (1, 4)\)
- **Region 2:** \((1, 4), (3, 4), (4, 0)\)
- **Region 3:** \((4, 0), (5, 2), (1, 0)\)
- **Region 4:** \((1, 0), (2, 0), (2, 2)\)

---

### Study Notebook

- **Have students—**
  - add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 3.
  - include any other item(s) that they find helpful in mastering the skills in this lesson.

---

### About the Exercises...

**Organization by Objective**

- **Graph Systems of Inequalities:** 12–23, 32–37
- **Find Vertices of a Polygonal Region:** 24–29

**Odd/Even Assignments**

Exercises 12–31 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**

- **Basic:** 13–27 odd, 33, 34, 38–54
- **Average:** 13–31 odd, 33, 34, 38–54
- **Advanced:** 12–32 even, 33–48 (optional: 49–54)
Find the coordinates of the vertices of the figure formed by each system of inequalities.

24. \((0, 0), (0, 4), (8, 0)\)
25. \((-3, -4), (5, -4), (1, 4)\)
26. \((0, 4), (3, 0), (3, 5)\)
27. \((-6, -9), (2, 7), (10, -1)\)
28. \((-11, -3), (-1, -3), (6, 4), (6, \frac{5}{2})\)
29. \((-4, 3), (-2, 7), (4, -1), \left(\frac{7}{3}, \frac{2}{3}\right)\)

Find the area of the region defined by the system of inequalities.

24. \(y \geq 0\) \(x + 2y \leq 8\)
25. \(y \leq -4\) \(x + 2y \geq 2\)
26. \(x \geq 3\) \(-x + 3y \leq 12\)
27. \(x + y \leq 9\)
28. \(y \leq -3\)
29. \(y \leq -5\)

Find the area of the region defined by the system of inequalities \(y - x \geq 3\), \(y - x \leq -3\), and \(y - x \geq -8\).

30. Find the area of the region defined by the system of inequalities \(y - x \leq 3\), \(y - x \leq -3\), and \(y - x \geq -8\).

31. Find the area of the region defined by the system of inequalities \(y - x \leq 3\), \(y - x \leq -3\), and \(y - x \geq -8\).

32. PART-TIME JOBS Bryan Clark makes $10 an hour cutting grass and $12 an hour for raking leaves. He cannot work more than 15 hours per week. Graph two inequalities that Bryan can use to determine how many hours he needs to work at each job if he wants to earn at least $120 per week. See pp. 151A–151F.

HURRICANES For Exercises 33 and 34, use the following information. Hurricanes are divided into categories according to their wind speed and storm surge.

33. Write and graph the system of inequalities that represents the range of wind speeds \(s\) and storm surges \(h\) for a category 3 hurricane. See pp. 151A–151F.
34. On September 16, 1999, Hurricane Floyd hit the United States with winds of 140 mph. Classify Hurricane Floyd, and identify the heights of its storm surges. Category 4: 13–18 ft
35. See pp. 151A–151F.

36. Sample answer: 2 pumpkin, 8 soda; 4 pumpkin, 6 soda; 8 pumpkin, 4 soda

37. Which combination uses all of the available flour and baking soda?

38. CRITICAL THINKING Find the area of the region defined by \(|x| + |y| \leq 5\) and \(|x| + |y| \geq 2\). 42 units²

39. Writing in Math Answer the question that was posed at the beginning of the lesson. See margin.

How can you determine whether your blood pressure is in a normal range?

Include the following in your answer:
• an explanation of how to use the graph, and
• a description of the regions that indicate high blood pressure, both systolic and diastolic.

40. Choose the system of inequalities whose solution is represented by the graph. B

A. \(y < -2\)

B. \(y \leq -2\)

C. \(x < -3\)

D. \(x < -3\)

E. \(y > -3\)

F. \(y < -3\)

41. OPEN ENDED Create a system of inequalities for which the graph will be a square with its interior located in the first quadrant. Sample answer: \(y \leq 6\), \(x \leq 5\), \(x \geq 1\)

45–47. See margin for graphs.

45. \(y = 2x + 1\)

46. \(2x + y = -3\) infinitely many

47. \(-x + 8y = 12\) (4, 2)

48. Write an equation in slope-intercept form of the line that passes through \((-4, 4)\) and \((6, 9)\).

\(y = \frac{1}{2}x + 6\)

PREREQUISITE SKILL Find each value if \(f(x) = 4x + 3\) and \(g(x) = 5x - 7\).

49. \(f(-2) = -5\)

50. \(g(-1) = -12\)

51. \(g(3) = 8\)

52. \(f(6) = 27\)

53. \(f(0.5) = 5\)

54. \(g(-0.25) = -8.25\)

39. The range for normal blood pressure satisfies four inequalities that can be graphed to find their intersection. Answers should include the following.
• Graph the blood pressure as an ordered pair; if the point lies in the shaded region, it is in the normal range.
• High systolic pressure is represented by the region to the right of \(x = 140\) and high diastolic pressure is represented by the region above \(y = 90\).
A Follow-Up of Lesson 3-3

Getting Started

Error Source One possible source of an error message when entering the expression in Step 1 is that the student used the subtract key \( \begin{array}{c} \texttt{-} \end{array} \) rather than the negative key \( \texttt{(-)} \).

Teach

• Explain to students that when they sketch the graph from their calculator window on paper, they need to draw the axes, the intercepts, and the points of intersection, as well as the lines.
• Have students complete Exercises 1–8.

Assess

Ask students:
• How many points are in the solution set of a system of inequalities such as those in this activity? infinitely many
• When is the point of intersection of the two boundary lines included in the solution set? The point of intersection is in the solution set when both boundary lines on the graph are solid and when the symbol in both inequalities is either \( \geq \) or \( \leq \).

Exercises 1–8. See pp. 151A–151F.

Solve each system of inequalities. Sketch each graph on a sheet of paper.

1. \( y \geq 4 \)
   \( y \leq -x \)

2. \( y \geq -2x \)
   \( y \leq -3 \)

3. \( y \geq 1 - x \)
   \( y \leq x + 5 \)

4. \( y \geq x + 2 \)
   \( y \leq -2x - 1 \)

5. \( 3y \geq 6x - 15 \)
   \( 2y \leq -x + 3 \)

6. \( y + 3x \geq 6 \)
   \( y - 2x \leq 9 \)

7. \( 6y + 4x \geq 12 \)
   \( 5y - 3x \leq -10 \)

8. \( \frac{1}{3}y - x \geq -2 \)
   \( \frac{2}{3}y + 2x \leq 4 \)

www.algebra2.com/other_calculator_keystrokes
**3-4 Linear Programming**

**What You'll Learn**
- Find the maximum and minimum values of a function over a region.
- Solve real-world problems using linear programming.

**How is linear programming used in scheduling work?**

One of the primary tasks of the U.S. Coast Guard is to maintain the buoys that ships use to navigate. The ships that service buoys are called buoy tenders. They check the buoys in their area, make repairs, and replace any damaged buoys.

Suppose a certain buoy tender can carry up to 8 new buoys for making replacements. Their crew can check and repair a buoy in one hour. It takes the crew $\frac{1}{2}$ hours to replace a buoy. The captain can use linear programming to find the maximum number of buoys this buoy tender can repair or replace in 24 hours at sea.

**MAXIMUM AND MINIMUM VALUES** The buoy tender captain can use a system of inequalities to represent the limitations of time and the number of replacement buoys on the ship. If these inequalities are graphed, all of the points in the intersection are the combinations of repairs and replacements that the buoy tender can schedule. The inequalities are called the **constraints**. The intersection of the graphs is called the **feasible region**. When the graph of a system of constraints is a polygonal region like the one graphed at the right, we say that the region is **bounded**.

Sometimes it is necessary to find the maximum or minimum values that a linear function has for the points in a feasible region. For example, the buoy tender captain wishes to maximize the total number of buoys serviced. The maximum or minimum value of a related function **always** occurs at one of the **vertices** of the feasible region.

**Example 1 Bounded Region**

Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x + y$ for this region.

$x \geq 1$

$y \geq 0$

$2x + y \leq 6$

**Step 1** Find the vertices of the region.

Graph the inequalities.

The polygon formed is a triangle with vertices at (1, 4), (3, 0), and (1, 0).

(continued on the next page)

**Workbook and Reproducible Masters**

- **Chapter 3 Resource Masters**
  - Study Guide and Intervention, pp. 137–138
  - Skills Practice, p. 139
  - Practice, p. 140
  - Reading to Learn Mathematics, p. 141
  - Enrichment, p. 142
  - Assessment, p. 164

- **Graphing Calculator and Spreadsheet Masters**, p. 31

**Resource Manager**

- **Transparencies**
  - 5-Minute Check Transparency 3-4
  - Real-World Transparency 3
  - Answer Key Transparencies

- **Technology**
  - Interactive Chalkboard
Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 2x + 3y$ for this region.

$-x + 2y \leq 2$
$x - 2y \leq 4$
$x + y \geq -2$

There is no maximum value. The minimum value is $-6$ at $(0, -2)$.

