Pacing suggestions for the entire year can be found on pages T20–T21.
# Chapter Resource Manager

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*Key to Abbreviations:* GCS = Graphing Calculator and Spreadsheet Masters, SC = School-to-Career Masters, SM = Science and Mathematics Lab Manual
Chapter 6
Quadratic Functions and Inequalities

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge
In the previous chapter students explored how to factor quadratic expressions, how to work with radicals, and how to perform operations on complex numbers. They have also solved linear equations and inequalities, and they are familiar with using formulas.

This Chapter
Students solve quadratic equations by graphing, by factoring, by completing the square, and by using the Quadratic Formula. They explore how values of a quadratic equation are reflected in the parabola that represents it, and use equations and graphs to explore quadratic equations that have 0, 1, or 2 roots. They relate the value of the discriminant to the number of roots and to whether the roots are rational, irrational, or complex.

Future Connections
Students will continue to explore roots and zeros of equations, examining higher-order polynomial equations in Chapter 7. Students will learn to recognize other types of equations that can be solved using the Quadratic Formula. They will explore systems of quadratic inequalities in Chapter 8.

6-1 Graphing Quadratic Functions
When graphing a quadratic function, use all the available information to produce the most accurate possible graph. This includes a table of values containing the vertex and the y-intercept. Be sure the points on the graph are connected with a smooth curve and that the graph has a U-shape and not a V-shape at the vertex of the graph. Unlike the letter U, however, the graph should become progressively wider and have arrows indicating that the graph continues to infinity.

6-2 Solving Quadratic Equations by Graphing
It is important to distinguish between finding solutions or roots of an equation and finding zeros of its related function. An equation of the form $ax^2 + bx + c = 0$ has a related function, $f(x) = ax^2 + bx + c$. The zeros of $f(x)$ are the $x$-coordinates of the points where the graph crosses the $x$-axis. These $x$ values are the solutions of the related quadratic equation. Without the use of a graphing calculator, this method of solving quadratic equations will usually provide only an estimate of solutions. Solutions that appear to be integers should be verified by substituting them into the original equation.

6-3 Solving Quadratic Equations by Factoring
When solving a quadratic equation by factoring, it is important to review factoring techniques. These include the techniques for factoring general trinomials, perfect square trinomials, and a difference of squares. Also remember to look for a greatest common factor (GCF) that might be factored out or the possibility of factoring by grouping. Before factoring, the equation should be rewritten so that one side of the equation is 0. If the GCF of the terms of the polynomial being factored is a variable or the product of a number and a variable, such as $3x$, one solution to the equation is 0.

6-4 Complete the Square
Completing the square is a technique most often used to solve quadratic equations that are not factorable. To use this technique it is desirable to rewrite the equation so that it is equal to a constant. Then divide the coefficient of the linear term by 2 and square the result. Add this value to both sides of the equation. One side of the equation will now be a perfect square trinomial that can be rewritten as the square of a binomial. To help isolate the variable on one side of the equation, take the square root
The Quadratic Formula and the Discriminant

While the technique of completing the square can be used to solve any quadratic equation, implementing this technique can lead to operations involving unwieldy fractions. The Quadratic Formula can also be used to solve any quadratic equation. Using this formula, the variable is isolated in the very first step and the other steps involve simplifying the solutions by simplifying radicals and fractions. The discriminant is simply the portion of the Quadratic Formula that appears underneath the radical, \( b^2 - 4ac \). This value alone will determine the number and type of roots (solutions) of the equation. This is because the square root of this value could result in a rational number, such as 6, an irrational number, such as \( \sqrt{2} \), the value 0, or an imaginary number, such as \( 3i \). By finding and examining just the value of the discriminant, you can tell very quickly what type of solutions a quadratic equation will have. This can serve as a check when solving the equation.

Analyzing Graphs of Quadratic Functions

To write a quadratic equation in the form \( y = a(x - h)^2 + k \), called vertex form, it is important to remember that an equation is a statement of equality. When an equation is rewritten in a different form, this equality must be maintained. In previous lessons, if a value was added to one side of an equation, it was also added to the other side of the equation in order to maintain equality. Another way to maintain equality is to add a value to one side and then subtract that same value from that side. For example, if an addition of 5 is shown on the right side of an equation, a subtraction of 5 would also be shown on the right side. So in essence this would be an overall addition of \( 5 - 5 \) or 0, which does not affect the equality of the statement. For example, \( y = 3x \) is equivalent to the statement \( y = 3x + 5 - 5 \). Be especially careful when adding a value inside a set of parentheses, since the use of parentheses often involves multiplication. For example \( y = 3(x) \) is not equivalent to \( y = 3(x + 1) - 1 \), but instead to \( y = 3(x + 1) - 3(1) \).

Graphing and Solving Quadratic Inequalities

One way of solving a linear inequality not discussed in Chapter 1 is to first solve its related linear equation and then test values on either side of this value in the original inequality. For example, to solve \( 3x + 2 > -4 \), you would solve the equation \( 3x + 2 = -4 \) and find that \( x = -2 \). Testing a value less than \(-2\) and a value greater than \(-2\) in the inequality reveals that the solution to the inequality is the set of values greater than \(-2\). The approach to solving a quadratic inequality algebraically is similar. The difference lies in the fact that many quadratic inequalities have not one but two solutions. This means that your number line is divided in three possible solution sets. Testing a value from each interval on the number line reveals which solution set or sets are correct. The solution set of a quadratic inequality will often be a compound inequality, so you will want to review this topic from Chapter 1.

Additional mathematical information and teaching notes are available in Glencoe’s Algebra 2 Key Concepts: Mathematical Background and Teaching Notes, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.
- Solving Quadratic Equations by Graphing (Lesson 31)
- Solving Equations by Factoring (Lesson 27)
- Solving Quadratic Equations by Completing the Square (Lesson 39)
- Solving Quadratic Equations by Using the Quadratic Formula (Lesson 32)
- Graphing Technology: Parent and Family Graphs (Lesson 29)
- Graphing Quadratic Functions (Lesson 28)
- More on Axis of Symmetry and Vertices (Lesson 30)
## Additional Intervention Resources

- The Princeton Review's *Cracking the SAT & PSAT*
- The Princeton Review’s *Cracking the ACT*
- ALEKS

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### TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

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### Key to Abbreviations

- **TWE** = Teacher Wraparound Edition
- **CRM** = Chapter Resource Masters

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### DAILY INTERVENTION and Assessment

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**Key to Abbreviations:** TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters
ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student’s needs. Subscribe at www.k12aleks.com.

For more information on Reading and Writing in Mathematics, see pp. T6–T7.
Notes

What You’ll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It’s Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Quadratic Functions and Inequalities

What You’ll Learn

- Lesson 6-1 Graph quadratic functions.
- Lessons 6-2 through 6-5 Solve quadratic equations.
- Lesson 6-3 Write quadratic equations and functions.
- Lesson 6-6 Analyze graphs of quadratic functions.
- Lesson 6-7 Graph and solve quadratic inequalities.

Key Vocabulary

- root (p. 294)
- zero (p. 294)
- completing the square (p. 307)
- Quadratic Formula (p. 313)
- discriminant (p. 316)

Why It’s Important

Quadratic functions can be used to model real-world phenomena like the motion of a falling object. They can also be used to model the shape of architectural structures such as the supporting cables of a suspension bridge. You will learn to calculate the value of the discriminant of a quadratic equation in order to describe the position of the supporting cables of the Golden Gate Bridge in Lesson 6-5.

Vocabulary Builder

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 6 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 6 test.
Prerequisite Skills  To be successful in this chapter, you’ll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lessons 6-1 and 6-2  Graph Functions
Graph each equation by making a table of values. (For review, see Lesson 2-1.)
1. \( y = 2x + 3 \)  2. \( y = -x - 5 \)  3. \( y = x^2 + 4 \)  4. \( y = -x^2 - 2x + 1 \)
1–4. See pp. 343A–343F.

For Lessons 6-1, 6-2, and 6-5  Simplify Radical Expressions
For Lessons 6-4 and 6-5
Simplify. (For review, see Lessons 5-6 and 5-9.)
17. \( \sqrt{225} \)  18. \( \sqrt{48} \)  19. \( \sqrt{180} \)  20. \( \sqrt{68} \)
15. \( 4\sqrt{3} \)  6. \( \sqrt{5} \)  15. \( 2\sqrt{17} \)
21. \( \sqrt{-25} \)  22. \( \sqrt{-32} \)  23. \( \sqrt{-270} \)
9. \( x + 6)(x + 5) \)  10. \( (x - 4)(x - 9) \)  11. \( (x - 8)(x + 7) \)  12. \( (x + 2)(x - 7) \)

For Lessons 6-3 and 6-4  Multiply Polynomials
Find each product. (For review, see Lesson 5-2.)
5. \( (x - 4)(7x + 12) \)  6. \( (x + 5)^2 \)  7. \( (3x - 1)^2 \)  8. \( (3x - 4)(2x - 9) \)
7\( x^2 - 16x - 48 \)  9. \( x^2 + 10x + 25 \)  10. \( 6x^2 - 6x + 1 \)  11. \( 6x^2 - 35x + 36 \)

For Lessons 6-3 and 6-4  Factor Polynomials
Factor completely. If the polynomial is not factorable, write prime. (For review, see Lesson 5-4.)
9. \( x^2 + 11x + 30 \)  10. \( x^2 - 13x + 36 \)  11. \( x^2 - x - 56 \)  12. \( x^2 - 5x - 14 \)
13. \( x^2 + x + 2 \)  14. \( x^2 + 10x + 25 \)  15. \( x^2 - 22x + 121 \)  16. \( x^2 - 9 \)
prime  prime  \( (x + 5)^2 \)  \( (x - 11)^2 \)  \( (x + 3)(x - 3) \)

For Lessons 6-4 and 6-5  Simplify Radical Expressions
Simplify. (For review, see Lessons 5-5 and 5-9.)
17. \( \sqrt{225} \)  18. \( \sqrt{48} \)  19. \( \sqrt{180} \)  20. \( \sqrt{68} \)
15. \( 4\sqrt{3} \)  6. \( \sqrt{5} \)  15. \( 2\sqrt{17} \)
21. \( \sqrt{-25} \)  22. \( \sqrt{-32} \)  23. \( \sqrt{-270} \)
9. \( x + 6)(x + 5) \)  10. \( (x - 4)(x - 9) \)  11. \( (x - 8)(x + 7) \)  12. \( (x + 2)(x - 7) \)

Foldables
Make this Foldable to record information about quadratic functions and inequalities. Begin with one sheet of 11” × 17” paper.

Step 1  Fold and Cut
Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.

Step 2  Refold and Label
Refold along lengthwise fold and staple uncut section at top. Label the section with a lesson number and close to form a booklet.

Reading and Writing  As you read and study the chapter, fill the journal with notes, diagrams, and examples for each lesson.

Sequencing Information and Progression of Knowledge
After students make their Foldable, have them label a section for each lesson in Chapter 6 and a section for vocabulary. As students progress through the lessons, have them summarize key concepts and note the order in which they are presented. Ask students to write about why the concepts were presented in this sequence. If they cannot see the logic in the sequence, have them reorder the key concepts and justify their reasoning.
5-Minute Check Transparency 6-1 Use as a quiz or a review of Chapter 5.

Mathematical Background notes are available for this lesson on p. 284C.

Building on Prior Knowledge

In Chapter 5, students wrote and solved various equations and inequalities. In this lesson, they will relate quadratic equations to their graphs.

How can income from a rock concert be maximized?

Ask students:

- How is the income represented in the given function? by \( P(x) \)
- What is significant about the value of \( P(x) \) when \( x = 40 \) (the ticket price of $40)? The value of \( P(x) \) is greatest when \( x = 40 \), or the income is at its maximum value when \( x = 40 \).

Rock music managers handle publicity and other business issues for the artists they manage. One group’s manager has found that based on past concerts, the predicted income for a performance is

\[
P(x) = -50x^2 + 4000x - 7500,
\]

where \( x \) is the price per ticket in dollars. The graph of this quadratic function is shown at the right. Notice that at first the income increases as the price per ticket increases, but as the price continues to increase, the income declines.

Graphing Quadratic Functions

A quadratic function is described by an equation of the following form.

\[
f(x) = ax^2 + bx + c, \text{ where } a \neq 0
\]

The graph of any quadratic function is called a parabola. One way to graph a quadratic function is to graph ordered pairs that satisfy the function.

Example 1 Graph a Quadratic Function

Graph \( f(x) = 2x^2 - 8x + 9 \) by making a table of values.

First, choose integer values for \( x \). Then, evaluate the function for each \( x \) value. Graph the resulting coordinate pairs and connect the points with a smooth curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2x^2 - 8x + 9 )</th>
<th>( f(x) )</th>
<th>( (x, f(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( 2(0)^2 - 8(0) + 9 )</td>
<td>9</td>
<td>((0, 9))</td>
</tr>
<tr>
<td>1</td>
<td>( 2(1)^2 - 8(1) + 9 )</td>
<td>3</td>
<td>((1, 3))</td>
</tr>
<tr>
<td>2</td>
<td>( 2(2)^2 - 8(2) + 9 )</td>
<td>1</td>
<td>((2, 1))</td>
</tr>
<tr>
<td>3</td>
<td>( 2(3)^2 - 8(3) + 9 )</td>
<td>3</td>
<td>((3, 3))</td>
</tr>
<tr>
<td>4</td>
<td>( 2(4)^2 - 8(4) + 9 )</td>
<td>9</td>
<td>((4, 9))</td>
</tr>
</tbody>
</table>

Resource Manager

Workbook and Reproducible Masters

- Chapter 6 Resource Masters
  - Study Guide and Intervention, pp. 313–314
  - Skills Practice, p. 315
  - Practice, p. 316
  - Reading to Learn Mathematics, p. 317
  - Enrichment, p. 318

Transparencies

- 5-Minute Check Transparency 6-1
- Answer Key Transparencies

Technology

- Alge2PASS: Tutorial Plus, Lesson 10
- Interactive Chalkboard
All parabolas have an **axis of symmetry**. If you were to fold a parabola along its axis of symmetry, the portions of the parabola on either side of this line would match.

The point at which the axis of symmetry intersects a parabola is called the **vertex**. The y-intercept of a quadratic function, the equation of the axis of symmetry, and the x-coordinate of the vertex are related to the equation of the function as shown below.