**Step 2**
Use a table to find the maximum and minimum values of $f(x, y)$. Substitute the coordinates of the vertices into the function.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$3x + y$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 4)$</td>
<td>$3(1) + 4$</td>
<td>7</td>
</tr>
<tr>
<td>$(3, 0)$</td>
<td>$3(3) + 0$</td>
<td>9</td>
</tr>
<tr>
<td>$(1, 0)$</td>
<td>$3(1) + 0$</td>
<td>3</td>
</tr>
</tbody>
</table>

The maximum value is 9 at $(3, 0)$. The minimum value is 3 at $(1, 0)$.

**Study Tip**

**Common Misconception**

Always test a point contained in the feasible region when the graph is unbounded. Do not assume that there is no minimum value if the feasible region is unbounded below the line, or that there is no maximum value if the feasible region is unbounded above the line.

**Example 2** Unbounded Region

Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 5x + 4y$ for this region.

$2x + y \geq 3$
$3y - x \leq 9$
$2x + y \leq 10$

Graph the system of inequalities. There are only two points of intersection, $(0, 3)$ and $(3, 4)$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$5x + 4y$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 3)$</td>
<td>$5(0) + 4(3)$</td>
<td>12</td>
</tr>
<tr>
<td>$(3, 4)$</td>
<td>$5(3) + 4(4)$</td>
<td>31</td>
</tr>
</tbody>
</table>

The maximum is 31 at $(3, 4)$.

Although $f(0, 3)$ is 12, it is not the minimum value since there are other points in the solution that produce lesser values. For example, $f(3, 2) = 7$ and $f(20, -35) = -40$. It appears that because the region is unbounded, $f(x, y)$ has no minimum value.

**REAL-WORLD PROBLEMS**

The process of finding maximum or minimum values of a function for a region defined by inequalities is called **linear programming**. The steps used to solve a problem using linear programming are listed below.

**Key Concept**

**Linear Programming Procedure**

- **Step 1** Define the variables.
- **Step 2** Write a system of inequalities.
- **Step 3** Graph the system of inequalities.
- **Step 4** Find the coordinates of the vertices of the feasible region.
- **Step 5** Write a function to be maximized or minimized.
- **Step 6** Substitute the coordinates of the vertices into the function.
- **Step 7** Select the greatest or least result. Answer the problem.

Linear programming can be used to solve many types of real-world problems. These problems have certain restrictions placed on the variables, and some function of the variable must be maximized or minimized.

**Teaching Tip**

Help students see that the constraints and feasible region represent a way to model the facts in a complex situation. Within those constraints, the function $f(x, y)$ represents the relationship for which you need to find a maximum or minimum value.
**In-Class Example**

**REAL-WORLD PROBLEMS**

**LANDSCAPING**

A landscaping company has crews who mow lawns and prune shrubbery. The company schedules 1 hour for mowing jobs and 3 hours for pruning jobs. Each crew is scheduled for no more than 2 pruning jobs per day. Each crew’s schedule is set up for a maximum of 9 hours per day. On the average, the charge for mowing a lawn is $40 and the charge for pruning shrubbery is $120. Find a combination of mowing lawns and pruning shrubs that will maximize the income the company receives per day.

**Lesson 3-4**

**Linear Programming**

**Example** 3

**VETERINARY MEDICINE**

As a receptionist for a veterinarian, one of Dolores Alvarez’s tasks is to schedule appointments. She allots 20 minutes for a routine office visit and 40 minutes for a surgery. The veterinarian cannot do more than 6 surgeries per day. The office has 7 hours available for appointments. If an office visit costs $55 and most surgeries cost $125, find a combination of office visits and surgeries that will maximize the income the veterinarian practice receives per day.

**Step 1** Define the variables.

- $v =$ the number of office visits
- $s =$ the number of surgeries

**Step 2** Write a system of inequalities.

- Since the number of appointments cannot be negative, $v$ and $s$ must be nonnegative numbers.
- $v \geq 0$ and $s \geq 0$
- An office visit is 20 minutes, and a surgery is 40 minutes. There are 7 hours available for appointments.
- $20v + 40s \leq 420 \quad 7 \text{ hours} = 420 \text{ minutes}$
- The veterinarian cannot do more than 6 surgeries per day.
- $s \leq 6$

**Step 3** Graph the system of inequalities.

**Step 4** Find the coordinates of the vertices of the feasible region.

From the graph, the vertices of the feasible region are at (0, 0), (6, 0), (6, 9), and (6, 0). If the vertices could not be read from the graph easily, we could also solve a system of equations using the boundaries of the inequalities.

**Step 5** Write a function to be maximized or minimized.

The function that describes the income is $f(v, s) = 125s + 55v$. We wish to find the maximum value for this function.

**Step 6** Substitute the coordinates of the vertices into the function.

<table>
<thead>
<tr>
<th>$(s, v)$</th>
<th>$125s + 55v$</th>
<th>$f(s, v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>125(0) + 55(0)</td>
<td>0</td>
</tr>
<tr>
<td>(6, 0)</td>
<td>125(6) + 55(0)</td>
<td>750</td>
</tr>
<tr>
<td>(6, 9)</td>
<td>125(6) + 55(9)</td>
<td>1245</td>
</tr>
<tr>
<td>(0, 21)</td>
<td>125(0) + 55(21)</td>
<td>1155</td>
</tr>
</tbody>
</table>

**Step 7** Select the greatest or least result. Answer the problem.

The maximum value of the function is 1245 at (6, 9). This means that the maximum income is $1245 when Dolores schedules 6 surgeries and 9 office visits.

**Differentiated Instruction**

**Visual/Spatial**

Have students use different colored pencils to shade the different regions of a graph defined by the inequalities in a linear programming problem. This should help students clarify the relationship between the various regions in these graphs.

**www.algebra2.com/extra_examples**

Lesson 3-4  Linear Programming  131
Check for Understanding

Concept Check
1. Determine whether the following statement is always, sometimes, or never true. A feasible region has a minimum and a maximum value. **sometimes**

2. OPENENDED Give an example of a system of inequalities that forms a bounded region. **Sample answer:** $y \geq -x, y \geq x - 5, y \leq 0$

Guided Practice
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region. 3–8. See margin.

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<td>9–14</td>
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</table>

3. $y \geq 2$
   - $x \geq 1$
   - $x + 2y \leq 9$
   - $(x, y) = 2x - 3y$
4. $x \geq -3$
   - $y \leq 1$
   - $3x + y \leq 6$
   - $(x, y) = 5x - 2y$
5. $y \leq 2x + 1$
   - $1 \leq y \leq 3$
   - $x + 2y \leq 12$
   - $(x, y) = 3x + y$
6. $y \geq -x + 2$
   - $2 \leq x \leq 7$
   - $x - 2y \geq -3$
   - $(x, y) = x - y$
7. $y \leq \frac{1}{2}x + 5$
   - $2x - y \leq 7$
   - $x + 6y \geq -9$
   - $(x, y) = x - y$
8. $y = x - 3y \geq -7$
   - $5x + y \leq 13$
   - $3x - 2y \geq -7$
   - $(x, y) = x - y$

Application
**MANUFACTURING** For Exercises 9–14, use the following information.
The students in the Future Homemakers Club are making canvas tote bags and leather tote bags for a money making project. They will line both types of tote bags with canvas and use leather for the handles of both bags. For the canvas tote bags, they need 4 yards of canvas and 1 yard of leather. For the leather tote bags, they need 3 yards of leather and 2 yards of canvas. Their faculty advisor has purchased 56 yards of leather and 104 yards of canvas.

9. Let $c$ represent the number of canvas tote bags and let $\ell$ represent the number of leather tote bags. Write a system of inequalities to represent the number of tote bags that can be produced. $c \geq 0, \ell \geq 0, c + 3\ell \leq 56, 4c + 2\ell \leq 104$

10. Draw the graph showing the feasible region. **See margin.**

11. List the coordinates of the vertices of the feasible region.

12. If the club plans to sell the canvas bags at a profit of $20 each and the leather bags at a profit of $35 each, write a function for the total profit on the bags.

13. Determine the number of canvas and leather bags that they need to make for a maximum profit. **20 canvas tote bags and 12 leather tote bags**

14. What is the maximum profit? **$820**

Practice and Apply
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region. 15–20. See pp. 151A–151F.

<table>
<thead>
<tr>
<th>HOMEWORK HELP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For Excerises</strong></td>
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</tr>
<tr>
<td>31–36</td>
</tr>
<tr>
<td>38–42</td>
</tr>
</tbody>
</table>

Extra Practice
See page 833.

15. $y \geq 1$
   - $x \leq 6$
   - $y \leq 2x + 1$
   - $(x, y) = x + y$
16. $y \geq -4$
   - $x \leq 3$
   - $y \leq 3x - 4$
   - $(x, y) = x - y$
17. $y \geq 2$
   - $1 \leq x \leq 5$
   - $y \leq x + 3$
   - $(x, y) = 3x - 2y$
18. $y \geq 1$
   - $2 \leq x \leq 4$
   - $x - 2y \geq -4$
   - $(x, y) = 3y + x$
19. $y \leq x + 2$
   - $y \leq 11 - 2x$
   - $2x + y \geq -7$
   - $(x, y) = 4x - 3y$
20. $y \leq x + 6$
   - $y + 2x \geq 6$
   - $2 \leq x \leq 6$
   - $(x, y) = -x + 3y$

Answers

3. Vertices: $(1, 2), (1, 4), (5, 2)$; max: $f(5, 2) = 4$, min: $f(1, 4) = -10$

4. Vertices: $(-3, 1), \left(\frac{5}{3}, 1\right)$; no maximum; min: $f(-3, 1) = -17$
21. \( x + y \leq 3 \)  
   \( x + 2y \leq 4 \)  
   \( x \geq 0, y \geq 0 \)  
   \( f(x, y) = 3y - 4x \)

22. \( y \leq 7 - x \)  
   \( 3x - 2y \leq 6 \)  
   \( x \geq 0, y \geq 0 \)  
   \( f(x, y) = 5x - 2y \)

23. \( y \geq 3 - x \)  
   \( y \leq 6 - 2x \)  
   \( 2x + y \geq -3 \)  
   \( f(x, y) = 3x + 4y \)

24. \( x + y \leq 4 \)  
   \( 3x - 2y \leq 12 \)  
   \( x - 4y \leq -16 \)  
   \( f(x, y) = x - 2y \)

25. \( x + y \geq 2 \)  
   \( 4y \leq x + 8 \)  
   \( y \geq 2x - 5 \)  
   \( f(x, y) = 4x + 3y \)

26. \( 2x + 2y \geq 4 \)  
   \( 2y \geq 3x - 6 \)  
   \( 4y \leq x + 8 \)  
   \( f(x, y) = 3x + y \)

27. \( 2x + 3y \geq 6 \)  
   \( 3x - 2y \geq -4 \)  
   \( 5x + y \geq 15 \)  
   \( f(x, y) = x + 3y \)

28. \( x \geq 0 \)  
   \( y \geq 0 \)  
   \( x + 2y \leq 6 \)  
   \( 2y - x \leq 2 \)  
   \( x + y \leq 5 \)  
   \( f(x, y) = 3x - 5y \)

29. \( x \geq 2 \)  
   \( y \geq 1 \)  
   \( x - 2y \geq -4 \)  
   \( x + y \leq 8 \)  
   \( 2x - y \leq 7 \)  
   \( f(x, y) = x - 4y \)

30. **CRITICAL THINKING** The vertices of a feasible region are \( A(1, 2), B(5, 2), \) and \( C(1, 4) \). Write a function that satisfies each condition. **Sample answers given.**
   a. \( A \) is the maximum and \( B \) is the minimum. \( f(x, y) = -2x - y \)
   b. \( C \) is the maximum and \( B \) is the minimum. \( f(x, y) = 3y - 2x \)
   c. \( B \) is the maximum and \( A \) is the minimum. \( f(x, y) = x + y \)
   d. \( A \) is the maximum and \( C \) is the minimum. \( f(x, y) = -x - 3y \)
   e. \( B \) and \( C \) are both maxima and \( A \) is the minimum. \( f(x, y) = x + 2y \)

**PRODUCTION** For Exercises 31–36, use the following information.
There are a total of 85 workers’ hours available per day for production at a calculator manufacturer. There are 40 workers’ hours available for encasement and quality control each day. The table below shows the number of hours needed in each department for two different types of calculators.

<table>
<thead>
<tr>
<th>Calculator Type</th>
<th>Production Time</th>
<th>Encasement and Quality Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>graphing calculator</td>
<td>( \frac{1}{2} ) hour</td>
<td>2 hours</td>
</tr>
<tr>
<td>computer-algebra systems (CAS)</td>
<td>1 hour</td>
<td>( \frac{1}{2} ) hour</td>
</tr>
</tbody>
</table>

31. Let \( g \) represent the number of graphing calculators and let \( c \) represent the number of CAS calculators. Write a system of inequalities to represent the number of calculators that can be produced. \( g \geq 0, c \geq 0, 1.5g + 2c \leq 85, g + 0.5c \leq 40 \)

32. Draw the graph showing the feasible region. **See margin.**

33. List the coordinates of the vertices of the feasible region.

34. If the profit on a graphing calculator is $50 and the profit on a CAS calculator is $65, write a function for the total profit on the calculators. \( f(g, c) = 50g + 65c \)

35. Determine the number of each type of calculator that is needed to make a maximum profit. **30 graphing calculators, 20 CAS calculators**

36. What is the maximum profit? **$28000**

37. **RESEARCH** Use the Internet or other reference to find an industry that uses linear programming. Describe the restrictions or constraints of the problem and explain how linear programming is used to help solve the problem. **See students’ work.**

38. **WEB TOOLS** Visit the Web site for additional help and practice problems.

39. **REVIEW** Use a graphing utility to graph the following set of parallel lines. \( x + 2y = 5 \), \( x + 2y = 3 \), \( x + 2y = 1 \)

40. **REVIEW** Use a graphing utility to graph the following set of parallel lines. \( 3x - 2y = 5 \), \( 3x - 2y = 3 \), \( 3x - 2y = 1 \)
FARMING  For Exercises 38–41, use the following information.
Dean Stadler has 20 days in which to plant corn and soybeans. The corn can be planted at a rate of 250 acres per day and the soybeans at a rate of 200 acres per day. He has 4500 acres available for planting these two crops.

38.  Let c represent the number of acres of corn and let s represent the number of acres of soybeans. Write a system of inequalities to represent the possible ways Mr. Stadler can plant the available acres.

39.  Draw the graph showing the feasible region and list the coordinates of the vertices of the feasible region. See pp. 151A–151F.

40.  If the profit on corn is $26 per acre and the profit on soybeans is $30 per acre, how much of each should Mr. Stadler plant? What is the maximum profit?

41.  How much of each should Mr. Stadler plant if the profit on corn is $29 per acre and the profit on soybeans is $24 per acre? What is the maximum profit?

4000 acres corn, 0 acres soybeans; $130,500

42.  PACKAGING  The Cookie Factory’s best selling items are chocolate chip cookies and peanut butter cookies. They want to sell both types of cookies together in combination packages. The different-sized packages will contain between 6 and 12 cookies, inclusively. At least three of each type of cookie should be in each package. The cost of making a chocolate chip cookie is 19¢, and the selling price is 44¢ each. The cost of making a peanut butter cookie is 13¢, and the selling price is 39¢. How many of each type of cookie should be in each package to maximize the profit? 3 chocolate chip, 9 peanut butter

43.  WRITING IN MATH  Answer the question that was posed at the beginning of the lesson. See pp. 151A–151F.

How is linear programming used in scheduling work?
Include the following in your answer:
• a system of inequalities that represents the constraints that are used to schedule busy repair and replacement,
• an explanation of the linear function that the buoy tender captain would wish to maximize, and
• a demonstration of how to solve the linear programming problem to find the maximum number of buoys the buoy tender could service in 24 hours at sea.

44.  A feasible region has vertices at (0, 0), (4, 0), (5, 5), and (0, 8). Find the maximum and minimum of the function \( f(x, y) = x + 3y \) over this region.

A  maximum: \((0, 8)\) 24
B  minimum: \((0, 0)\) 0
C  maximum: \((5, 5)\) 20
D  maximum: \((0, 8)\) 8
E  maximum: \((4, 0)\) 4
F  maximum: \((0, 0)\) 0

45.  What is the area of square \(ABCD?\)
C  25 units
D  \(4\sqrt{29}\) units
C  29 units
D  \(25 + \sqrt{2}\) units

Enrichment, p. 142

Computer Circuits and Logic  Computers operate according to the laws of logic. The circuits of a computer can be described using logic notation.  Truth tables are used to describe the flow of current. A circuit is a 1, or a circuit is a 0. If both inputs are the same, output is the same. If both inputs are different, output is different.
Mixed Review

Solve each system of inequalities by graphing. (Lesson 3-3) 46–47. See margin.

46. \(2y + x \geq 4\)
\(y \geq x - 4\)

47. \(3x - 2y \leq -6\)
\(y \leq \frac{3}{2}x - 1\)

Solve each system of equations by using either substitution or elimination. (Lesson 3-2)

48. \(4x + 5y = 20\)
\(5x + 4y = 7\)

49. \(6x + y = 15\)
\(x - 4y = -10\)

50. \(3x + 8y = 23\)
\(5x - y = 24\)

SCHOOLS  For Exercises 51 and 52, use the graph at the right.

(Lesson 1-3)

51. Define a variable and write an equation that can be used to determine on average how much the annual per-pupil spending has increased from 1986 to 2001.

52. Solve the problem.

Name the property illustrated by each equation. (Lesson 1-2)

53. \(4x + (-4x) = 0\) Add. Inv.

54. \((2 \cdot 5) \cdot 6 = 2 \cdot (5 \cdot 6)\) Assoc. (\(\times\))

55. \(\left(-\frac{3}{2}\right) - \left(-\frac{1}{3}\right) = 1\) Mult. Inv.

56. \(6(x + 9) = 6x + 6(9)\) Distributive

USA TODAY Snapshots®

Per-pupil spending is climbing
How annual per-pupil spending on public elementary and secondary school students has risen:

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>$3,479</td>
</tr>
<tr>
<td>1996</td>
<td>$5,689</td>
</tr>
<tr>
<td>2001</td>
<td>$7,489</td>
</tr>
</tbody>
</table>

Source: National Center for Education Statistics (2001 figure is a projection.)