### Key Concept

**Graph of a Quadratic Function**

- **Words**
  - Consider the graph of \( y = ax^2 + bx + c \), where \( a \neq 0 \).
  - The y-intercept is \( a(0)^2 + b(0) + c \) or \( c \).
  - The equation of the axis of symmetry is \( x = -\frac{b}{2a} \).
  - The x-coordinate of the vertex is \( -\frac{b}{2a} \).

- **Model**

Knowing the location of the axis of symmetry, y-intercept, and vertex can help you graph a quadratic function.

### Example 2 Axis of Symmetry, y-Intercept, and Vertex

Consider the quadratic function \( f(x) = x^2 + 9 + 8x \).

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

Begin by rearranging the terms of the function so that the quadratic term is first, the linear term is second, and the constant term is last. Then identify \( a \), \( b \), and \( c \).

\[
f(x) = ax^2 + bx + c
\]

\[
f(x) = x^2 + 9 + 8x \quad \rightarrow \quad f(x) = 1x^2 + 8x + 9
\]

So, \( a = 1 \), \( b = 8 \), and \( c = 9 \).

The y-intercept is 9. You can find the equation of the axis of symmetry using \( a \) and \( b \).

\[
x = -\frac{b}{2a} \quad \text{Equation of the axis of symmetry}
\]

\[
x = -\frac{8}{2(1)} \quad a = 1, b = 8
\]

\[
x = -4 \quad \text{Simplify.}
\]

The equation of the axis of symmetry is \( x = -4 \). Therefore, the x-coordinate of the vertex is \( -4 \).

---

**In-Class Examples**

1. Graph \( f(x) = x^2 + 3x - 1 \) by making a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-1</td>
<td>-3</td>
<td>-3</td>
<td>-1</td>
<td>3</td>
</tr>
</tbody>
</table>

2. Consider the quadratic function \( f(x) = 2 - 4x + x^2 \).

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex. \( 2; x = 2; 2 \)

b. Make a table of values that includes the vertex.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

c. Use this information to graph the function.
**MAXIMUM AND MINIMUM VALUES**

**In-Class Example**

3. Consider the function $f(x) = -x^2 + 2x + 3$.

a. Determine whether the function has a maximum or a minimum value. Maximum

b. State the maximum or minimum value of the function. Minimum

**Teaching Tip** Remind students to use the equation for the axis of symmetry to help determine the $x$-coordinate of the vertex and then find the $y$-coordinate of the vertex.

**Study Tip**

**Symmetry**

Sometimes it is convenient to use symmetry to help find other points on the graph of a parabola. Each point on a parabola has a mirror image located the same distance from the axis of symmetry on the other side of the parabola.

**Example 3** Maximum or Minimum Value

Consider the function $f(x) = x^2 - 4x + 9$.

a. Determine whether the function has a maximum or a minimum value.

For this function, $a = 1$, $b = -4$, and $c = 9$. Since $a > 0$, the graph opens up and the function has a minimum value.

**Key Concept**

**Maximum and Minimum Value**

- **Words** The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$, can be used interchangeably, and also that the maximum or minimum value of the function is given by the $y$-coordinate of the vertex of the parabola.

- **Models**
  - $a$ is positive.
  - $a$ is negative.

**Example 3** Maximum or Minimum Value

Consider the function $f(x) = x^2 - 4x + 9$.

a. Determine whether the function has a maximum or a minimum value.

For this function, $a = 1$, $b = -4$, and $c = 9$. Since $a > 0$, the graph opens up and the function has a minimum value.

**Unlocking Misconceptions**

**Minimum and Maximum Values** Make sure students understand that a parabola which opens upward is the graph of a function with a minimum value and that a parabola which opens downward is the graph of a function with a maximum value. Compare these parabolas to valleys (where the altitude of the valley floor is a minimum) and hills (where the peak of the hill is the maximum altitude).
b. State the maximum or minimum value of the function.
The minimum value of the function is the $y$-coordinate of the vertex.
The $x$-coordinate of the vertex is $\frac{b}{2a}$ or 2.
Find the $y$-coordinate of the vertex by evaluating the function for $x = 2$.

$$f(x) = x^2 - 4x + 9 \quad \text{Original function}$$

$$f(2) = (2)^2 - 4(2) + 9 \quad \text{or} \quad 5 \quad x = 2$$

Therefore, the minimum value of the function is 5.

When quadratic functions are used to model real-world situations, their maximum or minimum values can have real-world meaning.

**Example 4 Find a Maximum Value**

**FUND-RAISING** Four hundred people came to last year’s winter play at Sunnybrook High School. The ticket price was $5. This year, the Drama Club is hoping to earn enough money to take a trip to a Broadway play. They estimate that for each $0.50 increase in the price, 10 fewer people will attend their play.

a. How much should the tickets cost in order to maximize the income from this year’s play?

**Words** The income is the number of tickets multiplied by the price per ticket.

**Variables** Let $x$ = the number of $0.50 price increases.

Then $5 + 0.50x$ = the price per ticket and $400 - 10x$ = the number of tickets sold.

Let $I(x)$ = income as a function of $x$.

**Equation**

$$I(x) = (400 - 10x) \cdot (5 + 0.50x)$$

= $400(5) + 400(0.50x) - 10x(5) - 10x(0.50x)$

= $2000 + 200x - 50x - 5x^2$ **Multiply.**

= $2000 + 150x - 5x^2$ **Simplify.**

= $-5x^2 + 150x + 2000$ **Rewrite in $ax^2 + bx + c$ form.**

$I(x)$ is a quadratic function with $a = -5, b = 150$, and $c = 2000$. Since $a < 0$, the function has a maximum value at the vertex of the graph.

Use the formula to find the $x$-coordinate of the vertex.

$$x\text{-coordinate of the vertex} = \frac{-b}{2a} \quad \text{Formula for the } x\text{-coordinate of the vertex}$$

$$= \frac{150}{2(-5)} \quad a = -5, b = 150$$

$$= 15 \quad \text{Simplify.}$$

This means the Drama Club should make 15 price increases of $0.50 to maximize their income. Thus, the ticket price should be $5 + 0.50(15)$ or $12.50.

(continued on the next page)

**Differentiated Instruction**

**Auditory/Musical** Ask students to suggest the kinds of musical sounds they might associate with a parabola that has a maximum value, and have them contrast this to a sound that might be associated with a parabola that has a minimum value. For example, an orchestra piece that rises to a crescendo and then gradually returns to the previous volume might be seen as being related to a parabola with a maximum value.
b. What is the maximum income the Drama Club can expect to make?

To determine maximum income, find the maximum value of the function by evaluating \( f(x) \) for \( x = 15 \).

\[
\begin{align*}
\text{Income function} \\
I(x) &= -5x^2 + 150x + 2000 \\
I(15) &= -5(15)^2 + 150(15) + 2000 \\
\text{Use a calculator.}
\end{align*}
\]

Thus, the maximum income the Drama Club can expect is $3125.

CHECK

Graph this function on a graphing calculator, and use the \textsc{calc} menu to confirm this solution.

\[
\text{KEYSTROKES:} \quad \textsc{2nd} \quad [\textsc{calc}] \quad 4 \quad \text{ENTER} \quad 25 \quad \text{ENTER} \quad \text{ENTER}
\]

At the bottom of the display are the coordinates of the maximum point on the graph of \( y = -5x^2 + 150x + 2000 \). The \( y \)-value of these coordinates is the maximum value of the function, or 3125. \( \checkmark \)

### Guided Practice

**GUIDED PRACTICE KEY**

<table>
<thead>
<tr>
<th>Exercises</th>
<th>Examples</th>
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<tbody>
<tr>
<td>4–9</td>
<td>1, 2</td>
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<tr>
<td>10–12</td>
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</tr>
<tr>
<td>13</td>
<td>4</td>
</tr>
</tbody>
</table>

**Guided Practice**

Complete parts a–c for each quadratic function.

a. Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function. 4–9. See margin.

4. \( f(x) = -4x^2 \)
5. \( f(x) = x^2 + 2x \)
6. \( f(x) = x^2 + 4x - 1 \)
7. \( f(x) = x^2 + 8x + 3 \)
8. \( f(x) = 2x^2 - 4x + 1 \)
9. \( f(x) = 3x^2 + 10x \)

### Check for Understanding

#### Concept Check

1. **OPEN ENDED** Give an example of a quadratic function. Identify its quadratic term, linear term, and constant term.

2. **Identify** the vertex and the equation of the axis of symmetry for each function graphed below. \( a. \quad (2, 1); \ x = 2 \quad b. \quad (-3, -2); \ x = -3 \)

a. 

b. 

3. **State** whether the graph of each quadratic function opens \textit{up} or \textit{down}. Then state whether the function has a \textit{maximum} or \textit{minimum} value.

a. \( f(x) = 3x^2 + 4x - 5 \) \textit{up; min.} 

b. \( f(x) = -2x^2 + 9 \) \textit{down; max.} 

c. \( f(x) = -5x^2 - 8x + 2 \) \textit{down; max.} 

d. \( f(x) = 6x^2 - 5x \) \textit{up; min.} 

### About the Exercises...

**Organization by Objective**
- Graph Quadratic Functions: 14–31, 44
- Maximum and Minimum Values: 32–43, 45–53

**Odd/Even Assignments**
- Exercises 14–31 and 32–43 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Alert!** Exercises 58–63 require a graphing calculator.

**Assignment Guide**
- **Basic:** 15–27 odd, 33–43 odd, 44–47, 54–57, 64–78
- **Average:** 15–43 odd, 46–50, 53–57, 64–78 (optional: 58–63)
- **Advanced:** 14–42 even, 46–74 (optional: 75–78)

### Answers

4a. \( x = 0; \ y = 0 \)

4b. 

\[
\begin{array}{c|c}
\hline
x & f(x) \\
\hline
-1 & -4 \\
0 & 0 \\
1 & -4 \\
\hline
\end{array}
\]

5a. \( x = -1; \ -1 \)

5b. 

\[
\begin{array}{c|c}
\hline
x & f(x) \\
\hline
-3 & 3 \\
-2 & 0 \\
-1 & -1 \\
0 & 0 \\
1 & 3 \\
\hline
\end{array}
\]

5c. 

\[
\begin{array}{c|c}
\hline
x & f(x) \\
\hline
-1 & -1 \\
\hline
\end{array}
\]

290 Chapter 6 Quadratic Functions and Inequalities
Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

10. \( f(x) = -x^2 + 7 \) \hspace{1cm} 11. \( f(x) = x^2 - x - 6 \) \hspace{1cm} 12. \( f(x) = 4x^2 + 12x + 9 \)

\( \text{max.}; 7 \) \hspace{1cm} \( \text{min.}; -\frac{25}{4} \) \hspace{1cm} \( \text{min.}; 0 \)

13. **NEWSPAPERS** Due to increased production costs, the Daily News must increase its subscription rate. According to a recent survey, the number of subscriptions will decrease by about 1250 for each 25¢ increase in the subscription rate. What weekly subscription rate will maximize the newspaper’s income from subscriptions? $8.75

**EXTRA PRACTICE**

See page 859.

For **Exercises** | **See Examples**
--- | ---
14–19 | 1
20–31 | 2
32–43, 54 | 3
44–53 | 4

**ARCHITECTURE**

The Exchange House in London, England, is supported by two interior and two exterior steel arches. V-shaped braces add stability to the structure.

Source: Council on Tall Buildings and Urban Habitat

www.algebra2.com/self_check_quiz

Lesson 6-1 Graphing Quadratic Functions

291
1. a.

Gravitational: From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 20 ft per second. The height \( h(t) \) of the object \( t \) seconds after firing is given by

\[
h(t) = -16t^2 + 20t + 4.
\]

46. Find the maximum height reached by the object and the time that the height is reached. 300 ft, 2.5 s

47. Interpret the meaning of the \( y \)-intercept in the context of this problem.

The \( y \)-intercept is the initial height of the object.

CONSTRUCTION For Exercises 48–50, use the following information.

A tour bus in the historic district of Savannah, Georgia, serves 300 customers a day. The charge is $8 per person. The owner estimates that the company would lose 20 passengers a day for each $1 fare increase.

48. Write an algebraic expression for the market’s length. \( 120 - 2x \)

49. What dimensions produce a kernel with the greatest area?

50. Find the maximum area of the kernel. 1800 ft²

TOURISM For Exercises 51 and 52, use the following information.

A tour bus in the historic district of Savannah, Georgia, serves 300 customers a day. The charge is $8 per person. The owner estimates that the company would lose 20 passengers a day for each $1 fare increase.

51. What charge would give the most income for the company? $11.50

52. If the company raised their fare to this price, how much daily income should they expect to bring in? $2645

54. CRITICAL THINKING Write an expression for the minimum value of a function of the form

\[
y = ax^2 + c,
\]

where \( a > 0 \). Explain your reasoning.

Then use this function to find the minimum value of

\[
y = 8.6x^2 - 12.5.
\]

See margin for explanation; \(-12.5\).

55. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How can income from a rock concert be maximized?

Include the following in your answer:

• an explanation of how income increases and then declines as the ticket price increases;

• an explanation of how to algebraically and graphically determine what ticket price should be charged to achieve maximum income.

56. The graph of which of the following equations is symmetrical about the \( y \)-axis? C

\[
A) y = x^2 + 3x - 1
\]

\[
B) y = x^2 + x
\]

\[
C) y = 6x^2 + 9
\]

\[
D) y = 3x^2 - 3x + 1
\]
55. If a quadratic function can be used to model ticket price versus profit, then by finding the x-coordinate of the vertex of the parabola you can determine the price per ticket that should be charged to achieve maximum profit. Answers should include the following.

- If the price of a ticket is too low, then you won’t make enough money to cover your costs, but if the ticket price is too high fewer people will buy them.

- You can locate the vertex of the parabola on the graph of the function. It occurs when \( x = \frac{-b}{2a} \) which, for this case, is \( x = \frac{-4000}{2(-50)} = 40 \). Thus the ticket price should be set at $40 each to achieve maximum profit.
Focus
5-Minute Check Transparency 6-2 Use as a quiz or a review of Lesson 6-1.

Mathematical Background 
notes are available for this lesson on p. 284C.

How does a quadratic function model a free-fall ride?
Ask students:
• The acceleration of a free-falling object due to Earth’s gravity is −32 ft/sec². It is given as a negative value because the acceleration is downward, toward Earth’s surface. How is this fact represented in the height function? The coefficient −16 is the one half of the acceleration due to gravity in a downward direction.
• How far has a person fallen 1 second after beginning a free fall? after 2 seconds? after 3 seconds? 16 ft; 64 ft; 144 ft

Solving Quadratic Equations by Graphing

What You’ll Learn
• Solve quadratic equations by graphing.
• Estimate solutions of quadratic equations by graphing.