By Bob Land, USA Today

Getting Ready for

the Next Lesson

PREREQUISITE SKILL  Evaluate each expression if \(x = -2, y = 6,\) and \(z = 5.\) (To review evaluating expressions, see Lesson 1-1.)

57. \(x + y + z\)
58. \(2x - y + 3z\)
59. \(-x + 4y - 2z\)
60. \(5x + 2y - z\)
61. \(3x - y + 4z\)
62. \(-2x - 3y + 2z\)

Practice Quiz 2

Lessons 3-3 and 3-4

Solve each system of inequalities by graphing. (Lesson 3-3) 1–3. See pp. 151A–151F.

1. \(y - x > 0\)
\(y + x < 4\)

2. \(y \geq 3x - 4\)
\(y \leq x + 3\)

3. \(x + 3y \geq 15\)
\(4x + y \leq 16\)

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region. (Lesson 3-4) 4–5. See pp. 151A–151F for graphs.

4. \(x \geq 0\)
\(y \geq 0\)
\(y \leq 2x + 4\)
\(3x + y \leq 9\)
\(f(x, y) = 2x + y\)

vertices: \((0, 0), (0, 4), (1, 6), (3, 0)\); max: \(f(1, 6) = 8\), min: \(f(0, 0) = 0\)

5. \(x \leq 5\)
\(y \geq 0\)

vertices: \((1, -3)\), \((-1, 3), (5, 6), (5, 1)\); max: \(f(5, 1) = 17\), min: \(f(-1, 3) = -13\)

Answers

46.

Graph: \(y = x + 4\)

47.

Graph: \(3x - 2y = -6\)

Experience TODAY

USA TODAY Education’s Online site offers resources and interactive features connected to each day’s newspaper. Experience TODAY, USA TODAY’s daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.
Graphing Equations in Three Variables

To graph an equation in three variables, it is necessary to add a third dimension to our coordinate system. The graph of an equation of the form $Ax + By + Cz = D$, where $A$, $B$, $C$, and $D$ can not all be equal to zero is a plane.

When graphing in three-dimensional space, begin with the $xy$-coordinate plane in a horizontal position. Then draw the $z$-axis as a vertical line passing through the origin. There are now three coordinate planes: the $xy$-plane, the $xz$-plane, and the $yz$-plane. These planes intersect at right angles and divide space into eight regions, called octants.

A point in space (three dimensions) has three coordinates and is represented by an ordered triple $(x, y, z)$.

The first octant contains the points in space for which all three coordinates are positive. The octants are numbered as shown.

Activity 1

Use isometric dot paper to graph $(3, 4, 2)$ on a three-dimensional coordinate system. Name the octant in which it lies.

Draw the $x$, $y$, and $z$-axes as shown. Begin by finding the point $(3, 4, 0)$ in the $xy$-plane. The $z$-coordinate is 2, so move the point up two units parallel to the $z$-axis. The point lies in octant 1.

To graph a linear equation in three variables, first find the intercepts of the graph. Connect the intercepts on each axis. This forms a portion of a plane that lies in a single octant.

Resource Manager

**Teaching Algebra with Manipulatives**

- p. 19 (master for isometric dot paper)
- p. 225 (student recording sheet)

**Glencoe Mathematics Classroom Manipulative Kit**

- isometric dot grid stamp
- ink pad
- ruler
Activity 2

Graph \(2x + 3y + 4z = 12\).

Begin by finding the \(x\)-, \(y\)-, and \(z\)-intercepts.

\[\begin{array}{ccc}
\text{x-intercept} & \text{y-intercept} & \text{z-intercept} \\
\text{Let } y = 0 \text{ and } z = 0. & \text{Let } x = 0 \text{ and } z = 0. & \text{Let } x = 0 \text{ and } y = 0. \\
2x = 12 & 3y = 12 & 4z = 12 \\
x = 6 & y = 4 & z = 3 \\
\end{array}\]

To sketch the plane, graph the intercepts, which have coordinates \((6, 0, 0)\), \((0, 4, 0)\), and \((0, 0, 3)\). Then connect the points. Remember this is only a portion of the plane that extends indefinitely.

Graph each equation. Name the coordinates for the \(x\)-, \(y\)-, and \(z\)-intercepts.

1. \((5, 3, 6)\)
2. \((-2, 4, 3)\)
3. \((1, -5, 7)\)

Graph each ordered triple in a three-dimensional coordinate system.

4. \((2, 0, 0), (0, 1, 0), (0, 0, 6)\)
5. \((10, 0, 0), (0, -4, 0), (0, 0, 5)\)
6. \((3, 0, 0), (0, 1, 0), (0, 0, -0.5)\)
7. \((-5, 0, 0), (0, 3, 0), (0, 0, 1.5)\)
8. \((3, 0, 0), \text{none}, (0, 0, 2)\)
9. \((6, 0, 0), \text{none}, (0, -4, 0)\)

10–12. Sample answers are given.

Model and Analyze ★ indicates increased difficulty

Graph each ordered triple on a three-dimensional coordinate system.

Name the octant in which each point lies. 1–3. See pp. 151A–151F for graphs.

1. \((5, 3, 6)\)
2. \((-2, 4, 3)\)
3. \((1, -5, 7)\)

Graph each equation. Name the coordinates for the \(x\)-, \(y\)-, and \(z\)-intercepts.

4. \(3x + 6y + z = 6\)
5. \(2x - 5y + 4z = 20\)
6. \(x + 3y - 6z = 3\)
7. \(-3x + 5y + 10z = 15\)
8. \(6x + 9z = 18\)
9. \(4x - 6y = 24\)

4–9. See margin for graphs.

Write an equation of the plane given its \(x\)-, \(y\)-, and \(z\)-intercepts, respectively.

\[\begin{array}{ccc}
\text{x-intercept} & \text{y-intercept} & \text{z-intercept} \\
\text{Let } y = 0 \text{ and } z = 0. & \text{Let } x = 0 \text{ and } z = 0. & \text{Let } x = 0 \text{ and } y = 0. \\
2x = 12 & 3y = 12 & 4z = 12 \\
x = 6 & y = 4 & z = 3 \\
\end{array}\]

10. \(8, -3, 6\)
11. \(10, 4, -5\)
12. \(
\frac{1}{2}, 4, -12
\)

13. Describe the values of \(x\), \(y\), and \(z\) as either positive or negative for each octant. See margin.

14. Consider the graph \(x = -3\) in one, two, and three dimensions. a–e. See pp. 151A–151F.

a. Graph the equation on a number line.

b. Graph the equation on a coordinate plane.

c. Graph the equation in a three-dimensional coordinate axis.

d. Describe and compare the graphs in parts a, b, and c.

e. Make a conjecture about the graph of \(x > -3\) in one, two, and three dimensions.
At the 2000 Summer Olympics in Sydney, Australia, the United States won 97 medals. They won 6 more gold medals than bronze and 8 fewer silver medals than bronze.

You can write and solve a system of three linear equations to determine how many of each type of medal the U.S. Olympians won.

Let \( g \) represent the number of gold medals, let \( s \) represent the number of silver medals, and let \( b \) represent the number of bronze medals.

\[
\begin{align*}
g + s + b &= 97 & \text{The U.S. won a total of 97 medals.} \\
g &= b + 6 & \text{They won 6 more gold medals than bronze.} \\
s &= b - 8 & \text{They won 8 fewer silver medals than bronze.}
\end{align*}
\]

**SYSTEMS IN THREE VARIABLES**

The system of equations above has three variables. The graph of an equation in three variables, all to the first power, is a plane. The solution of a system of three equations in three variables can have one solution, infinitely many solutions, or no solution.

<table>
<thead>
<tr>
<th>Key Concept</th>
<th>System of Equations in Three Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Solution</td>
<td>Infinite Solutions</td>
</tr>
<tr>
<td>• planes intersect in one point</td>
<td>• planes intersect in a line</td>
</tr>
<tr>
<td>( (x, y, z) )</td>
<td>• planes intersect in the same plane</td>
</tr>
<tr>
<td>No Solution</td>
<td>• planes have no point in common</td>
</tr>
</tbody>
</table>

Ask students:

- What do you know about the values for each of the variables? They are greater than or equal to zero.
- Did the U.S. Olympians win more bronze medals or more silver medals? bronze
Solving systems of equations in three variables is similar to solving systems of equations in two variables. Use the strategies of substitution and elimination. The solution of a system of equations in three variables \( x, y, \) and \( z \) is called an 
ordered triple
 and is written as \((x, y, z)\).

**Example 1 One Solution**

Solve the system of equations.

\[
\begin{align*}
x + 2y + z &= 10 \\
2x - y + 3z &= -5 \\
2x - 3y - 5z &= 27
\end{align*}
\]

**Step 1** Use elimination to make a system of two equations in two variables.

\[
\begin{align*}
x + 2y + z &= 10 \quad \text{Multiply by 2.} \\
2x - y + 3z &= -5 \\
2x - y + 3z &= -5 \quad \text{Second equation} \\
\hline
5y - z &= 25 \quad \text{Subtract to eliminate} \ x.
\end{align*}
\]

\[
\begin{align*}
(-) 2x - 3y - 5z &= -27 \\
2y + 8z &= -32 \quad \text{Third equation} \\
\hline
2y + 8z &= -32 \quad \text{Subtract to eliminate} \ z.
\end{align*}
\]

Notice that the \( x \) terms in each equation have been eliminated. The result is two equations with the same two variables \( y \) and \( z \).

**Step 2** Solve the system of two equations.

\[
\begin{align*}
5y - z &= 25 \quad \text{Multiply by 6.} \\
2y + 8z &= -32 \quad \text{Add to eliminate} \ z.
\end{align*}
\]

For \( z \):

\[
\begin{align*}
5y - z &= 25 \quad \text{Equation with two variables} \\
5(4) - z &= 25 \quad \text{Replace} \ z \ \text{with} \ 4. \\
20 - z &= 25 \quad \text{Multiply.} \\
z &= -5 \quad \text{Simplify.}
\end{align*}
\]

The result is \( y = 4 \) and \( z = -5 \).

**Step 3** Substitute 4 for \( y \) and \( -5 \) for \( z \) in one of the original equations with three variables.

\[
\begin{align*}
x + 2y + z &= 10 \quad \text{Original equation with three variables} \\
x + 2(4) + (-5) &= 10 \quad \text{Replace} \ y \ \text{with} \ 4 \ \text{and} \ z \ \text{with} \ -5. \\
x + 8 - 5 &= 10 \quad \text{Multiply.} \\
x &= 7 \quad \text{Simplify.}
\end{align*}
\]

The solution is \((7, 4, -5)\). You can check this solution in the other two original equations.
In-Class Examples

2 Solve the system of equations.
\[2x + y - 3z = 5\]
\[x + 2y - 4z = 7\]
\[6x + 3y - 9z = 15\]
There are an infinite number of solutions.

Teaching Tip Ask students to sketch a drawing of three planes whose equations form a system with an infinite number of solutions.

3 Solve the system of equations.
\[3x - y - 2z = 4\]
\[6x + 4y + 8z = 11\]
\[9x + 6y + 12z = -3\]
There is no solution of this system.

Teaching Tip Ask students to sketch a drawing of three planes whose equations form a system with no solutions.

Example 2 Infinite Solutions
Solve the system of equations.
\[4x - 6y + 4z = 12\]
\[6x - 9y + 6z = 18\]
\[5x - 8y + 10z = 20\]
Eliminate \(x\) in the first two equations.
\[4x - 6y + 4z = 12\]
\[6x - 9y + 6z = 18\]
\[\text{Multiply by 3.}\]
\[12x - 18y + 12z = 36\]
\[6x - 9y + 6z = 18\]
\[\text{Multiply by -2.}\]
\[(-) 12x + 18y - 12z = -36\]
\[0 = 0\]
The equation \(0 = 0\) is always true. This indicates that the first two equations represent the same plane. Check to see if this plane intersects the third plane.
\[4x - 6y + 4z = 12\]
\[5x - 8y + 10z = 20\]
\[\text{Multiply by 5.}\]
\[20x - 30y + 20z = 60\]
\[5x - 14y = 20\]
\[\text{Divide by the GCF, 2.}\]
The planes intersect in the line. So, there are an infinite number of solutions.

Example 3 No Solution
Solve the system of equations.
\[6a + 12b - 8c = 24\]
\[9a + 18b - 12c = 30\]
\[4a + 8b - 7c = 26\]
Eliminate \(a\) in the first two equations.
\[6a + 12b - 8c = 24\]
\[9a + 18b - 12c = 30\]
\[\text{Multiply by 3.}\]
\[18a + 36b - 24c = 72\]
\[9a + 18b - 12c = 30\]
\[\text{Multiply by 2.}\]
\[(-) 18a + 36b - 24c = 60\]
\[0 = 12\]
The equation \(0 = 12\) is never true. So, there is no solution of this system.

REAL-WORLD PROBLEMS When solving problems involving three variables, use the four-step plan to help organize the information.

Example 4 Write and Solve a System of Equations
INVESTMENTS Andrew Chang has $15,000 that he wants to invest in certificates of deposit (CDs). For tax purposes, he wants his total interest per year to be $800. He wants to put $1000 more in a 2-year CD than in a 1-year CD and invest the rest in a 3-year CD. How much should Mr. Chang invest in each type of CD?

<table>
<thead>
<tr>
<th>Number of Years</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate</td>
<td>3.4%</td>
<td>5.0%</td>
<td>6.0%</td>
</tr>
</tbody>
</table>

Explore Read the problem and define the variables.
\(a\) = the amount of money invested in a 1-year certificate
\(b\) = the amount of money in a 2-year certificate
\(c\) = the amount of money in a 3-year certificate

Unlocking Misconceptions
Systems in Three Variables Some students may think that any ordered triple will be a solution to a system in three variables where there are an infinite number of solutions (as in Example 2). Explain that only the infinite set of ordered triples that names points on the line of intersection contains solutions to the system of equations.
**Plan**

Mr. Chang has $15,000 to invest.

\[a + b + c = 15,000\]

The interest he earns should be $800. The interest equals the rate times the amount invested.

\[0.034a + 0.05b + 0.06c = 800\]

There is $1000 more in the 2-year certificate than in the 1-year certificate.

\[b = a + 1000\]

**Solve**

Substitute \(b = a + 1000\) in each of the first two equations.

\[a + (a + 1000) + c = 15,000 \quad \text{Replace } b \text{ with } (a + 1000)\]

\[2a + 1000 + c = 15,000 \quad \text{Simplify}\]

\[2a + c = 14,000 \quad \text{Subtract 1000 from each side}\]

\[0.034a + 0.05(a + 1000) + 0.06c = 800 \quad \text{Replace } b \text{ with } (a + 1000)\]

\[0.034a + 0.05a + 50 + 0.06c = 800 \quad \text{Distributive Property}\]

\[0.084a + 0.06c = 750 \quad \text{Simplify}\]

Now solve the system of two equations in two variables.

\[2a + c = 14,000 \quad \text{Multiply by 0.06}, \quad 0.12a + 0.06c = 840\]

\[0.084a + 0.06c = 750 \quad (\quad ) 0.084a + 0.06c = 750\]

\[0.036a = 90 \quad \frac{-}{0.036a} = 90 \quad a = 2500\]

Substitute 2500 for \(a\) in one of the original equations.

\[b = a + 1000 \quad \text{Third equation}\]

\[= 2500 + 1000 \quad a = 2500\]

\[= 3500 \quad \text{Add}\]

Substitute 2500 for \(a\) and 3500 for \(b\) in one of the original equations.

\[a + b + c = 15,000 \quad \text{First equation}\]

\[2500 + 3500 + c = 15,000 \quad a = 2500, b = 3500\]

\[6000 + c = 15,000 \quad \text{Add}\]

\[c = 9000 \quad \text{Subtract 6000 from each side}\]

So, Mr. Chang should invest $2500 in a 1-year certificate, $3500 in a 2-year certificate, and $9000 in a 3-year certificate.

**Examine**

Check to see if all the criteria are met.

The total investment is $15,000.

\[2500 + 3500 + 9000 = 15,000 \quad \checkmark\]

The interest earned will be $800.

\[0.034(2500) + 0.05(3500) + 0.06(9000) = 800\]

\[85 + 175 + 540 = 800 \quad \checkmark\]

There is $1000 more in the 2-year certificate than the 1-year certificate.

\[3500 = 2500 + 1000 \quad \checkmark\]
4. \[ x + 2y = 12 \]
   \[ 3y - 4z = 25 \]
   \[ x + 6y + z = 20 \]
   \[ (6, 3, -4) \]

5. \[ 9x + 7b = -30 \]
   \[ 8b + 5c = 11 \]
   \[ -3y + 10c = 73 \]
   \[ (-1, -3, 7) \]

6. \[ r - 3x + t = 4 \]
   \[ 3r - 6s + 9t = 5 \]
   \[ 4r - 9s + 10t = 9 \]
   \[ \text{infinitely many} \]

7. \[ 2r + 3s - 4t = 20 \]
   \[ 4r - s + 5t = 13 \]
   \[ 3r + 2s + 4t = 15 \]
   \[ (5, 2, -1) \]

8. \[ 2x - y + z = 1 \]
   \[ x + 2y - 4z = 3 \]
   \[ 4x + 3y - 7z = -8 \]
   \[ \text{no solution} \]

9. \[ x + y + z = 12 \]
   \[ 6x - 2y - z = 16 \]
   \[ 3x + 4y + 2z = 28 \]
   \[ (4, 0, 8) \]

Is she correct? Explain your reasoning.

3. OPEN ENDED Give an example of a system of three equations in three variables that has \((-3, 5, 2)\) as a solution. Show that the ordered triple satisfies all three equations. See margin.

COOKING For Exercises 10 and 11, use the following information.

Jambalaya is a Cajun dish made from chicken, sausage, and rice. Simone is making a large pot of jambalaya for a party. Chicken costs \$6 per pound, sausage costs \$3 per pound, and rice costs \$1 per pound. She spends \$42 on \(13\frac{1}{2}\) pounds of food. She buys twice as much rice as sausage.