Vocabulary
• quadratic function
• root
• zero

How does a quadratic function model a free-fall ride?
As you speed to the top of a free-fall ride, you are pressed against your seat so that you feel like you’re being pushed downward. Then as you free-fall, you fall at the same rate as your seat. Without the force of your seat pressing on you, you feel weightless. The height above the ground (in feet) of an object in free-fall can be determined by the quadratic function
\[ h(t) = -16t^2 + h_0, \]
where \( t \) is the time in seconds and the initial height is \( h_0 \) feet.

Solve Quadratic Equations
When a quadratic function is set equal to a value, the result is a quadratic equation. A quadratic equation can be written in the form
\[ ax^2 + bx + c = 0, \]
where \( a \neq 0 \).

The solutions of a quadratic equation are called the roots of the equation. One method for finding the roots of a quadratic equation is to find the zeros of the related quadratic function. The zeros of the function are the x-intercepts of its graph. These are the solutions of the related equation because \( f(x) = 0 \) at those points. The zeros of the function graphed at the right are 1 and 3.

Example 1 Two Real Solutions
Solve \( x^2 + 6x + 8 = 0 \) by graphing.
Graph the related quadratic function \( f(x) = x^2 + 6x + 8 \). The equation of the axis of symmetry is \( x = -\frac{6}{2(1)} = -3 \). Make a table using \( x \) values around −3. Then, graph each point.

<table>
<thead>
<tr>
<th>( x )</th>
<th>−5</th>
<th>−4</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3</td>
<td>0</td>
<td>−1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

From the table and the graph, we can see that the zeros of the function are −4 and −2. Therefore, the solutions of the equation are −4 and −2.

CHECK Check the solutions by substituting each solution into the equation to see if it is satisfied.

\[
\begin{align*}
(x + 4)(x + 2) &= 0 \\
(x + 4)(x + 2) &= 0 \\
-4 - 2 &= 0 \\
0 &= 0 \checkmark
\end{align*}
\]

The graph of the related function in Example 1 had two zeros; therefore, the quadratic equation had two real solutions. This is one of the three possible outcomes when solving a quadratic equation.
**Key Concept**

**Solutions of a Quadratic Equation**

- **Words**: A quadratic equation can have one real solution, two real solutions, or no real solution.
- **Models**
  - One Real Solution
  - Two Real Solutions
  - No Real Solution

---

**Example 2**  
**One Real Solution**

Solve $8x - x^2 = 16$ by graphing.

Write the equation in $ax^2 + bx + c = 0$ form.

$8x - x^2 = 16 \rightarrow -x^2 + 8x - 16 = 0$  
Subtract 16 from each side.

Graph the related quadratic function $f(x) = -x^2 + 8x - 16$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-4</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-4</td>
</tr>
</tbody>
</table>

Notice that the graph has only one $x$-intercept, 4. Thus, the equation’s only solution is 4.

---

**Example 3**  
**No Real Solution**

**NUMBER THEORY** Find two real numbers whose sum is 6 and whose product is 10 or show that no such numbers exist.

**Explore**  
Let $x$ = one of the numbers. Then $6 - x$ = the other number.

**Plan**  
Since the product of the two numbers is 10, you know that $x(6 - x) = 10$.

\[
\begin{align*}
x(6 - x) &= 10 & \text{Original equation} \\
6x - x^2 &= 10 & \text{Distributive Property} \\
-x^2 + 6x - 10 &= 0 & \text{Subtract 10 from each side.}
\end{align*}
\]

**Solve**  
You can solve $-x^2 + 6x - 10 = 0$ by graphing the related function $f(x) = -x^2 + 6x - 10$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-5</td>
<td>-2</td>
<td>-1</td>
<td>-2</td>
<td>-5</td>
</tr>
</tbody>
</table>

Notice that the graph has no $x$-intercepts. This means that the original equation has no real solution. Thus, it is not possible for two numbers to have a sum of 6 and a product of 10.

**Examine**  
Try finding the product of several pairs of numbers whose sum is 6. Is the product of each pair less than 10 as the graph suggests?

---

**Unlocking Misconceptions**

**Equations and Functions** Some students may notice that the equation derived in Example 2, $-x^2 + 8x - 16 = 0$, is equivalent to the equation $x^2 - 8x + 16 = 0$. Either equation can be produced from the other by multiplying each side by $-1$. These two equations have the same solution, 4. However, stress that the related functions, $f(x) = -x^2 + 8x - 16$ and $f(x) = x^2 - 8x + 16$ are not equivalent. This can be seen by looking at their graphs, which open in opposite directions.

---

**Lesson 6-2**  
Solving Quadratic Equations by Graphing
ESTIMATE SOLUTIONS

**Example 4** Estimate Roots

Solve \(-x^2 + 4x - 1 = 0\) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The equation of the axis of symmetry of the related function is \(x = \frac{4}{2(-1)} = 2\).

\[
\begin{array}{c|cccc}
\text{x} & 0 & 1 & 2 & 3 & 4 \\
\hline
f(x) & -1 & 2 & 3 & 2 & -1 \\
\end{array}
\]

The \(x\)-intercepts of the graph are between 0 and 1 and between 3 and 4. So, one solution is between 0 and 1, and the other is between 3 and 4.

For many applications, an exact answer is not required, and approximate solutions are adequate. Another way to estimate the solutions of a quadratic equation is by using a graphing calculator.

**Example 5** Write and Solve an Equation

EXTREME SPORTS On March 12, 1999, Adrian Nicholas broke the world record for the longest human flight. He flew 10 miles from his drop point in 4 minutes 55 seconds using a specially designed, aerodynamic suit. Using the information at the right and ignoring air resistance, how long would Mr. Nicholas have been in free-fall had he not used this special suit? Use the formula \(h(t) = -16t^2 + h_0\), where the time \(t\) is in seconds and the initial height \(h_0\) is in feet.

We need to find \(t\) when \(h_0 = 35,000\) and \(h(t) = 500\). Solve 500 = \(-16t^2 + 35,000\).

500 = \(-16t^2 + 35,000\) Original equation

0 = \(-16t^2 + 34,500\) Subtract 500 from each side.

Graph the related function \(y = -16t^2 + 34,500\) using a graphing calculator. Adjust your window so that the \(x\)-intercepts of the graph are visible.

Use the ZERO feature, \(2^{nd}\) [CALC], to find the positive zero of the function, since time cannot be negative. Use the arrow keys to locate a left bound for the zero and press ENTER.

Then, locate a right bound and press ENTER twice. The positive zero of the function is approximately 46.4. Mr. Nicholas would have been in free-fall for about 46 seconds.
Check for Understanding

Concept Check

1. Define each term and explain how they are related. **See margin.**
   a. solution  
   b. root  
   c. zero of a function  
   d. x-intercept

2. OPEN ENDED Give an example of a quadratic function and state its related quadratic equation. **Sample answer:** \( f(x) = 3x^2 + 2x - 1; 3x^2 + 2x - 1 = 0 \)

3. Explain how you can estimate the solutions of a quadratic equation by examining the graph of its related function. **See margin.**

Guided Practice

Use the related graph of each equation to determine its solutions.

4. \( x^2 + 3x - 4 = 0 \) \(-4, 1\)  
5. \( 2x^2 + 2x - 4 = 0 \) \(-2, 1\)  
6. \( x^2 + 8x + 16 = 0 \) \(-4\)

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

7. \(-x^2 + 7x = 0 \) \(-7, 0\)  
8. \( x^2 - 2x - 24 = 0 \) \(-4, 6\)  
9. \( x^2 + 3x = 28 \) \(-7, 4\)  
10. \( 25 + x^2 + 10x = 0 \) \(-5\)  
11. \( 4x^2 - 7x - 15 = 0 \) \(-2, -1; 3\)  
12. \( 2x^2 - 2x - 3 = 0 \) \([-2, -1]; 3\)

Application

13. **NUMBER THEORY** Use a quadratic equation to find two real numbers whose sum is 5 and whose product is -14, or show that no such numbers exist. \(-2, 7\)

Practice and Apply

Use the related graph of each equation to determine its solutions.

14. \( x^2 - 6x = 0 \) \(0, 6\)  
15. \( x^2 - 6x + 9 = 0 \) \(3\)  
16. \(-2x^2 - x + 6 = 0 \) \(-2, 1\)

Exercises 14–16 odd, 17–19 even

17. \(-0.5x^2 = 0 \) \(0\)  
18. \( 2x^2 - 5x - 3 = 0 \) \(-1, 3\)  
19. \(-3x^2 - 1 = 0 \) \(\text{no real solutions}\)

About the Exercises...

**Organization by Objective**

- Solve Quadratic Equations: 14–21, 24–31, 36–41

**Odd/Even Assignments**

Exercises 14–41 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Alert!** Exercises 51–56 require a graphing calculator.

**Assignment Guide**

**Basic:** 15–45 odd, 47–50, 57–72  
**Average:** 15–45 odd, 47–50, 57–72 (optional: 51–56)  
**Advanced:** 14–46 even, 47–66 (optional: 67–72)

**Answers**

1a. The solution is the value that satisfies an equation.  
1b. A root is a solution of an equation.  
1c. A zero is the \( x \) value of a function that makes the function equal to 0.

1d. An \( x \)-intercept is the point at which a graph crosses the \( x \)-axis. The solutions, or roots, of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the \( x \)-intercepts of its graph.

3. The \( x \)-intercepts of the related function are the solutions to the equation. You can estimate the solutions by stating the consecutive integers between which the \( x \)-intercepts are located.
Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

22. \( x^2 - 3x = 0 \) \( \text{0, 3} \)

23. \( x^2 + 2x - 3 = 0 \) \( \text{1, 3} \)

24. \( x^2 + x = 20 \) \( \text{-5, 4} \)

25. \( x^2 - 9x = -18 \) \( \text{3, 6} \)

26. \( 14x^2 + x^2 + 49 = 0 \) \( \text{-7} \)

27. \( -12x + x^2 = -36 \) \( \text{6} \)

28. \( 2x^2 = 3x + 9 \) \( \text{1, 3} \)

29. \( 2x^2 = -5x + 12 \) \( \text{-4, 1} \)

30. \( 3x^2 = 12x - 4 \) \( \text{1, 3} \)

31. \( x^2 = x + 15 \) \( \text{-3, 5} \)

32. \( x^2 - 4x + 2 = 0 \) \( \text{-1, 3} \)

33. \( x^2 - 4x + 2 = 0 \) \( \text{-1, 3} \)

34. \( -2x^2 + 3x + 3 = 0 \) \( \text{no real solutions} \)

35. \( 0.5x^2 - 3 = 0 \) \( \text{no real solutions} \)

36. \( x^2 + 2x + 5 = 0 \) \( \text{no real solutions} \)

37. \( -x^2 + 4x - 6 = 0 \) \( \text{no real solutions} \)

**Number Theory** Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

38. Their sum is -17, and their product is 72. \( \text{-8, -9} \)

39. Their sum is 7, and their product is 14. \( \text{See pp. 343A–343F.} \)

40. Their sum is -9, and their product is 24. \( \text{See pp. 343A–343F.} \)

41. Their sum is 12, and their product is -28. \( \text{-2, 14} \)

For Exercises 42–44, use the formula \( h(t) = v_0t - 16t^2 \) where \( h(t) \) is the height of an object in feet, \( v_0 \) is the object’s initial velocity in feet per second, and \( t \) is the time in seconds.

42. **Archery** An arrow is shot upward with a velocity of 64 feet per second. Ignoring the height of the archer, how long after the arrow is released does it hit the ground? \( 4 \text{ s} \)

43. **Tennis** A tennis ball is hit upward with a velocity of 48 feet per second. Ignoring the height of the tennis player, how long does it take for the ball to fall to the ground? \( 3 \text{ s} \)

44. **Boating** A boat in distress launches a flare straight up with a velocity of 190 feet per second. Ignoring the height of the boat, how many seconds will it take for the flare to hit the water? \( 12 \text{ s} \)

45. **Law Enforcement** Police officers can use the length of skid marks to help determine the speed of a vehicle before the brakes were applied. If the skid marks are on dry concrete, the formula \( s^2 = 24d \) can be used. In the formula, \( s \) represents the speed in miles per hour, and \( d \) represents the length of the skid marks in feet. If the length of the skid marks on dry concrete are 50 feet, how fast was the car traveling? \( 35 \text{ mph} \)

46. **Empire State Building** Suppose you could conduct an experiment by dropping a small object from the Observatory of the Empire State Building. How long would it take for the object to reach the ground, assuming there is no air resistance? Use the information at the left and the formula \( h(t) = -16t^2 + h_0 \) where \( t \) is the time in seconds and the initial height \( h_0 \) is in feet. \( 8 \text{ s} \)

47. **Critical Thinking** A quadratic function has values \( f(-4) = -11, f(-2) = 9 \), and \( f(0) = 5 \). Between which two \( x \) values must \( f(x) \) have a zero? Explain your reasoning. \( -4 \) and \( -2; \text{ See margin for explanation.} \)
48. **Writing in Math** Answer the question that was posed at the beginning of the lesson. See pp. 343A–343B.

How does a quadratic function model a free-fall ride?
Include the following in your answer:
- a graph showing the height at any given time of a free-fall ride that lifts riders to a height of 185 feet, and
- an explanation of how to use this graph to estimate how long the riders would be in free-fall if the ride were allowed to hit the ground before stopping.