10. Write a system of three equations that represents how much food Simone purchased.
   \[ 6c + 3s + r = 42, c + s + r = 13\frac{1}{2}, r = 2s \]

11. How much chicken, sausage, and rice will she use in her dish?
   \[ 4\frac{1}{2} \text{ lb chicken, } 3 \text{ lb sausage, } 6 \text{ lb rice} \]
20. (1, 2, -1)
21. (1/2, 1/3, 1/6)
22. (1/3, 2/3, 1/3)

24. The sum of three numbers is 20. The second number is 4 times the first, and the sum of the first and third is 8. Find the numbers. 3, 12, 5

25. The sum of three numbers is 12. The first number is twice the second and third. The third number is 5 less than the first. Find the numbers. 8, 1, 3

26. TRAVEL. Jonathan and members of his Spanish Club are going to Costa Rica over spring break. Before his trip, he purchases 10 traveler checks in denominations of $20, $50, and $100, totaling $370. He has twice as many $20 checks as $50 checks. How many of each type of denomination of traveler checks does he have? 1 - $100, 3 - $50, and 6 - $20 checks

27. enchilada, $2.50; taco, $1.95; burrito, $2.65

28. If Maka wants 2 burritos and 1 enchilada, how much should she plan to spend? $7.80

Basketball. In 2001, Katie Smith was ranked first in the WNBA for total points and three-point goals made. She scored 646 points making 355 shots, including 3-point field goals, 2-point field goals, and 1-point free throws. She made 27 more 2-point field goals than 3-point field goals.

29. Write a system of three equations that represents the number of goals Katie Smith made. x + y + z = 355, x + 2y + 3z = 646, y = z + 27

30. Find the number of each type of goal she made. 83 3-point goals, 115 2-point goals, 152 1-point free throws

31. CRITICAL THINKING. The general form of an equation for a parabola is y = ax^2 + bx + c, where (x, y) is a point on the parabola. Determine the values of a, b, c for the parabola at the right. Write the general form of the equation.

a = 3, b = 1, c = 3; y = 4/3x^2 + 1/3x + 3

Lunch Combo Meals

1. Two Tacos, One Burrito $6.95
2. One Enchilada, One Taco, One Burrito $7.10
3. Two Enchiladas, Two Tacos $8.90

Study Guide and Intervention, p. 143 (shown) and p. 144

Solving Systems of Linear Equations in Three Variables

Three systems of linear equations in three variables can be solved by the same techniques used to solve systems of linear equations in two variables. A system of three equations in three variables can have a unique solution, infinitely many solutions, or no solution. A solution is an ordered triple.

1. Solve this system of equations:

\[ \begin{align*}
2x + y - 3z &= 1 \\
-3x + 2y + z &= 4 \\
4x - y - 2z &= 3
\end{align*} \]

2. Solve the system of three equations.

\[ \begin{align*}
2x + y - 3z &= 1 \\
-3x + 2y + z &= 4 \\
4x - y - 2z &= 3
\end{align*} \]

3. Solve each system of equations.

\[ \begin{align*}
2x + y - 3z &= 1 \\
-3x + 2y + z &= 4 \\
4x - y - 2z &= 3
\end{align*} \]
Open-Ended Assessment
Speaking Ask students to state the various ways that three planes can intersect, explaining what will occur when solving the system of equations in three variables in each situation.

Intervention
Some students have difficulty visualizing situations in three dimensions. Encourage them to use paper, pencils, and other objects to help them model the situations they encounter in this lesson.

Assessment Options
Quiz (Lesson 3-5) is available on p. 164 of the Chapter 3 Resource Masters.

Answers
32. You can write a system of three equations in three variables to find the number of each type of medal. Answers should include the following.
- You can substitute \( b + 6 \) for \( g \) and \( b - 8 \) for \( s \) in the equation \( g + s + b = 97 \). This equation is now in terms of \( b \). Once you find \( b \), you can substitute again to find \( g \) and \( s \). The U.S. Olympians won 39 gold medals, 25 silver medals, and 33 bronze medals.
- Another situation involving three variables is winning times of the first, second, and third place finishers of a race.

33. If \( a + b = 16 \), \( a - c = 4 \), and \( b - c = -4 \), which statements are true? (Lesson 3-4) 
   - I. \( b + c = 12 \)
   - II. \( a - b = 8 \)
   - III. \( a + c = 20 \)
   - \( \text{A} \) I only  
   - \( \text{B} \) II only  
   - \( \text{C} \) I and II only  
   - \( \text{D} \) I, II, and III

34. If \( x + y = 1 \), \( y + z = 10 \), and \( x + z = 3 \), what is \( x + y + z \)? (Lesson 3-4) 
   - \( \text{A} \) 7  
   - \( \text{B} \) 8  
   - \( \text{C} \) 13  
   - \( \text{D} \) 14

35. PAPER Wood pulp can be converted to either notebook paper or newsprint. The Canyon Pulp and Paper Mill can produce at most 200 units of paper a day. Regular customers require at least 10 units of notebook paper and 80 units of newspaper daily. If the profit on a unit of notebook paper is $500 and the profit on a unit of newsprint is $350, how many units of each type of paper should the mill produce each day to maximize profits? (Lesson 3-4) 
   120 units of notebook paper and 80 units of newsprint

36. \( y \leq x + 2 \)
   \( y \geq 7 - 2x \)

37. \( 4y - 2x > 4 \)
   \( 3x + y > 3 \)
   \( 2y - x \leq -4 \)

38. Solve each system of inequalities by graphing. (Lesson 3-3) 

39. Sample answer using \((7, 15)\) and \((14, 22)\): \( y = x + 8 \)

40. Write a prediction equation for this relationship.
41. Predict the price for a first-class stamp issued in the year 2010. \( \text{about 47¢} \)

32. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How can you determine the number and type of medals U.S. Olympians won?
Include the following in your answer:
- a demonstration of how to find the number of each type of medal won by the U.S. Olympians, and
- a description of another situation where you can use a system of three equations in three variables to solve a problem.
Vocabulary and Concept Check

Choose the letter of the term that best matches each phrase.

1. the inequalities of a linear programming problem  
2. a system of equations that has an infinite number of solutions  
3. the region of a graph where every constraint is met  
4. a method of solving equations in which one equation is solved for one variable in terms of the other variable  
5. a system of equations that has at least one solution  
6. a method of solving equations in which one variable is eliminated when the two equations are combined  
7. the solution of a system of equations in three variables \((x, y, z)\)  
8. a method for finding the maximum or minimum value of a function  
9. a system of equations that has no solution  
10. a region in which no maximum value exists

Lesson-by-Lesson Review

3-1 Solving Systems of Equations by Graphing

Concept Summary

- The solution of a system of equations can be found by graphing the two lines and determining if they intersect and at what point they intersect.

Example

Solve the system of equations by graphing.

\[ \begin{align*}
    x + y &= 3 \\
    3x - y &= 1
\end{align*} \]

Graph both equations on the same coordinate plane.

The solution of the system is \((1, 2)\).

Exercises

Solve each system of equations by graphing.

11. \(3x + 2y = 12\)  \(x - 2y = 4\) \((4, 0)\)
12. \(8x - 10y = 7\)  \(4x - 5y = 7\) no solution
13. \(y - 2x = 8\)  \(y = \frac{1}{2}x - 4\) \((-8, -8)\)
14. \(20y + 13x = 10\)  \(0.65x + y = 0.5\) infinitely many

For each lesson,
- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students’ mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Foldables Study Organizer

Have students compare the length of their summaries for each lesson. Discuss what might make a summary too long (excessive detail) and what might make it too short (entries without enough detail). Have students work in small groups to compare their summaries for each lesson.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.
Solving Systems of Equations Algebraically

Concept Summary

- In the substitution method, one equation is solved for a variable and substituted to find the value of another variable.
- In the elimination method, one variable is eliminated by adding or subtracting the equations.

Examples

1 Use substitution to solve the system of equations.

\[
\begin{align*}
x &= 4y + 7 \\
y &= -3 - x
\end{align*}
\]

Substitute \(-3 - x\) for \(y\) in the first equation.

\[
\begin{align*}
x &= 4y + 7 & \text{First equation} \\
x &= 4(-3 - x) + 7 & \text{Substitute } -3 - x \text{ for } y. \\
x &= -12 - 4x + 7 & \text{Distributive Property} \\
5x &= -5 & \text{Add 4x to each side.} \\
x &= -1 & \text{Divide each side by 5.}
\end{align*}
\]

Now substitute the value for \(x\) in either original equation.

\[
\begin{align*}
y &= -3 - x & \text{Second equation} \\
y &= -3 - (-1) & \text{or } -2 \text{ The solution is } (-1, -2).
\end{align*}
\]

2 Use the elimination method to solve the system of equations.

\[
\begin{align*}
3x - 2y &= 8 \\
-x + y &= 9
\end{align*}
\]

Multiply the second equation by 2. Then add the equations to eliminate the \(y\) variable.

\[
\begin{align*}
3x - 2y &= 8 & \text{Multiply by 2.} \\
-2x + 2y &= 18 & \text{Multiply.} \\
x &= 26 & \text{Add the equations.}
\end{align*}
\]

Replace \(x\) with 26 and solve for \(y\).

\[
\begin{align*}
3x - 2y &= 8 & \text{Original equation.} \\
3(26) - 2y &= 8 & \text{Replace } x \text{ with } 26. \\
78 - 2y &= 8 & \text{Multiply.} \\
-2y &= -70 & \text{Subtract 78 from each side.} \\
y &= 35 & \text{The solution is } (26, 35).
\end{align*}
\]

Exercises  Solve each system of equations by using either substitution or elimination.  See Examples 1–4 on pages 116–119.

\[
\begin{align*}
15. & \quad x + y = 5 \\
& \quad 2x - y = 4 \quad (3, 2) \\
16. & \quad 2x - 3y = 9 \\
& \quad 4x + 2y = -22 \quad (-3, -5) \\
& \quad -3y + x = -3 \quad (9, 4) \\
17. & \quad 7y - 2x = 10 \\
18. & \quad -2x - 6y = 0 \\
& \quad 3x + 11y = 4 \quad (-2, 6) \\
19. & \quad 3x - 5y = -13 \\
& \quad 4x + 2y = 0 \quad (-1, 2) \\
& \quad x + y = 4 \\
20. & \quad x - y = 8.5 \quad (6.25, -2.25)
\end{align*}
\]
Solving Systems of Inequalities by Graphing

Concept Summary
- A solution of a system of inequalities is found by graphing the inequalities and determining the intersection of the graphs.

Example
Solve the system of inequalities by graphing.

\[ y \leq x + 2 \]
\[ y \geq -4 - \frac{1}{2}x \]

Graph each inequality and shade the intersection.

Exercises
Solve each system of inequalities by graphing.

21. \( y \leq 4 \) \( y > -3 \)
22. \( |y| > 3 \) \( x \leq 1 \)
23. \( y < x + 1 \) \( x > 5 \)
24. \( y \leq x + 4 \) \( 2y \geq x - 3 \)

Linear Programming

Concept Summary
- The maximum and minimum values of a function are determined by linear programming techniques.

Example
The available parking area of a parking lot is 600 square meters. A car requires 6 square meters of space, and a bus requires 30 square meters of space. The attendant can handle no more than 60 vehicles. If a car is charged $3 to park and a bus is charged $8, how many of each should the attendant accept to maximize income?

Let \( c \) = the number of cars and \( b \) = the number of buses.
\[ c \geq 0, b \geq 0, 6c + 30b \leq 600, \text{ and } c + b \leq 60 \]

Graph the inequalities. The vertices of the feasible region are (0, 0), (0, 20), (50, 10), and (60, 0).

The profit function is \( f(c, b) = 3c + 8b \). The maximum value of $230 occurs at (50, 10). So the attendant should accept 50 cars and 10 buses.

Exercise
See Example 3 on page 131.

25. MANUFACTURING  A toy manufacturer is introducing two new dolls, My First Baby and My Real Baby. In one hour, the company can produce 8 First Babies or 20 Real Babies. Because of demand, the company produces at least twice as many First Babies as Real Babies. The company spends no more than 48 hours per week making these two dolls. The profit on each First Baby is $3.00, and the profit on each Real Baby is $7.50. Find the number and type of dolls that should be produced to maximize profit.

160 My Real Babies, 320 My First Babies
Solving Systems of Equations in Three Variables

**Summary**
- A system of three equations in three variables can be solved algebraically by using the substitution method or the elimination method.

**Example**
Solve the system of equations.

\[
\begin{align*}
x + 3y + 2z &= 1 \\
2x + y - z &= 2 \\
x + y + z &= 2
\end{align*}
\]

**Step 1** Use elimination to make a system of two equations in two variables.

\[
\begin{align*}
x + 3y + 2z &= 1 & \text{First equation} \\
(-) 5y + 5z &= 0 & \text{Subtract.}
\end{align*}
\]

\[
\begin{align*}
(-) x + y + z &= 2 & \text{Third equation} \\
2y + z &= -1 & \text{Subtract to eliminate } x.
\end{align*}
\]

**Step 2** Solve the system of two equations.

\[
\begin{align*}
5y + 5z &= 0 \quad \text{Multiply by 5.} \\
2y + z &= -1 \quad \text{Add } 2 \text{ to each side.}
\end{align*}
\]

\[
\begin{align*}
5y + 5z &= 0 \\
(-) 10y + 5z &= -5 \\
-5y &= 5 \\
y &= -1 \quad \text{Divide by } -5.
\end{align*}
\]

Substitute \(-1\) for \(y\) in one of the equations with two variables and solve for \(z\).

\[
\begin{align*}
5y + 5z &= 0 \quad \text{Equation with two variables} \\
5(-1) + 5z &= 0 \quad \text{Replace } y \text{ with } -1. \\
5z &= 5 \quad \text{Add } 5 \text{ to each side.} \\
z &= 1 \quad \text{Divide each side by } 5.
\end{align*}
\]

**Step 3** Substitute \(-1\) for \(y\) and \(1\) for \(z\) in one of the equations with three variables.

\[
\begin{align*}
2x + y - z &= 2 \quad \text{Original equation with three variables} \\
2x + (-1) - 1 &= 2 \quad \text{Replace } y \text{ with } -1 \text{ and } z \text{ with } 1. \\
2x &= 4 \quad \text{Add } 2 \text{ to each side.} \\
x &= 2 \quad \text{Divide each side by } 2.
\end{align*}
\]

The solution is \((2, -1, 1)\).

**Exercises** Solve each system of equations. See Examples 2–4 on pages 140–141.

26. \(x + 4y - z = 6\) \hspace{1cm} 27. \(2a + b - c = 5\) \hspace{1cm} 28. \(e + f = 4\)

\[
\begin{align*}
3x + 2y + 3z &= 16 \\
2x - y + z &= 3 \quad (1, 2, 3) \\
3a - 6c &= 6 \quad (4, -2, 1) \\
3e - 3 &= 3 \quad (3, -1, 5)
\end{align*}
\]
Choose the word or term that best completes each statement or phrase.

1. Finding the maximum and minimum value of a linear function subject to constraints is called (linear, polygonal) programming.
2. The process of adding or subtracting equations to remove a variable and simplify solving the system of equations is called (substitution, elimination).
3. If a system of three equations in three variables has one solution, the graphs of the equations intersect in a (point, plane).

Solve each system of equations by graphing, substitution, or elimination.

4. \(-4x + y = -5\)  
   \(2x + y = 7\) \((2, 3)\)

5. \(x + y = -8\)  
   \(-3x + 2y = 9\) \((-5, -3)\)

6. \(3x + 2y = 18\)  
   \(y = 6x - 6\) \((2, 6)\)

7. \(-6x + 3y = 33\)  
   \(-4x + y = 16\) \((-2.5, 6)\)

8. \(-7x + 6y = 42\)  
   \(3x + 4y = 28\) \((0, 7)\)

9. \(2y = 5x - 1\)  
   \(x + y = -1\) \((-1, -6)\) \((-7, -7)\)

Solve each system of inequalities by graphing. 10–12. See margin.

10. \(y \geq x - 3\)  
    \(y \leq -x + 1\) \((2, 3)\)

11. \(x + 2y \geq 7\)  
    \(3x - 4y < 12\) \((7, 3)\)

12. \(3x + y \leq -5\)  
    \(2x - 4y \geq 6\) \((-6, 6)\)

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and the minimum values of the given function. 13–14. See margin.

13. \(5 \geq y \geq -3\)  
    \(4x + y \leq 5\)  
    \(-2x + y \leq 5\)  
    \(f(x, y) = 4x - 3y\)

14. \(x \geq -10\)  
    \(1 \geq y \geq -6\)  
    \(3x + 4y \leq -8\)  
    \(2y \geq x - 10\)  
    \(f(x, y) = 2x + y\)

MANUFACTURING  For Exercises 15 and 16, use the following information.
A sporting goods manufacturer makes a $5 profit on soccer balls and a $4 profit on volleyballs. Cutting requires 2 hours to make 75 soccer balls and 3 hours to make 60 volleyballs. Sewing needs 3 hours to make 75 soccer balls and 2 hours to make 60 volleyballs. Cutting has 500 hours available, and Sewing has 450 hours available.
15. How many soccer balls and volleyballs should be made to maximize the profit? 16. What is the maximum profit the company can make from these two products?

Solve each system of equations.

17. \(x + y + z = -1\)  
   \(2x + 4y + 2z = 1\)  
   \(x + 2y - 3z = -3\) \((-4, 2, 1)\)

18. \(x + z = 7\)  
   \(2y - z = -3\)  
   \(-x + 3y + 2z = 11\) \((-2, 3, 9)\)

SHOPPING  Carla bought 3 shirts, 4 pairs of pants, and 2 pairs of shoes for a total of $149.79. Beth bought 5 shirts, 3 pairs of pants, and 3 pairs of shoes totaling $183.19. Kayla bought 6 shirts, 5 pairs of pants, and a pair of shoes for $181.14. Assume that all of the shirts were the same price, all of the pants were the same price, and all of the shoes were the same price. What was the price of each item?  
   shirt, $12.95; pants, $15.99; shoes, $23.49

20. STANDARDIZED TEST PRACTICE  Find the point at which the graphs of \(2x + 3y = 7\) and \(3x - 4y = 2\) intersect. \((2, 1)\)

Portoflio Suggestion

Introduction  Your portfolio represents the mathematics you have done in this course. It shows you, your family, and your teacher a sampling of what you have learned and accomplished.