---

49. If one of the roots of the equation \( x^2 + kx - 12 = 0 \) is 4, what is the value of \( k? \)

\[ \begin{array}{ccc}
\text{A} & -1 & \text{B} \\
\text{C} & 0 & \text{D} \\
\text{E} & 3 & \end{array} \]

50. For what value of \( x \) does \( f(x) = x^2 + 5x + 6 \) reach its minimum value?

\[ \begin{array}{ccc}
\text{A} & -3 & \text{B} \\
\text{C} & -\frac{5}{2} & \text{D} \\
\text{E} & -2 & \end{array} \]

---

51. \( |x + 1| = 0 \)

52. \( |x| - 3 = 0 \)

53. \( |x - 4| - 1 = 0 \)

54. \( -|x + 4| + 5 = 0 \)

55. \( 2|3x| - 8 = 0 \)

56. \( 2|x - 3| + 1 = 0 \)

---

57. \( f(x) = x^2 - 6x + 4 \)

58. \( f(x) = -4x^2 + 8x - 1 \)

59. \( f(x) = \frac{1}{4}x^2 + 3x + 4 \)

Simplify.

\[ \begin{array}{ccc}
\text{A} & 4; \; x = 3; & 3 \\
\text{B} & -1; & x = 1; \; 1 \\
\text{C} & 4; & x = -6; \; -6 \\
\end{array} \]

---

60. \( \frac{2i}{3 + i} \)

61. \( \frac{4}{5 - i} \)

62. \( \frac{1 + i}{3 - 2i} \)

Evaluate the determinant of each matrix.

\[ \begin{array}{ccc}
\text{A} & \begin{vmatrix} 8 & 5 & -2 \\
2 & -1 & -6 \\
5 & 0 & 3 \\
\end{vmatrix} & \text{B} \\
\text{C} & \begin{vmatrix} 6 & 5 & -2 \\
-3 & 0 & 6 \\
-3 & 1 & 4 \\
\end{vmatrix} & \text{C} \\
\text{D} & \begin{vmatrix} 6 & 5 & -2 \\
-3 & 0 & 6 \\
-3 & 1 & 4 \\
\end{vmatrix} \end{array} \]

---

57. \( f(x) = x^2 - 6x + 4 \)

58. \( f(x) = -4x^2 + 8x - 1 \)

59. \( f(x) = \frac{1}{4}x^2 + 3x + 4 \)

---

**Maintain Your Skills**

**Mixed Review**

Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex for each quadratic function. Then graph the function by making a table of values. (Lesson 6-1) 57–59. See margin for graphs.

57. \( f(x) = x^2 - 6x + 4 \)

58. \( f(x) = -4x^2 + 8x - 1 \)

59. \( f(x) = \frac{1}{4}x^2 + 3x + 4 \)

Simplify. (Lesson 5-9)

\[ \begin{array}{ccc}
\text{A} & 4; \; x = 3; & 3 \\
\text{B} & -1; & x = 1; \; 1 \\
\text{C} & 4; & x = -6; \; -6 \\
\end{array} \]

---

60. \( \frac{2i}{3 + i} \)

61. \( \frac{4}{5 - i} \)

62. \( \frac{1 + i}{3 - 2i} \)

Evaluate the determinant of each matrix. (Lesson 4-3)

\[ \begin{array}{ccc}
\text{A} & \begin{vmatrix} 8 & 5 & -2 \\
2 & -1 & -6 \\
5 & 0 & 3 \\
\end{vmatrix} & \text{B} \\
\text{C} & \begin{vmatrix} 6 & 5 & -2 \\
-3 & 0 & 6 \\
-3 & 1 & 4 \\
\end{vmatrix} & \text{C} \\
\text{D} & \begin{vmatrix} 6 & 5 & -2 \\
-3 & 0 & 6 \\
-3 & 1 & 4 \\
\end{vmatrix} \end{array} \]

---

66. **Community Service** A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised? (Lesson 3-4) $500

---

68. \( (x - 10)(x + 10) \)

69. \( (x - 7)(x + 4) \)

**Getting Ready for the Next Lesson**

(To review factoring trinomials, see Lesson 5-4.)

67. \( x^2 + 5x \) \( (x + 5) \)

68. \( x^2 - 100 \)

69. \( x^2 - 11x + 28 \)

70. \( x^2 - 18x + 81 \) \( (x - 9)^2 \)

71. \( 3x^2 + 8x + 4 \)

72. \( 6x^2 - 14x - 12 \)

---

**4 Assess**

**Open-Ended Assessment**

Modeling Have students draw parabolas in various positions and label them to show how many real roots they have and approximately where those roots occur.

**Getting Ready for Lesson 6-3**

**Prerequisite skill** Lesson 6-3 presents solving quadratic equations by factoring. Frequently this involves factoring a trinomial expression on one side of an equation. Exercises 67–72 should be used to determine your students’ familiarity with factoring trinomials.

**Assessment Options**

**Quiz** (Lessons 6-1 and 6-2) is available on p. 369 of the Chapter 6 Resource Masters.

**Answers**

57. \( f(x) = x^2 - 6x + 4 \)

58. \( f(x) = -4x^2 + 8x - 1 \)

59. \( f(x) = \frac{1}{4}x^2 + 3x + 4 \)
Graphing Calculator Investigation

Getting Started

Know Your Calculator When students use the procedure in Step 2 to copy the regression equation from Step 1 to the Y= list, the coefficients will have several more digits than the coefficients displayed on the home screen. The coefficients on the home screen are rounded versions of those in the Y= list.

Scientific Notation In Step 1, the value of the coefficient \(a\) is displayed as \(2.1035215 \times 10^{-4}\). Point out that this is how the calculator displays the scientific notation \(2.1035215 \times 10^{-4}\).

Teach

- Make sure students have cleared the L1 and L2 lists before entering new data. Also have them enter the WINDOW dimensions shown.
- For Step 1, point out that you can use the same keystrokes shown in Step 2, substituting 5 for the first 5, to select LinReg.
- If an error message appears in Step 2, have students clear the Y= list before trying Step 2 again.
- If students need to review entering data or selecting statistical plots, refer them to p. 87.
- Have students complete Exercises 1–4.

Assess

Ask students:

- What does it mean when the points on a scatter plot appear to lie along a curved path? The equation that best models the situation may be quadratic, and is probably not linear.

Modeling Real-World Data

You can use a TI-83 Plus to model data points whose curve of best fit is quadratic.

FALLING WATER Water is allowed to drain from a hole made in a 2-liter bottle. The table shows the level of the water \(y\) measured in centimeters from the bottom of the bottle after \(x\) seconds. Find and graph a linear regression equation and a quadratic regression equation. Determine which equation is a better fit for the data.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
<th>200</th>
<th>220</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water level (cm)</td>
<td>42.6</td>
<td>40.7</td>
<td>38.9</td>
<td>37.2</td>
<td>35.8</td>
<td>34.3</td>
<td>33.3</td>
<td>32.3</td>
<td>31.5</td>
<td>30.8</td>
<td>30.4</td>
<td>30.1</td>
</tr>
</tbody>
</table>

Step 1 Find a linear regression equation.
- Enter the times in L1 and the water levels in L2. Then find a linear regression equation.
  KEYS: Review lists and finding a linear regression equation on page 87.
- Graph a scatter plot and the regression equation.
  KEYS: Review graphing a regression equation on page 87.

Step 2 Find a quadratic regression equation.
- Find the quadratic regression equation. Then copy the equation to the Y= list and graph.
  KEYS: 5 ENTER Y=

The graph of the linear regression equation appears to pass through just two data points. However, the graph of the quadratic regression equation fits the data very well.

Exercises 1–4. See margin.

For Exercises 1–4, use the graph of the braking distances for dry pavement.

1. Find and graph a linear regression equation and a quadratic regression equation for the data. Determine which equation is a better fit for the data.
2. Use the CALC menu with each regression equation to estimate the braking distance at speeds of 100 and 150 miles per hour.
3. How do the estimates found in Exercise 2 compare?
4. How might choosing a regression equation that does not fit the data well affect predictions made by using the equation?

Average Braking Distance on Dry Pavement

Source: Missouri Department of Revenue

www.algebra2.com/other_calculator_keystrokes

Answers

1. See pp. 343A–343F.
2. linear: (100, 345), (150, 562); quadratic: (100, 440), (150, 990)
3. The quadratic estimates are much greater.
4. Sample answer: Choosing a model that does not fit the data well may cause inaccurate predictions when the data are very large or small.
Lesson 6-3
Solving Quadratic Equations by Factoring

What You’ll Learn
- Solve quadratic equations by factoring.
- Write a quadratic equation with given roots.

How is the Zero Product Property used in geometry?
The length of a rectangle is 5 inches more than its width, and the area of the rectangle is 24 square inches. To find the dimensions of the rectangle you need to solve the equation \( x(x + 5) = 24 \) or \( x^2 + 5x = 24 \).

SOLVE EQUATIONS BY FACTORING In the last lesson, you learned to solve a quadratic equation like the one above by graphing. Another way to solve this equation is by factoring. Consider the following products.

\[
\begin{align*}
7(0) &= 0 \\
0(-2) &= 0 \\
(6 - 6)(0) &= 0 \\
-4(-5 + 5) &= 0
\end{align*}
\]

Notice that in each case, at least one of the factors is zero. These examples illustrate the Zero Product Property.

<table>
<thead>
<tr>
<th>Key Concept</th>
<th>Zero Product Property</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Words</strong></td>
<td>For any real numbers ( a ) and ( b ), if ( ab = 0 ), then either ( a = 0 ), ( b = 0 ), or both ( a ) and ( b ) equal zero.</td>
</tr>
<tr>
<td><strong>Example</strong></td>
<td>If ( (x + 5)(x - 7) = 0 ), then ( x + 5 = 0 ) and/or ( x - 7 = 0 ).</td>
</tr>
</tbody>
</table>

**Example 1 Two Roots**

Solve each equation by factoring.

**a.** \( x^2 = 6x \)

\[
\begin{align*}
x^2 &= 6x \\
x^2 - 6x &= 0 \\
x(x - 6) &= 0 \\
x &= 0 \text{ or } x - 6 &= 0 \\
x &= 0 \text{ or } x &= 6
\end{align*}
\]

Zero Product Property

The solution set is \( \{0, 6\} \).

**CHECK** Substitute 0 and 6 for \( x \) in the original equation.

\[
\begin{align*}
(0)^2 &= 6(0) \\
6(6) &= 6(6) \\
0 &= 0 \\
36 &= 36
\end{align*}
\]

Resource Manager

**Workbook and Reproducible Masters**

- **Chapter 6 Resource Masters**
  - Study Guide and Intervention, pp. 325–326
  - Skills Practice, p. 327
  - Practice, p. 328
  - Reading to Learn Mathematics, p. 329
  - Enrichment, p. 330

**Transparencies**

- 5-Minute Check Transparency 6-3
- Real-World Transparency 6
- Answer Key Transparencies

**Technology**

- Interactive Chalkboard
Teach

SOLVE EQUATIONS BY FACTORING

In-Class Examples

Teaching Tip In Example 1a, some students may suggest solving the equation by dividing both sides by \( x \). Point out that this cannot be done because the value of \( x \) could be zero, and division by zero is undefined.

1 Solve each equation by factoring.
   a. \( x^2 = -4x \) \( \{0, -4\} \)
   b. \( 3x^2 = 5x + 2 \) \( \left\{-\frac{1}{3}, 2\right\} \)

2 Solve \( x^2 - 6x = -9 \) by factoring. \( \text{[3]} \)

Teaching Tip Point out that the term repeated root is sometimes used as a substitute for the term double root.

3 What is the positive solution of the equation \( 2x^2 - 8x - 42 = 0 \)?
   a. \(-3\) \( B \) \( 5 \)
   c. \(6\) \( D \) \( 7 \)

Teaching Tip Ask students why dividing each side of the equation in this example results in an equivalent equation, without the possibility of losing a root. (The right side of the equation is 0, not \( f(x) \) or \( y \), and dividing by 2 means that you can be sure that you are not dividing by zero.)

Write Quadratic Equations

In-Class Example

4 Write a quadratic equation with \(-\frac{2}{3}\) and 6 as its roots.
   Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a, b, \) and \( c \) are integers. Sample answer: \( 3x^2 - 16x - 12 = 0 \)

Example 2 Double Root

Solve \( x^2 - 16x + 64 = 0 \) by factoring.

\[
\begin{align*}
\text{Original equation} & : x^2 - 16x + 64 = 0 \\
(x - 8)(x - 8) & = 0 \\
\text{Factor.} & \\
x - 8 = 0 \quad \text{or} \quad x - 8 = 0 & \quad \text{Zero Product Property} \\
x = 8 & \quad x = 8 \\
\text{Solve each equation.} & \\
\text{The solution set is \( \{8\} \).}
\end{align*}
\]

CHECK The graph of the related function, \( f(x) = x^2 - 16x + 64 \), intersects the \( x \)-axis only once. Since the zero of the function is 8, the solution of the related equation is 8.

Example 3 Greatest Common Factor

Multiple-Choice Test Item

What is the positive solution of the equation \( 3x^2 - 3x - 60 = 0 \)?

| A | -4           | B | 2           | C | 5           | D | 10          |

Read the Test Item

You are asked to find the positive solution of the given quadratic equation. This implies that the equation also has a solution that is not positive. Since a quadratic equation can either have one, two, or no solutions, we should expect to find two solutions to this equation.

Solve the Test Item

Solve this equation by factoring. But before trying to factor \( 3x^2 - 3x - 60 \) into two binomials, look for a greatest common factor. Notice that each term is divisible by 3.

\[
\begin{align*}
3x^2 - 3x - 60 & = 0 \\
(3x^2 - x - 20) & = 0 \\
(x + 4)(x - 5) & = 0 \\
\text{Divide each side by 3.} & \\
x + 4 = 0 \quad \text{or} \quad x - 5 = 0 & \quad \text{Zero Product Property} \\
x = -4 & \quad x = 5 \\
\text{Solve each equation.} & \\
\text{Both solutions, -4 and 5, are listed among the answer choices. Since the question asked for the positive solution, the answer is C.}
\end{align*}
\]

Example 3 Point out to students that by reading the question carefully and noting exactly what is asked for (the positive solution), they can quickly eliminate answer choice A because it is negative.

Choice A is an attractive (though incorrect) choice because it is indeed a solution of the equation, just not the positive one.
**WRITE QUADRATIC EQUATIONS** You have seen that a quadratic equation of the form \((x - p)(x - q) = 0\) has roots \(p\) and \(q\). You can use this pattern to find a quadratic equation for a given pair of roots.

**Example 4** Write an Equation Given Roots

Write a quadratic equation with \(\frac{1}{2}\) and \(-5\) as its roots. Write the equation in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers.

\[
\begin{align*}
(x - p)(x - q) &= 0 \\
(x - \frac{1}{2})(x + 5) &= 0
\end{align*}
\]

**Check for Understanding**

**Concept Check**

1. **Write** the meaning of the Zero Product Property.
2. **OPEN ENDED** Choose two integers. Then, write an equation with those roots in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers.
3. **FIND THE ERROR** Lina and Kristin are solving \(x^2 + 2x = 8\).

A quadratic equation with roots \(\frac{1}{2}\) and \(-5\) and integral coefficients is \(2x^2 + 9x - 5 = 0\). You can check this result by graphing the related function.

**Guided Practice**

Solve each equation by factoring.