Ask Students  Select one of the assignments from this chapter that you found especially challenging and place it in your portfolio. Write a short paragraph explaining why you found the assignment challenging and discuss how you were able to complete the assignment.

www.algebra2.com/chapter_test
Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. What is the slope of any line parallel to the graph of $6x + 5y = 9$?\[ \text{A} \ -6 \quad \text{B} \ -\frac{6}{5} \quad \text{C} \ 2 \quad \text{D} \ 6 \]

2. In the figure, $\triangle MOQ$ is similar to $\triangle NOP$. What is the length of $MO$? \[ \text{D} \]

3. If $3x - y = -3$ and $x + 5y = 15$, what is the value of $y$?\[ \text{D} \]

4. When $3$ times $x$ is increased by $4$, the result is less than $16$. Which of the following is a graph of the real numbers $x$ that satisfy this relationship?\[ \text{D} \]

5. What is the area of the square $ABCD$? \[ \text{C} \]
   \[ \text{A} \ 27 \text{ units}^2 \quad \text{B} \ 9\sqrt{2} \text{ units}^2 \quad \text{C} \ 18 \text{ units}^2 \quad \text{D} \ 12\sqrt{2} \text{ units}^2 \]

6. Twenty-seven white cubes of the same size are put together to form a larger cube. The larger cube is painted red. How many of the smaller cubes have exactly one red face? \[ \text{B} \]

7. Find the value of $| -4 | - | 3 |$. \[ \text{D} \]

8. If two sides of a triangle measure $30$ and $60$, which of the following cannot be the measure of the third side? \[ \text{A} \]

9. Marcus tried to compute the average of his $8$ test scores. He mistakenly divided the correct total $S$ of his scores by $7$. The result was $12$ more than what it should have been. Which equation would determine the value of $S$? \[ \text{D} \]

10. If $x = -2$, then $15 - 3(x + 1) = \[ \text{C} \]
   \[ \text{A} \ 6. \quad \text{B} \ 12. \quad \text{C} \ 18. \quad \text{D} \ 21. \]

Additional Practice

See pp. 167–168 in the Chapter 3 Resource Masters for additional standardized test practice.

Log On for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.
11. Six of the 13 members of a club are boys, and the rest are girls. What is the ratio of girls to boys in the club? 7:6

12. The integer $k$ is greater than 50 and less than 100. When $k$ is divided by 3, the remainder is 1. When $k$ is divided by 8, the remainder is 2. What is one possible value of $k$? 58 or 82

13. The area of the base of the rectangular box shown at the right is 35 square units. The area of one of the faces is 56 square units. Each of the dimensions $a$, $b$, and $c$ is an integer greater than 1. What is the volume of the rectangular box? 280 cubic units

14. Four lines on a plane intersect in one point, forming 8 equal angles that are nonoverlapping. What is the measure, in degrees, of one of these angles? 45

15. What is the greatest of five consecutive integers if the sum of these integers equals 135? 29

16. If the perimeter of a rectangle is 12 times the width of the rectangle, then the length of the rectangle is how many times the width? 5

17. Points $A$, $B$, $C$, and $D$ lie in consecutive order on a line. If $AC = \frac{4}{3}AB$ and $BD = 6BC$, then what is $\frac{AB}{CD}$? 3/5 or .6

18. The average (arithmetic mean) of the test scores of a class of $x$ students is 74, and the average of the test scores of a class of $y$ students is 88. When the scores of both classes are combined, the average is 76. What is the value of $\frac{x}{y}$? 6

19. Compare the quantity in Column A and the quantity in Column B. Then determine whether:

A. the quantity in Column A is greater,
B. the quantity in Column B is greater,
C. the two quantities are equal, or
D. the relationship cannot be determined from the information given.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>the percent increase from 75 to 100</td>
<td>the percent decrease from 100 to 75</td>
</tr>
<tr>
<td>$3x + 12 = 5x - 3$</td>
<td>$2x$</td>
</tr>
<tr>
<td>the sum of two different prime numbers if each number is less than 8</td>
<td>the sum of two different positive even integers if each integer is less than 8</td>
</tr>
<tr>
<td>$u - p = 40$</td>
<td>$t - s$</td>
</tr>
</tbody>
</table>

Test-Taking Tip

Questions 8, 12, 16, and 17: If the question involves a geometric object but does not include a figure, draw one. A diagram can help you see relationships among the given values that will help you answer the question.
12. $y = 3 x = 2$

13. $y = -4 x = -1$

14. $y = 2 - x y = x + 4$

15. $y = 2 y = -3$

16. \[
\begin{align*}
3x + 2y &= 6 \\
4x - y &= 2
\end{align*}
\]

17. \[
\begin{align*}
4x - 3y &= 7 \\
2y - x &= -6
\end{align*}
\]

18. \[
\begin{align*}
y &= \frac{1}{2}x + 1 \\
y &= 2x - 3
\end{align*}
\]

19. no solution

20. \[
\begin{align*}
y &= 1 \\
x &= 2
\end{align*}
\]

21. \[
\begin{align*}
x + 3y &= 6 \\
x &= -4
\end{align*}
\]

22. no solution

23. \[
\begin{align*}
2x - y &= 4 \\
2x + 4y &= -7 \\
x - 3y &= 2
\end{align*}
\]

32. Hours Raking Leaves

33. $s \geq 111, s \leq 130, h \geq 9, h \leq 12$

35. Swedish Soda

Page 128, Follow-Up of Lesson 3-3
Graphing Calculator Investigation

1. $y = 4 y = -x$

2. $y = -2x y = -3$

[[-10, 10] scl: 1 by [-10, 10] scl: 1]
3. \[ y = 1 - x \]
\[ y = x + 5 \]

4. \[ y = x + 2 \]
\[ y = -2x - 1 \]

5. \[ 3y = 6x - 15 \]
\[ 2y = -x + 3 \]

6. \[ y + 3x = 6 \]
\[ y - 2x = 9 \]

7. \[ 6y + 4x = 12 \]
\[ 5y - 3x = -10 \]

8. \[ \frac{1}{4}y - x = -2 \]
\[ \frac{1}{3}y + 2x = 4 \]

9. \[ y = \frac{1}{2}x + 2 \]
\[ y = -x - 1 \]

10. \[ y = -x + 3 \]
\[ y = 4x - 6 \]

11. \[ y = \frac{1}{3}x + 1 \]
\[ y = -\frac{1}{2}x - 2 \]

12. \[ y = 2x - 1 \]
\[ y = -x + 3 \]

13. \[ y = x + 5 \]
\[ y = -x + 1 \]

14. \[ y = \frac{1}{2}x + 3 \]
\[ y = -x - 5 \]

15. \[ y = x + 4 \]
\[ y = -2x - 3 \]

16. \[ y = \frac{1}{3}x + 2 \]
\[ y = -\frac{1}{2}x - 5 \]

17. \[ y = x + 3 \]
\[ y = -x - 2 \]

18. \[ y = \frac{1}{2}x + 1 \]
\[ y = -x - 3 \]

19. \[ y = \frac{1}{3}x + 2 \]
\[ y = -x - 1 \]

20. \[ y = \frac{1}{4}x + 3 \]
\[ y = -\frac{1}{2}x - 2 \]
21. 22.
vertices: (0, 0), (0, 2),
(2, 1), (3, 0);
max: \( f(0, 2) = 6; \)
min: \( f(3, 0) = -12 \)

23. 24.
vertices: (3, 0), (0, -3);
no maximum;
min: \( f(0, -3) = -12 \)

vertices: (0, 2), (4, 3),
(7, -1), (3, -3);
max: \( f(4, 3) = 25; \)
min: \( f(0, 2) = 6 \)

27. 28.
vertices: (2, 5), (3, 0);
no maximum;
no minimum

29. 30.
vertices: (2, 1), (2, 3),
(4, 1), (4, 4), (5, 3);
max: \( f(4, 1) = 0; \)
min: \( f(4, 4) = -12 \)

43. There are many variables in scheduling tasks. Linear programming can help make sure that all the requirements are met. Answers should include the following.

- Let \( x \) = the number of buoy replacements and let \( y \) = the number of buoy repairs. Then, \( x \geq 0, y \geq 0, x \leq 8 \) and \( x + 2.5y \leq 24 \).

- The captain would want to maximize the number of buoys that a crew could repair and replace so \( f(x, y) = x + y \).

- Graph the inequalities and find the vertices of the intersection of the graphs. The coordinate (0, 24) maximizes the function. So the crew can service the maximum number of buoys if they replace 0 and repair 24 buoys.
1. One is a point (one-dimensional), one is a line (two-dimensional), and one is a plane (three-dimensional).

2. The graph of $x > -3$ in one dimension includes all of the numbers that lie to the right of the point $x = -3$ on a number line. The graph of $x > -3$ in two dimensions is a half-plane and includes all of the ordered pairs that lie to the right of the line $x = -3$. The graph of $x > -3$ in three dimensions includes all of the space that lies in front of the plane $x = -3$.

14a. $y = 2x + 3$

14b. $y = x + 1$

14c. $y = -x + 2$

14d. $y = 0$

14e. The graph of $x > -3$ in one dimension includes all of the numbers that lie to the right of the point $x = -3$ on a number line. The graph of $x > -3$ in two dimensions is a half-plane and includes all of the ordered pairs that lie to the right of the line $x = -3$. The graph of $x > -3$ in three dimensions includes all of the space that lies in front of the plane $x = -3$. 