4. \(x^2 - 11x = 0\) \((0, 11)\)
5. \(x^2 + 6x = 16 = 0\) \((-8, 2)\)
6. \(x^2 = 49\) \((-7, 7)\)
7. \(x^2 + 9 = 6x\) \((3)\)
8. \(4x^2 - 13x = 12\) \((-\frac{3}{4}, 4)\)
9. \(5x^2 - 5x - 60 = 0\) \((-3, 4)\)

Write a quadratic equation with the given roots. Write the equation in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\), and \(c\) are integers.

10. \(-4, 7\)
11. \(\frac{1}{2}, \frac{4}{3}\)
12. \(-\frac{3}{5}, \frac{1}{3}\)
13. Which of the following is the sum of the solutions of \(x^2 - 2x - 8 = 0?\) \(D\)

\(-6\) \(\text{A}\) 
\(-4\) \(\text{B}\) 
\(-2\) \(\text{C}\) 
\(2\) \(\text{D}\)

**About the Exercises...**

**Organization by Objective**

- Solve Equations by Factoring: 14–33, 42–26
- Write Quadratic Equations: 34–41

**Odd/Even Assignments**

Exercises 14–41 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**

**Basic:** 15–31 odd, 35–43 odd, 47–68

**Average:** 15–43 odd, 47–68

**Advanced:** 14–44 even, 45–62 (optional: 63–68)

**All:** Practice Quiz 1 (1–5)

**Daily Intervention**

**FIND THE ERROR**

To show that neither factor on the left side of Lina’s second equation needs to be 8, ask students to name several pairs of numbers whose product is 8. Possible pairs are 2 and 4, 0.5 and 16, and \(-1\) and \(-8\).

**Differentiated Instruction**

**Visual/Spatial** Provide each student with a sheet of grid paper. Have students begin by first drawing a coordinate grid with two points on the \(x\)-axis plotted as the roots of a quadratic equation. Then ask students to draw several different parabolas that might be the graphs of different equations having those two points as their solutions. Point out that this demonstrates that the steps demonstrated in Example 4 yield just one of the possible equations having the given roots.
Solve each equation by factoring.

14. \( x^2 + 5x - 24 = 0 \)  
15. \( x^2 - 3x - 28 = 0 \)

16. \( x^2 = 25 \)  
17. \( x^2 = 81 \)

18. \( x^2 + 3x = 18 \)  
19. \( x^2 - 4x = 21 \)

20. \( 3x^2 = 5x \)  
21. \( 4x^2 = -3x \)  
22. \( x^2 + 36 = 12x \)  
23. \( x^2 + 64 = 16x \)

24. \( 4x^2 + 7x = 2 \)  
25. \( 4x^2 - 17x = -4 \)

26. \( x^2 + 8x = -3 \)  
27. \( 9x^2 + 30x = -16 \)

28. \( -2x^2 + 12x - 16 = 0 \)  
29. \( 16x^2 - 48x = -27 \)

30. \( -3x^2 - 6x + 9 = 0 \)  
31. \( 2x + 3y = 0 \)

32. Find the roots of \( x(x + 6)(x - 5) = 0 \).  
33. Solve \( x^2 = 9x \) by factoring.  
34. Solve \( x^2 = 9x \) by factoring.

Write a quadratic equation with the given roots. Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a, b, \) and \( c \) are integers.

34. \( x = 4, 5 \)  
35. \( x = -2, 7 \)  
36. \( x = -2, 14 \)

37. \( x = 2, 14 \)  
38. \( x = 3, 5 \)  
39. \( x = 3, 5 \)  
40. \( x = 3, 5 \)  
41. \( x = 3, 5 \)  
42. \( x = 3, 5 \)

43. NUMBER THEORY  
Find two consecutive even integers whose product is 224.  
44. PHOTOGRAPHY  
A rectangular photograph is 8 centimeters wide and 12 centimeters long. The photograph is enlarged by the length and width by an equal amount in order to double its area. What are the dimensions of the new photograph?  
45. Rewrite Doyle's formula for logs that are 16 feet long.  
46. Find the root(s) of the quadratic equation you wrote in Exercise 45. What do the root(s) tell you about the roots of which Doyle’s rule makes sense?  

47. CRITICAL THINKING  
For a quadratic equation of the form \( (x - p)(x - q) = 0 \), show that the axis of symmetry of the related quadratic function is located halfway between the \( x \)-intercepts \( p \) and \( q \).  

CRITICAL THINKING  
Find a value of \( k \) that makes each statement true.

48. \(-3\) is a root of \( 2x^2 + kx - 21 = 0 \).  
49. \( \frac{1}{2} \) is a root of \( 2x^2 + 11x = -k \).
50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 343A–343F.

How is the Zero Product Property used in geometry?

Include the following in your answer:

- an explanation of how to find the dimensions of the rectangle using the Zero Product Property, and
- why the equation \(x(x + 5) = 24\) is not solved by using \(x = 24\) and \(x + 5 = 24\).

51. Which quadratic equation has roots \(\frac{1}{2}\) and \(\frac{1}{3}\)?

A) \(x^2 - 5x - 2 = 0\)  
B) \(x^2 - 5x + 1 = 0\)  
C) \(6x^2 + 5x - 1 = 0\)  
D) \(6x^2 - 5x + 1 = 0\)

52. If the roots of a quadratic equation are 6 and -3, what is the equation of the axis of symmetry?

A) \(x = 1\)  
B) \(x = \frac{3}{2}\)  
C) \(x = \frac{1}{2}\)  
D) \(x = -2\)

### Maintain Your Skills

#### Mixed Review

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.  

- 53. \(f(x) = -x^2 - 4x + 5\)  
- 54. \(f(x) = 4x^2 + 4x + 1\)  
- 55. \(f(x) = 3x^2 - 10x - 4\)

56. Determine whether \(f(x) = 3x^2 - 12x - 7\) has a maximum or a minimum value. Then find the maximum or minimum value.  

- (Lesson 6-1 min.: -19)

- (Lesson 5-6)

- (Lesson 3-2)

- (To review simplifying radicals, see Lesson 5-5.)

57. \(\sqrt{3}\left(\sqrt{6} - 2\right)\)  
58. \(\sqrt{108} - \sqrt{48} + \left(\sqrt{3}\right)^3\)  
59. \((5 + \sqrt{8})^2\)

Solve each system of equations.

- 60. \(4x - 3b = -4\)  
- 61. \(2r + s = 1\)  
- 62. \(3x - 2y = -3\)

53. \(3x - 2b = -4\)  
54. \((-4, -4)\)  
55. \((3, -5)\)  
56. \(3x + y = 3\)

#### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Simplify.

- **Getting Ready for Lesson 6-4**

- 63. \(\sqrt{8}\)  
- 64. \(\sqrt{20}\)  
- 65. \(\sqrt{27}\)  
- 66. \(\sqrt{50}\)

**Practice Quiz 1**

### Lessons 6-1 through 6-3

1. Find the \(y\)-intercept, the equation of the axis of symmetry, and the \(x\)-coordinate of the vertex for \(f(x) = 3x^2 - 12x + 4\). Then graph the function by making a table of values.

- \(x = 2, 2\); See margin for graph.

2. Determine whether \(f(x) = 3 - x^2 + 5x\) has a maximum or minimum value. Then find this maximum or minimum value.

- \(x = \frac{37}{4}\) or \(9\frac{1}{4}\)

3. Solve \(2x^2 - 11x + 12 = 0\) by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

- \(x = \frac{11}{2}, 4\)

4. Solve \(2x^2 + 9x - 5 = 0\) by factoring.

- \(-5, \frac{1}{2}\)

5. Write a quadratic equation with roots \(-4\) and \(\frac{1}{3}\). Write the equation in the form \(ax^2 + bx + c = 0\), where \(a, b,\) and \(c\) are integers.

- \(3x^2 + 11x - 4 = 0\)

The axis of symmetry is the average of the \(x\)-intercepts. Therefore the axis of symmetry is located halfway between the \(x\)-intercepts.

47. \(y = (x - p)(x - q)\)

- \(y = x^2 - px - qx + pq\)

- \(a = 1, b = -(p + q), c = +pq\)

- axis of symmetry: \(x = -\frac{b}{2a}\)

- \(x = -\frac{(p + q)}{2(1)}\)

- \(x = \frac{p + q}{2}\)
Focus

5-Minute Check Transparency 6-4 Use as a quiz or review of Lesson 6-3.

Mathematical Background notes are available for this lesson on p. 284C.

Building on Prior Knowledge

In Lesson 6-3, students solved quadratic equations by factoring. In this lesson they use two other methods to solve equations: the Square Root Property and completing the square.

How can you find the time it takes an accelerating race car to reach the finish line?

Ask students:

• The number 121 is a perfect square. What is the square root of 121? 11
• How does the number 11 relate to the coefficient of the x term? The coefficient of x is twice 11.

Vocabulary

• completing the square

How

SQUARE ROOT PROPERTY You have solved equations like \( x^2 - 25 = 0 \) by factoring. You can also use the Square Root Property to solve such an equation. This method is useful with equations like the one above that describes the race car’s speed. In this case, the quadratic equation contains a perfect square trinomial set equal to a constant.

Example 1

Equation with Rational Roots

Solve \( x^2 + 10x + 25 = 49 \) by using the Square Root Property.

\[
\begin{align*}
x^2 + 10x + 25 &= 49 \\
(x + 5)^2 &= 49 \\
x + 5 &= \pm \sqrt{49} \\
x + 5 &= \pm 7 \\
x &= -5 \pm 7 \\
x &= -5 + 7 \quad \text{or} \quad x = -5 - 7 \\
x &= 2 \quad \text{or} \quad x = -12
\end{align*}
\]

The solution set is \( \{2, -12\} \). You can check this result by using factoring to solve the original equation.

Roots that are irrational numbers may be written as exact answers in radical form or as approximate answers in decimal form when a calculator is used.
Example 2  Equation with Irrational Roots

Solve $x^2 - 6x + 9 = 32$ by using the Square Root Property.

Original equation

$$(x - 3)^2 = 32$$

Factor the perfect square trinomial.

$x - 3 = ± √32$

Square Root Property

$x = 3 ± 4√2$

Add 3 to each side; $√32 = 4√2$

$x = 3 + 4√2$ or $x = 3 - 4√2$

Write as two equations.

$x = 8.7$ or $x = -2.7$

Use a calculator.

The exact solutions of this equation are $3 - 4√2$ and $3 + 4√2$. The approximate solutions are $-2.7$ and $8.7$. Check these results by finding and graphing the related quadratic function.

CHECK Use the ZERO function of a graphing calculator. The approximate zeros of the related function are $-2.7$ and $8.7$.

COMPLETE THE SQUARE   The Square Root Property can only be used to solve quadratic equations when the side containing the quadratic expression is a perfect square. However, few quadratic expressions are perfect squares. To make a quadratic expression a perfect square, a method called completing the square may be used.

In a perfect square trinomial, there is a relationship between the coefficient of the linear term and the constant term. Consider the pattern for squaring a sum.

$$(x + 7)^2 = x^2 + 2(7)x + 7^2$$

Square of a sum pattern

$= x^2 + 14x + 49$  Simplify.

$\frac{14}{2} \rightarrow 7^2$ Notice that $49$ is $7^2$ and $7$ is one-half of $14$.

You can use this pattern of coefficients to complete the square of a quadratic expression.

Teaching Tip In Example 1, point out that both constants in the equation, $25$ and $49$, are perfect squares.

1 Solve $x^2 + 14x + 49 = 64$ by using the Square Root Property. $(-15, 1)$

Teaching Tip Point out that the constant on the right side of the equation given in Example 2, is not a perfect square. Stress that this occurrence means the roots will be irrational numbers involving radicals. Also emphasize the use of the $±$ symbol in the step where the Square Root Property is utilized.

2 Solve $x^2 - 10x + 25 = 12$ by using the Square Root Property. $[5 ± 2\sqrt{3}]$
Example 3 Complete the Square

Find the value of \( c \) that makes \( x^2 + 12x + c \) a perfect square. Then write the trinomial as a perfect square.

Step 1 Find one half of 12. \( \frac{12}{2} = 6 \)
Step 2 Square the result of Step 1. \( 6^2 = 36 \)
Step 3 Add the result of Step 2 to \( x^2 + 12x \). \( x^2 + 12x + 36 \)

The trinomial \( x^2 + 12x + 36 \) can be written as \((x + 6)^2\).

You can solve any quadratic equation by completing the square. Because you are solving an equation, add the value you use to complete the square to each side.

Algebra Activity
Completing the Square

Use algebra tiles to complete the square for the equation \( x^2 + 2x - 3 = 0 \).

Step 1 Represent \( x^2 + 2x - 3 = 0 \) on an equation mat.

\[
\begin{array}{c}
\text{x}^2 \quad \text{x} \\
\frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{1}
\end{array} = 0
\]

\( x^2 + 2x - 3 = 0 \)

Step 2 Add 3 to each side of the mat. Remove the zero pairs.

\[
\begin{array}{c}
\text{x}^2 \quad \text{x} \\
\frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{1}
\end{array} = 0 + 3
\]

\( x^2 + 2x + 3 = 0 + 3 \)

Step 3 Begin to arrange the \( x^2 \) and \( x \) tiles into a square.

\[
\begin{array}{c}
\text{x}^2 \quad \text{x} \\
\frac{1}{1} \quad \frac{1}{1}
\end{array} = 3
\]

\( x^2 + 2x = 3 \)

Step 4 To complete the square, add 1 yellow 1 tile to each side. The completed equation is \( x^2 + 2x + 1 = 4 \) or \( (x+1)^2 = 4 \).

\[
\begin{array}{c}
\text{x}^2 \quad \text{x} \\
\frac{1}{1} \quad \frac{1}{1} \quad \frac{1}{1}
\end{array} = 1 + 1
\]

\( x^2 + 2x + 1 = 3 + 1 \)

Study Tip

Common Misconception
When solving equations by completing the square, don’t forget to add \( \left(\frac{b}{2}\right)^2 \) to each side of the equation.

Examples:
1. \( x^2 + 2x - 4 = 0 \) \( (x + 1)^2 = 5 \)
2. \( x^2 + 4x + 1 = 0 \) \( (x + 2)^2 = 3 \)
3. \( x^2 - 6x = -5 \) \( (x - 3)^2 = 4 \)
4. \( x^2 - 2x = -1 \) \( (x - 1)^2 = 0 \)

Example 4 Solve an Equation by Completing the Square

Solve \( x^2 + 8x - 20 = 0 \) by completing the square.

\[
x^2 + 8x - 20 = 0
\]
\[
x^2 + 8x = 20
\]
\[
x^2 + 8x + 16 = 20 + 16
\]
\[
(x + 4)^2 = 36
\]

Notice that \( x^2 + 8x - 20 \) is not a perfect square.
Rewrite so the left side is of the form \( x^2 + bx \).
Since \( \left(\frac{b}{2}\right)^2 = 16 \), add 16 to each side.
Write the left side as a perfect square by factoring.

Algebra Activity

Materials: algebra tiles, equation mat

- Ask students why the choice was made to add 3 unit tiles to each side of the equation mat in Step 2. Sample answer: In order to simplify the work arranging the tiles into a square in Step 3.
- Remind students that an \( x \) tile is \( x \) units long and 1 unit wide. Stress that the width is the same as the length of each side of a unit tile.
When the coefficient of the quadratic term is not 1, you must first divide the equation by that coefficient before completing the square.

**Example 5  Equation with a ≠ 1**

Solve $2x^2 - 5x + 3 = 0$ by completing the square.

\[
\begin{align*}
2x^2 &- 5x + 3 = 0 & \text{Notice that } 2x^2 - 5x + 3 \text{ is not a perfect square.} \\
x^2 &- \frac{5}{2}x + \frac{3}{2} = 0 & \text{Divide by the coefficient of quadratic term, } 2. \\
x^2 &- \frac{5}{2}x = -\frac{3}{2} & \text{Subtract } \frac{3}{2} \text{ from each side.} \\
x^2 &- \frac{5}{2}x + \frac{25}{16} = -\frac{3}{2} + \frac{25}{16} & \text{Since } \left( -\frac{5}{2} \right)^2 = \frac{25}{4} \text{ add } \frac{25}{16} \text{ to each side.} \\
\left(x - \frac{5}{4}\right)^2 & = \frac{16}{16} & \text{Write the left side as a perfect square by factoring.} \\
x - \frac{5}{4} & = \pm \frac{4}{4} & \text{Simplify the right side.} \\
x & = \frac{5}{4} \pm \frac{1}{4} & \text{Square Root Property} \\
x & = \frac{5}{4} + \frac{1}{4} \quad \text{or} \quad x = \frac{5}{4} - \frac{1}{4} & \text{Add } \frac{4}{4} \text{ to each side.} \\
x & = \frac{3}{2} & \text{Write as two equations.} \\
\end{align*}
\]

The solution set is \(\left\{1, \frac{3}{2}\right\}\).

Not all solutions of quadratic equations are real numbers. In some cases, the solutions are complex numbers of the form \(a + bi\), where \(b \neq 0\).

**Example 6  Equation with Complex Solutions**

Solve $x^2 + 4x + 11 = 0$ by completing the square.

\[
\begin{align*}
x^2 &+ 4x + 11 = 0 & \text{Notice that } x^2 + 4x + 11 \text{ is not a perfect square.} \\
x^2 &+ 4x = -11 & \text{Rewrite so the left side is of the form } x^2 + bx. \\
x^2 &+ 4x + 4 = -11 + 4 & \text{Since } \left(\frac{4}{2}\right)^2 = 4 \text{ add } 4 \text{ to each side.} \\
(x + 2)^2 & = -7 & \text{Write the left side as a perfect square by factoring.} \\
x + 2 & = \pm \sqrt{-7} & \text{Square Root Property} \\
\sqrt{-1} & = i & \text{Simplify the right side.} \\
x & = -2 \pm i\sqrt{7} & \text{Subtract } 2 \text{ from each side.} \\
\end{align*}
\]

The solution set is \(\left\{-2 + i\sqrt{7}, -2 - i\sqrt{7}\right\}\). Notice that these are imaginary solutions.

**CHECK** A graph of the related function shows that the equation has no real solutions since the graph has no x-intercepts. Imaginary solutions must be checked algebraically by substituting them in the original equation.

---

**In-Class Examples**

**Teaching Tip** Some students may notice that the left side of the equation in Example 5 can be factored into the product of two binomials: \((2x - 3)(x - 1)\). Then the Zero Product Property can be used to obtain the same solutions. This is a good time to point out that more than one method of solution is often possible when solving a quadratic equation.

5 Solve $3x^2 - 2x - 1 = 0$ by completing the square. \(\left\{-\frac{1}{3}, 1\right\}\)

6 Solve $x^2 + 2x + 3 = 0$ by completing the square. \(\{-1 \pm i\sqrt{2}\}\)

**Teaching Tip** Ask students to name three possible ways to check the solutions to an equation that they have solved by completing the square. graphing

the related function, factoring the equation, substituting the solutions into the original equation.

---

**Differentiated Instruction**

**Kinesthetic** Have students work with algebra tiles to help them write five equations that can be solved by completing the square. Provide each student with one \(x^2\) tile, several \(x\) tiles, and several unit tiles. Have students begin by creating a square arrangement of their tiles and then work backwards through the steps shown in the Algebra Activity on p. 308 to find a quadratic equation. After students have written their five equations, ask them to trade their equations with another student and then use their algebra tiles to find the solutions of the equations they receive.

---

**Lesson 6-4 Completing the Square** 309
Chapter 6
Quadratic Functions and Inequalities

1. Completing the square allows you to rewrite one side of a quadratic equation in the form of a perfect square. Once in this form, the equation is solved by using the Square Root Property.

2. Never; see margin for explanation.

3. Before completing the square, you must first check to see that the coefficient of the quadratic term is 1. If it is not, you must first divide the equation by that coefficient.
23. ENGINEERING In an engineering test, a rocket sled is propelled into a 10,000-foot track for target. The sled’s distance d in meters from the target is given by the formula d = −1.5t² + 120, where t is the number of seconds after rocket ignition. How many seconds have passed since rocket ignition when the sled is 10 meters from the target? About 8.56 s

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

24. x² + 16x + c 64; (x + 8)²
25. x² − 18x + c 81; (x − 9)²
26. x² − 15x + c 225; (x − 15/2)²
27. x² + 7x + c 49; (x + 7/2)²
28. x² + 0.6x + c 0.09; (x + 0.3)²
29. x² − 2.4x + c 1.44; (x − 1.2)²
30. x² − 8/3x + c 16; (x − 4/3)²

Solve each equation by completing the square.

32. x² − 8x + 15 = 0 [3, 5]
33. x² + 2x − 120 = 0 [−12, 10]
34. x² + 2x − 6 = 0 [−1 ± √7]
35. x² − 4x + 1 = 0 [2 ± √3]
36. x² − 4x + 5 = 0 [2 ± i]
37. x² + 6x + 13 = 0 [−3 ± 2i]
38. 2x² + 3x − 5 = 0 [−5/2, 1]
39. 2x² − 3x + 1 = 0 [1/2, 1]
40. 3x² − 5x + 1 = 0
41. 3x² − 4x − 2 = 0
42. 2x² − 7x + 12 = 0 [−3, 4]
43. 3x² + 5x + 4 = 0
44. x² + 1.4x = 1.2 [−0.6, 0.2]
45. x² − 4.7x = −2.8 [0.7, 4]
46. x² − 2/3x − 26/9 = 0 [1/3 ± √3]
47. x² − 3/2x − 23/16 = 0 [3/4 ± √2]

48. FRAMING A picture has a square frame that is 2 inches wide. The area of the picture is one-third of the total area of the picture and frame. What are the dimensions of the picture to the nearest quarter of an inch? 5 1/2 in. by 5 1/2 in.

GOLDEN RECTANGLE For Exercises 49–51, use the following information.
A golden rectangle is one that can be divided into a square and a second rectangle that is geometrically similar to the original rectangle. The ratio of the length of the longer side to the shorter side of a golden rectangle is called the golden ratio.

49. Find the ratio of the length of the longer side to the length of the shorter side for rectangle ABCD and for rectangle EBCF.
50. Find the exact value of the golden ratio by setting the two ratios in Exercise 49 equal and solving for x. (Hint: The golden ratio is a positive value.)
51. RESEARCH Use the Internet or other reference to find examples of the golden rectangle in architecture. What applications does the reciprocal of the golden ratio have in music? See margin.

52. CRITICAL THINKING Find all values of n such that x² + bx + [b/2]² = n has
a. one real root, n ≠ 0  
b. two real roots, n > 0  
c. two imaginary roots, n < 0

Lesson 6-4 Completing the Square 311

51. Sample answers: The golden rectangle is found in much of ancient Greek architecture, such as the Parthenon, as well as in modern architecture, such as in the windows of the United Nations building. Many songs have their climax at a point occurring 61.8% of the way through the piece, with 0.618 being about the reciprocal of the golden ratio. The reciprocal of the golden ratio is also used in the design of some violins.

Enrichment, p. 336

The Golden Quadratic Equations
A golden rectangle has the property that its length can be written as a + b, where a is the width of the rectangle and a² + b² = c². Any golden rectangle can be divided into a square and a smaller golden rectangle, as shown.

The properties used to solve golden rectangles can be used to derive two quadratic equations. These are sometimes called golden quadratic equations.

Solve each problem.
1. In the proportion for the golden rectangle, let x = 1. Write the resulting quadratic equation and solve for b.

b² + b − 1 = 0
b = −1 ± √5

Readings and Learning Materials, p. 335

Pre-Activity How can you find the time it takes an accelerating race car to reach the finish line?
Read the introduction to Lesson 6-4 at the top of page 306 in your textbook.
Explain what it means to say that the driver accelerated at a constant rate of 8 ft per second squared.
If the driver is traveling at a certain speed at a particular moment, then one second later, the driver is traveling 8 feet per second faster.

Reading the Lesson
1. Give the reason for each step in the following solution of an equation by using the Square Root Property:
   [In the proportion for the golden rectangle, let x = 1. Write the resulting quadratic equation and solve for b.]
   a. ± 1 ± √5
   b. −1 ± √5
   c. −1 ± √5

2. Explain how you can find the constant that would need to be added to make a binomial into a perfect square trinomial.
Sample answer: Find half of the coefficient of the linear term and square it.

3. Divide the equation by 3.

4. What is the first step in solving the equation x² + 7x + 12 = 0 by completing the square?

5. Solve each equation by completing the square.

6. What is the first step in solving the equation x² + 4x = −12 by completing the square?

Helping You Remember
Here are the rules for squaring a binomial to help you remember the procedure for changing a binomial into a perfect square trinomial.

One of the rules for squaring a binomial is (a + b)² = a² + 2ab + b². In completing the square, you are starting with a² + bx and need to find b². This shows you that b = 2y or y = ±b/2. That is why you must take half of the coefficient and square it to get the constant that must be added to complete the square.

Lesson 6-4 Completing the Square 311
53. **KENNEL**  A kennel owner has 164 feet of fencing with which to enclose a rectangular region. He wants to subdivide this region into three smaller rectangles of equal length, as shown. If the total area to be enclosed is 576 square feet, find the dimensions of the entire enclosed region. \(\text{Hint: Write an expression for } \ell \text{ in terms of } w.\) \(\text{Area} \leq 32 \text{ ft or 64 \text{ ft by 9 ft}}\)

54. **Writing in Math**  Answer the question that was posed at the beginning of the lesson. \See margin.

**How can you find the time it takes an accelerating race car to reach the finish line?**

Include the following in your answer:

- an explanation of why \(t^2 + 22t + 121 = 246\) cannot be solved by factoring, and

- a description of the steps you would take to solve the equation \(t^2 + 22t + 121 = 246.\)

55. What is the absolute value of the product of the two solutions for \(x\) in \(x^2 - 2x - 2 = 0?\)

- a) \(-1\)
- b) \(0\)
- c) \(1\)
- d) \(2\)

56. For which value of \(c\) will the roots of \(x^2 + 4x + c = 0\) be real and equal? \(\text{D}\)

- a) \(1\)
- b) \(2\)
- c) \(3\)
- d) \(4\)
- e) \(5\)

---

**Getting Ready for Lesson 6-5**

**PREREQUISITE SKILL**  Lesson 6-5 presents the Quadratic Formula. The first step in evaluating the formula is to evaluate the expression under the radical sign. Use Exercises 69–72 to determine your students’ familiarity with evaluating expressions.

**Assessment Options**

- **Quiz (Lessons 6-3 and 6-4)** is available on p. 369 of the Chapter 6 Resource Masters.
- **Mid-Chapter Test (Lessons 6-1 through 6-4)** is available on p. 371 of the Chapter 6 Resource Masters.

---

**Maintain Your Skills**

**Mixed Review**

Write a quadratic equation with the given root(s). Write the equation in the form \(ax^2 + bx + c = 0,\) where \(a, b,\) and \(c\) are integers. \(\text{(Lesson 6-3)}\)

- 57. \(2, 1\)
- 58. \(-3, 9\)
- 59. \(6, \frac{1}{3}\)
- 60. \(-3, \frac{3}{4}\)

\(x^2 - 3x + 2 = 0 \quad x^2 - 6x - 27 = 0 \quad 3x^2 - 19x + 6 = 0 \quad 12x^2 + 13x + 3 = 0\)

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. \(\text{(Lesson 6-2)}\)

- 61. between \(-4\) and \(-3;\) between \(0\) and \(1\)
- 62. \(x^2 + 48 = 14x\)
- 63. \(2x^2 + 11x = -12\)
- 64. Write the seventh root of 5 cubed using exponents. \(\text{(Lesson 5-7)}\) \(\sqrt[7]{5^3}\)

Solve each system of equations by using inverse matrices. \(\text{(Lesson 4-8)}\)

- 65. \(5x + 3y = -5\)
- 66. \(6x + 5y = 8\)
- 7x + 5y = -11 \((2, -5)\)
- 3x - y = 7 \((43, -6)\)

**CHEMISTRY**  For Exercises 67 and 68, use the following information.

For hydrogen to be a liquid, its temperature must be within 2°C of \(-257°C.\) \(\text{(Lesson 1-4)}\)

- 67. Write an equation to determine the greatest and least temperatures for this substance. \(x - (-257) = 2\)
- 68. Solve the equation. greatest: \(-255°C;\) least: \(-259°C\)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**  Evaluate \(b^2 - 4ac\) for the given values of \(a, b,\) and \(c.\)

\(\text{(To review evaluating expressions, see Lesson 1-1.)}\)

- 69. \(a = 1, b = 7, c = 3\)
- 70. \(a = 1, b = 2, c = 5\)
- 71. \(a = 2, b = -9, c = -5\)
- 72. \(a = 4, b = -12, c = 9\)

---

**Answer**

54. To find the distance traveled by the accelerating race car in the given situation, you must solve the equation \(t^2 + 22t + 121 = 246\) or \(t^2 + 22t - 125 = 0.\) Answers should include the following.

- Since the expression \(t^2 + 22t - 125\) is prime, the solutions of \(t^2 + 22t + 121 = 246\) cannot be obtained by factoring.
- Rewrite \(t^2 + 22t + 121\) as \((t + 11)^2.\) Solve \((t + 11)^2 = 246\) by applying the Square Root Property. Then, subtract 11 from each side. Using a calculator, the two solutions are about 4.7 or \(-26.7.\) Since time cannot be negative, the driver takes about 4.7 seconds to reach the finish line.
The Quadratic Formula and the Discriminant

**What You’ll Learn**
- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number and type of roots of a quadratic equation.

**Vocabulary**
- Quadratic Formula
- Discriminant

**How is blood pressure related to age?**
As people age, their arteries lose their elasticity, which causes blood pressure to increase. For healthy women, average systolic blood pressure is estimated by 
$$P = 0.01A^2 + 0.05A + 107,$$
where \(P\) is the average blood pressure in millimeters of mercury (mm Hg) and \(A\) is the person’s age. For healthy men, average systolic blood pressure is estimated by 
$$P = 0.006A^2 - 0.02A + 120.$$

**QUADRATIC FORMULA** You have seen that exact solutions to some quadratic equations can be found by graphing, by factoring, or by using the Square Root Property. While completing the square can be used to solve any quadratic equation, the process can be tedious if the equation contains fractions or decimals. Fortunately, a formula exists that can be used to solve any quadratic equation of the form 
$$ax^2 + bx + c = 0.$$ This formula can be derived by solving the general form of a quadratic equation.

$$\begin{align*}
ax^2 + bx + c &= 0 \\
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\
x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= -\frac{c}{a} + \frac{b^2}{4a^2} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
x + \frac{b}{2a} &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
\end{align*}$$

This equation is known as the **Quadratic Formula**.

**Key Concept**

The solutions of a quadratic equation of the form 
$$ax^2 + bx + c = 0,$$ where \(a \neq 0\), are given by the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Workbook and Reproducible Masters**
- **Chapter 6 Resource Masters**
  - Study Guide and Intervention, pp. 337–338
  - Skills Practice, p. 339
  - Practice, p. 340
  - Reading to Learn Mathematics, p. 341
  - Enrichment, p. 342
- **Graphing Calculator and Spreadsheet Masters**, p. 38

**Resource Manager**
- **Transparencies**
  - 5-Minute Check Transparency 6-5
  - Answer Key Transparencies
- **Technology**
  - Alge2PASS: Tutorial Plus, Lessons 11, 12
  - Interactive Chalkboard
**QUADRATIC FORMULA**

**In-Class Examples**

1. Solve $x^2 - 8x = 33$ by using the Quadratic Formula. $-3, 11$

**Teaching Tip** Encourage students to write down the values of $a$, $b$, and $c$ from the standard form of the quadratic equation before they begin substituting into the formula.

2. Solve $x^2 - 34x + 289 = 0$ by using the Quadratic Formula. $17$

**Study Tip**

**Example 1** Two Rational Roots

Solve $x^2 - 12x = 28$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify $a$, $b$, and $c$.

$$ax^2 + bx + c = 0$$

$$x^2 - 12x - 28 = 0$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)}$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{2}$$

$$x = \frac{12 \pm \sqrt{256}}{2}$$

$$x = \frac{12 \pm 16}{2}$$

Write as two equations.

$$x = \frac{12 + 16}{2} \text{ or } x = \frac{12 - 16}{2}$$

Simplify.

$$x = \frac{28}{2} \text{ or } x = \frac{-4}{2}$$

$$x = 14 \text{ or } x = -2$$

The solutions are $-2$ and $14$. Check by substituting each of these values into the original equation.

When the value of the radicand in the Quadratic Formula is $0$, the quadratic equation has exactly one rational root.

**Example 2** One Rational Root

Solve $x^2 + 22x + 121 = 0$ by using the Quadratic Formula.

Identify $a$, $b$, and $c$. Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-22 \pm \sqrt{(22)^2 - 4(1)(121)}}{2(1)}$$

$$x = \frac{-22 \pm \sqrt{0}}{2}$$

Simplify.

$$x = \frac{-22}{2} = -11$$

The solution is $-11$.

**CHECK** A graph of the related function shows that there is one solution at $x = -11$.

---

**Teacher to Teacher**

Lori Haldorson & Cathy Hokkanen  
Blaine H.S., Blaine, MN

“To help students memorize the Quadratic Formula, we sing it to the tune of ‘Pop Goes the Weasel.’”
You can express irrational roots exactly by writing them in radical form.

**Example 3 Irrational Roots**

Solve \(2x^2 + 4x - 5 = 0\) by using the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Quadratic Formula

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}
\]

Replace \(a\) with 2, \(b\) with 4, and \(c\) with -5.

\[
x = \frac{-4 \pm \sqrt{56}}{4}
\]

Simplify.

\[
x = \frac{-4 \pm 2\sqrt{14}}{4} \quad \text{or} \quad \frac{-2 \pm \sqrt{14}}{2}
\]

The exact solutions are \(-2 - \sqrt{14} / 2\) and \(-2 + \sqrt{14} / 2\). The approximate solutions are -2.9 and 0.9.

**CHECK** Check these results by graphing the related quadratic function, 
\(y = 2x^2 + 4x - 5\). Using the ZERO function of a graphing calculator, the approximate zeros of the related function are -2.9 and 0.9.

When using the Quadratic Formula, if the radical contains a negative value, the solutions will be complex. Complex solutions always appear in conjugate pairs.

**Example 4 Complex Roots**

Solve \(x^2 - 4x = -13\) by using the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Quadratic Formula

\[
x = \frac{4 \pm \sqrt{-36}}{2}
\]

Simplify.

\[
x = \frac{4 \pm 6i}{2}
\]

\[
x = 2 \pm 3i
\]

The solutions are the complex numbers \(2 + 3i\) and \(2 - 3i\).

A graph of the related function shows that the solutions are complex, but it cannot help you find them.

**In-Class Examples**

**Teaching Tip** Some students may have commented that the equations in Examples 1 and 2 could have been solved by factoring. Before beginning Example 3, stress that many quadratic equations cannot easily be solved by factoring. State that the quadratic equation presented in Example 3 is such an equation. Emphasize that the Quadratic Formula provides a way to find the roots for any quadratic equation.

3 Solve \(x^2 - 6x + 2 = 0\) by using the Quadratic Formula.

\[3 \pm \sqrt{7}, \text{or approximately}\ 0.4\ \text{and}\ 5.6\]

**Teaching Tip** Remind students that conjugate pairs are two complex numbers of the form \(a + bi\) and \(a - bi\).

4 Solve \(x^2 + 13 = 6x\) by using the Quadratic Formula.

\[3 \pm 2i\]

**Concept Check**

**Real Roots or Imaginary Roots**

Ask students to look back at the first four examples in this lesson and see how they might predict whether the roots will be real or imaginary. If the values of \(a\) and \(c\) have the same sign and 4 times their product is greater than the square of the value of \(b\), then the roots will be imaginary.
ROOTS AND THE DISCRIMINANT

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. \( x^2 + 6x + 9 = 0 \) 
   0; one rational root

b. \( x^2 + 3x + 5 = 0 \) 
   \(-11\); two complex roots

c. \( x^2 + 8x - 4 = 0 \) 
   \(80\); two irrational roots

d. \( x^2 - 11x + 10 = 0 \) 
   \(81\); two rational roots

**Study Tip**

Remember that the solutions of an equation are called roots.

**Key Concept**

Consider \( ax^2 + bx + c = 0 \).

<table>
<thead>
<tr>
<th>Value of Discriminant</th>
<th>Type and Number of Roots</th>
<th>Example of Graph of Related Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b^2 - 4ac &gt; 0 ); ( b^2 - 4ac ) is a perfect square</td>
<td>( 2 ) real, rational roots</td>
<td><img src="https://example.com/graph1.png" alt="Graph 1" /></td>
</tr>
<tr>
<td>( b^2 - 4ac &gt; 0 ); ( b^2 - 4ac ) is not a perfect square</td>
<td>( 2 ) real, irrational roots</td>
<td><img src="https://example.com/graph2.png" alt="Graph 2" /></td>
</tr>
<tr>
<td>( b^2 - 4ac = 0 )</td>
<td>( 1 ) real, rational root</td>
<td><img src="https://example.com/graph3.png" alt="Graph 3" /></td>
</tr>
<tr>
<td>( b^2 - 4ac &lt; 0 )</td>
<td>( 2 ) complex roots</td>
<td><img src="https://example.com/graph4.png" alt="Graph 4" /></td>
</tr>
</tbody>
</table>

**Example 5** Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. \( 9x^2 - 12x + 4 = 0 \)
   \( a = 9 \), \( b = -12 \), \( c = 4 \)
   \( b^2 - 4ac = (-12)^2 - 4(9)(4) \)
   \( = 144 - 144 \)
   \( = 0 \)

The discriminant is 0, so there is one rational root.

b. \( 2x^2 + 16x + 33 = 0 \)
   \( a = 2 \), \( b = 16 \), \( c = 33 \)
   \( b^2 - 4ac = (16)^2 - 4(2)(33) \)
   \( = 256 - 264 \)
   \( = -8 \)

The discriminant is negative, so there are two complex roots.

**CHECK** To check complex solutions, you must substitute them into the original equation. The check for \( 2 + 3i \) is shown below.

\[
\begin{align*}
\text{Original equation} & : x^2 - 4x = -13 \\
(2 + 3i)^2 - 4(2 + 3i) & = -13 \\
4 + 12i + 9i^2 - 8 - 12i & = -13 \\
-4 + 9i^2 & = -13 \\
-4 - 9 & = -13 \sqrt{-1} \\
\end{align*}
\]

**Logical Differentiated Instruction**

Have students use their classification skills to create a classroom poster listing the four different types of roots that can result when solving a quadratic equation. Each listing should include a sample equation that results in that type of roots and an explanation of how the value of the discriminant is indicative of the root type. Graphs like those shown on p. 316 can be added to the poster.
You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

### Concept Summary

<table>
<thead>
<tr>
<th>Method</th>
<th>Can be Used</th>
<th>When to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphing</td>
<td>sometimes</td>
<td>Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.</td>
</tr>
<tr>
<td>Factoring</td>
<td>sometimes</td>
<td>Use if the constant term is 0 or if the factors are easily determined. Example ( x^2 - 3x = 0 )</td>
</tr>
<tr>
<td>Square Root Property</td>
<td>sometimes</td>
<td>Use for equations in which a perfect square is equal to a constant. Example ( (x + 13)^2 = 9 )</td>
</tr>
<tr>
<td>Completing the Square</td>
<td>always</td>
<td>Useful for equations of the form ( x^2 + bx + c = 0 ), where ( b ) is even. Example ( x^2 + 14x - 9 = 0 )</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>always</td>
<td>Useful when other methods fail or are too tedious. Example ( 3.4x^2 - 2.5x + 7.9 = 0 )</td>
</tr>
</tbody>
</table>

You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

### Concept Check

1. **OPEN ENDED** Sketch the graph of a quadratic equation whose discriminant is a. positive. b. negative. c. zero. a–c. See margin.

2. Explain why the roots of a quadratic equation are complex if the value of the discriminant is less than 0.

3. Describe the relationship that must exist between \( a, b, \) and \( c \) in the equation \( ax^2 + bx + c = 0 \) in order for the equation to have exactly one solution. \( b^2 - 4ac \) must equal 0.

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant. 4–7. See margin.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

4. \( 8x^2 + 18x - 5 = 0 \)

5. \( 2x^2 - 4x + 1 = 0 \)

6. \( 4x^2 + 4x + 1 = 0 \)

7. \( x^2 + 3x + 8 = 5 \)

### Check for Understanding

### Guided Practice

<table>
<thead>
<tr>
<th>GUIDED PRACTICE KEY</th>
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<tr>
<td>Exercises</td>
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<tr>
<td>4–7</td>
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<td>8–11</td>
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<tr>
<td>12, 13</td>
</tr>
</tbody>
</table>

Lesson 6-5 The Quadratic Formula and the Discriminant 317
Solve each equation using the method of your choice. Find exact solutions.
8. \( x^2 + 8x = 0 \)  
9. \( x^2 + 5x + 6 = 0 \)  
10. \( x^2 - 2x - 2 = 0 \)  

**PHYSICS** For Exercises 12 and 13, use the following information.
The height \( h(t) \) in feet of an object \( t \) seconds after it is propelled straight up from the ground with an initial velocity of 85 feet per second is modeled by \( h(t) = -16t^2 + 85t \).
12. When will the object be at a height of 50 feet?
13. Will the object ever reach a height of 120 feet? Explain your reasoning.

**No; see margin for explanation.**

**Applying the Quadratic Formula and the Discriminant**

**Lesson 6-5**

**QUADRATIC EQUATIONS** To solve quadratic equations, use the quadratic formula.

**The Quadratic Formula**

If a quadratic equation is in the form \( ax^2 + bx + c = 0 \), then the solutions are given by:

\[
 x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Skills Practice, p. 339 and Practice, p. 340 (shown)**

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

**Reading to Learn Mathematics**

The Golden Gate, located in San Francisco, California, is the tallest bridge in the world, with its towers extending 746 feet above the water and the floor of the bridge extending 220 feet above water.

Source: [www.goldengatebridge.org](http://www.goldengatebridge.org)

**Bridges**

The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by the quadratic function \( y = 0.00012x^2 + 6 \), where \( x \) represents the distance from the axis of symmetry and \( y \) represents the height of the parabola. The related quadratic equation is \( 0.00012x^2 + 6 = 0 \).

40. Calculate the value of the discriminant. \( \Delta \approx -0.00288 \)

41. What does the discriminant tell you about the supporting cables of the Golden Gate Bridge? **See pp. 343A–343F.**

**FOOTBALL** For Exercises 42 and 43, use the following information.

The average NFL salary \( A(t) \) (in thousands of dollars) from 1975 to 2000 can be estimated using the function \( A(t) = 2.34t - 12.4t + 73.7 \), where \( t \) is the number of years since 1975.

42. Determine a domain and range for which this function makes sense.

43. According to this model, in what year did the average salary first exceed 1 million dollars? **1998**

**Online Research Data Update** What is the current average NFL salary? How does this average compare with the average given by the function used in Exercises 42 and 43? Visit [www.algebra2.com/data_update](http://www.algebra2.com/data_update) to learn more.
4. **HIGHWAY SAFETY** Highway safety engineers can use the formula $d = 0.05s^2 + 1.10s$ to estimate the minimum stopping distance $d$ in feet for a vehicle traveling $s$ miles per hour. If a car is able to stop after 125 feet, what is the fastest it could have been traveling when the driver first applied the brakes? About 40.2 mph.

5. **CRITICAL THINKING** Find all values of $k$ such that $x^2 - kx + 9 = 0$ has
   a. one real root. $k = \pm 6$
   b. two real roots. $k < -6$ or $k > 6$
   c. no real roots. $-6 < k < 6$

6. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 343A–343F.

   How is blood pressure related to age?

   Include the following in your answer:
   • an expression giving the average systolic blood pressure for a person of your age, and
   • an example showing how you could determine $A$ in either formula given a specific value of $P$.

47. If $2x^2 - 5x - 9 = 0$, then $x$ could equal which of the following? D
   A $1.12$  B $1.54$  C $2.63$  D $3.71$

48. Which best describes the nature of the roots of the equation $x^2 - 3x + 4 = 0$? C
   A real and equal  B real and unequal  C complex  D real and complex

**Standardized Test Practice**

**Mixed Review** Solve each equation by using the Square Root Property. (Lesson 6-4)

49. $x^2 + 18x + 81 = 25$

50. $x^2 - 8x + 16 = 7$

51. $4x^2 - 4x + 1 = 8$

   $-14, -4$

Solve each equation by factoring. (Lesson 6-3)

52. $4x^2 + 8x = 0$

53. $x^2 - 5x = 14$

54. $3x^2 + 10 = 17x$

   $2, 5$

Simplify. (Lesson 5-5)

55. $\sqrt{a^8b^{20}}$

56. $\sqrt[12]{q^2}$

57. $\sqrt[6]{4b^6}c^2$

58. **ANIMALS** The fastest-recorded physical action of any living thing is the wing beat of the common midge. This tiny insect normally beats its wings at a rate of 133,000 times per minute. At this rate, how many times would the midge beat its wings in an hour? Write your answer in scientific notation. (Lesson 5-1)

   $7.98 \times 10^6$

Solve each system of inequalities. (Lesson 3-3)

59. $x + y \leq 9$

60. $x \geq 1$

$x - y \leq 3$

$y \leq -1$

$y - x \geq 4$

$y \leq x$

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** State whether each trinomial is a perfect square. If it is, factor it. (To review perfect square trinomials, see Lesson 5-4.)

61. $x^2 - 5x - 10$ 62. $x^2 - 14x + 49$

63. $4x^2 + 12x + 9$ 64. $25x^2 + 20x + 4$

65. $9x^2 - 12x + 16$ 66. $36x^2 - 60x + 25$

   yes; $(2x + 3)^2$

   yes; $(5x + 2)^2$

   yes; $(6x - 5)^2$

   no

67. $x^2 - 4x - 5$

68. $x^2 - 9x + 8$

   yes; $(x - 9)^2$

   yes; $(x - 8)^2$

   yes; $(x - 4)^2$

69. $2x^2 - 2x - 12$

70. $3x^2 + 4x - 15$

   yes; $(2x - 3)^2$

   yes; $(3x + 5)^2$

   yes; $(x - 3)^2$

71. $6x^2 - 12x + 2$

72. $-3x^2 + 6x - 2$

   yes; $(3x - 2)^2$

   yes; $(2x - 3)^2$

   yes; $(3x + 2)^2$

73. $4x^2 + 4x - 16$

74. $2x^2 - 12x + 24$

   yes; $(2x - 4)^2$

   yes; $(4x - 6)^2$

   yes; $(2x + 3)^2$

**Answers**

14a. 21  16a. −16
14b. 2 irrational  16b. 2 complex
14c. $-3 \pm \sqrt{21}$  16c. $1 \pm 2i$
21a. −23  17a. 2 complex
15a. 240  17b. 2 complex
15b. 2 irrational  17b. $1 \pm i/2$
15c. $8 \pm 2\sqrt{15}$  19a. 49
18a. 121  19b. 2 rational
18b. 2 rational  19c. $-2, \frac{1}{3}$
18c. $-\frac{1}{2}, \frac{2}{3}$  20a. 20
20a. 20  20b. 2 irrational
20b. 2 irrational  20c. $-2 \pm \sqrt{5}$
21a. 24  21b. 2 irrational
21b. 2 irrational  21c. $-1 \pm \sqrt{6}$
27a. 1.48  27b. 2 irrational
27b. 2 irrational  27c. $-1 \pm 2\sqrt{0.37}$

4. **Assess**

**Open-Ended Assessment**

**Modeling** Ask students to sketch graphs of parabolas that illustrate each of the four types of roots for quadratic equations. Have them label each sketch with the type of value the discriminant of the corresponding quadratic equation would have.

**Getting Ready for Lesson 6-6**

**PREREQUISITE SKILL** Lesson 6-6 presents the analysis of the graphs of quadratic functions. To graph a quadratic function, it is helpful if the function is written in vertex form, which often requires students to complete the square. Recognition of perfect square trinomials is an important part of completing the square. Exercises 61–66 should be used to determine your students’ familiarity with perfect square trinomials.

**Answers**

22a. 0  22b. one rational
22c. $\frac{1}{3}$  23a. 0
23b. one rational  23c. $-\frac{5}{2}$
24a. $-\frac{3}{1}$  24b. 2 complex
24c. $\frac{9 \pm i\sqrt{31}}{8}$  25a. −135
25b. 2 complex  25c. $-1 \pm i\sqrt{15}$
26a. $\frac{28}{9}$  26b. 2 irrational
26c. $2 \pm 4\sqrt{7}$

27a. 1.48  27b. 2 irrational
27c. $-1 \pm 2\sqrt{0.37}$

0.8

Lesson 6-5 The Quadratic Formula and the Discriminant 319
Families of Parabolas

The general form of a quadratic equation is \( y = a(x - h)^2 + k \). Changing the values of \( a, h, \) and \( k \) results in a different parabola in the family of quadratic functions. You can use a TI-83 Plus graphing calculator to analyze the effects that result from changing each of these parameters.

**Example 1**

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

\[
y = x^2, \quad y = x^2 + 3, \quad y = x^2 - 5
\]

The graphs have the same shape, and all open up. The vertex of each graph is on the \( y \)-axis. However, the graphs have different vertical positions.

Example 1 shows how changing the value of \( k \) in the equation \( y = a(x - h)^2 + k \) translates the parabola along the \( y \)-axis. If \( k > 0 \), the parabola is translated \( k \) units up, and if \( k < 0 \), it is translated \( k \) units down.

How do you think changing the value of \( h \) will affect the graph of \( y = x^2 \)?

**Example 2**

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

\[
y = x^2, \quad y = (x + 3)^2, \quad y = (x - 5)^2
\]

These three graphs all open up and have the same shape. The vertex of each graph is on the \( x \)-axis. However, the graphs have different horizontal positions.

Example 2 shows how changing the value of \( h \) in the equation \( y = a(x - h)^2 + k \) translates the graph horizontally. If \( h > 0 \), the graph translates to the right \( h \) units. If \( h < 0 \), the graph translates to the left \( h \) units.

www.algebra2.com/other_calculator_keystrokes
Investigation

How does the value \( a \) affect the graph of \( y = x^2 \)?

Example 3

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

a. \( y = x^2, y = -x^2 \)

The graphs have the same vertex and the same shape. However, the graph of \( y = x^2 \) opens up and the graph of \( y = -x^2 \) opens down.

b. \( y = x^2, y = 4x^2, y = \frac{1}{4}x^2 \)

The graphs have the same vertex, \((0, 0)\), but each has a different shape. The graph of \( y = 4x^2 \) is narrower than the graph of \( y = x^2 \). The graph of \( y = \frac{1}{4}x^2 \) is wider than the graph of \( y = x^2 \).

Changing the value of \( a \) in the equation \( y = a(x - h)^2 + k \) can affect the direction of the opening and the shape of the graph. If \( a > 0 \), the graph opens up, and if \( a < 0 \), the graph opens down or is reflected over the \( x \)-axis. If \( |a| > 1 \), the graph is narrower than the graph of \( y = x^2 \). If \( |a| < 1 \), the graph is wider than the graph of \( y = x^2 \). Thus, a change in the absolute value of \( a \) results in a dilation of the graph of \( y = x^2 \).

Exercises 1–3. See margin.

Consider \( y = a(x - h)^2 - k \).

1. How does changing the value of \( h \) affect the graph? Give an example.
2. How does changing the value of \( k \) affect the graph? Give an example.
3. How does using \(-a\) instead of \( a \) affect the graph? Give an example.

Examine each pair of equations and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs. 4–15. See pp. 343A–343F.

4. \( y = x^2, y = x^2 + 2.5 \)
5. \( y = -x^2, y = x^2 - 9 \)
6. \( y = x^2, y = 3x^2 \)
7. \( y = x^2, y = -6x^2 \)
8. \( y = x^2, y = (x + 3)^2 \)
9. \( y = -\frac{1}{2}x^2, y = -\frac{1}{3}x^2 + 2 \)
10. \( y = x^2, y = (x - 7)^2 \)
11. \( y = x^2, y = 3(x + 4)^2 - 7 \)
12. \( y = x^2, y = -\frac{1}{4}x^2 + 1 \)
13. \( y = (x + 3)^2 - 2, y = (x + 3)^2 + 5 \)
14. \( y = 3(x + 2)^2 - 1, y = 6(x + 2)^2 - 1 \)
15. \( y = 4(x - 2)^2 - 3, y = \frac{1}{4}(x - 2)^2 - 1 \)

Assess

Ask students:

- In the general form of a quadratic equation, which constant would you change to move the graph left or right? \( h \)
- Which constant would you change to move the graph up or down? \( k \)
- Which constant would you change to make the graph wider or narrower? \( a \)

Answers

1. Changing the value of \( h \) moves the graph to the left and the right. If \( h > 0 \), the graph translates to the right, and if \( h < 0 \), it translates to the left. In \( y = x^2 \), the vertex is at \((0, 0)\) and in \( y = (x - 2)^2 \), the vertex is at \((2, 0)\). The graph has been translated to the right.
2. Changing the value of \( k \) moves the graph up and down. If \( k > 0 \), the graph translates upward, and if \( k < 0 \), it translates downward. In \( y = x^2 \), the vertex is at \((0, 0)\) and in \( y = x^2 - 3 \), the vertex is at \((0, -3)\). The graph has been translated downward.
3. Using \(-a\) instead of \( a \) reflects the graph over the \( x \)-axis. The graph of \( y = x^2 \) opens upward, while the graph of \( y = -x^2 \) opens downward.
Focus

5-Minute Check Transparency 6-6 Use as a quiz or review of Lesson 6-5.

Mathematical Background notes are available for this lesson on p. 284D.

How can the graph of \( y = x^2 \) be used to graph any quadratic function?

Ask students:

- For the function \( y = x^2 \), what value of \( x \) makes \( y \) equal 0? 0
- What value of \( x \) makes \( y \) equal 0 if the function is \( y = (x - 3)^2 \)? 3
- Compare the graph of \( y = x^2 + 2 \) with the graph of \( y = (x + 2)^2 \). What difference does adding the 2 within the parentheses make? Sample answer: Adding the 2 inside the parentheses moves the graph 2 units to the left rather than 2 units up when compared to the graph of \( y = x^2 \).

Vocabulary

- vertex form

How can the graph of \( y = x^2 \) be used to graph any quadratic function?

A family of graphs is a group of graphs that displays one or more similar characteristics. The graph of \( y = x^2 \) is called the parent graph of the family of quadratic functions. Study the graphs of \( y = x^2 \), \( y = x^2 + 2 \), and \( y = (x - 3)^2 \). Notice that adding a constant to \( x^2 \) moves the graph up. Subtracting a constant from \( x \) before squaring it moves the graph to the right.

Example 1 Graph a Quadratic Function in Vertex Form

Analyze \( y = (x + 2)^2 + 1 \). Then draw its graph.

This function can be rewritten as \( y = (x - (-2))^2 + 1 \). Then \( h = -2 \) and \( k = 1 \).

The vertex is at \((h, k)\) or \((-2, 1)\), and the axis of symmetry is \( x = -2 \). The graph has the same shape as the graph of \( y = x^2 \), but is translated 2 units left and 1 unit up.

Now use this information to draw the graph.

Step 1 Plot the vertex, \((-2, 1)\).

Step 2 Draw the axis of symmetry, \( x = -2 \).

Step 3 Find and plot two points on one side of the axis of symmetry, such as \((-1, 2)\) and \((0, 5)\).

Step 4 Use symmetry to complete the graph.
How does the value of $a$ in the general form $y = a(x - h)^2 + k$ affect a parabola? Compare the graphs of the following functions to the parent function, $y = x^2$.

- **a.** $y = 2x^2$
- **b.** $y = \frac{1}{2}x^2$
- **c.** $y = -2x^2$
- **d.** $y = -\frac{1}{2}x^2$

All of the graphs have the vertex $(0, 0)$ and axis of symmetry $x = 0$.

Notice that the graphs of $y = 2x^2$ and $y = \frac{1}{2}x^2$ are **dilations** of the graph of $y = x^2$. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$, while the graph of $y = \frac{1}{2}x^2$ is wider. The graphs of $y = -2x^2$ and $y = 2x^2$ are **reflections** of each other over the $x$-axis, as are the graphs of $y = -\frac{1}{2}x^2$ and $y = \frac{1}{2}x^2$.

Changing the value of $a$ in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph:

- If $a > 0$, the graph opens up.
- If $a < 0$, the graph opens down.
- If $|a| > 1$, the graph is narrower than the graph of $y = x^2$.
- If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

### Concept Summary

**Quadratic Functions in Vertex Form**

The vertex form of a quadratic function is $y = a(x - h)^2 + k$.  

<table>
<thead>
<tr>
<th>$h$ and $k$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vertex and Axis of Symmetry</strong></td>
<td><strong>Vertical Translation</strong></td>
</tr>
<tr>
<td>$y = a(x - h)^2 + k$</td>
<td>$x = h$</td>
</tr>
<tr>
<td>$y = x^2$, $k = 0$</td>
<td>$k &gt; 0$</td>
</tr>
</tbody>
</table>

**Horizontal Translation**

- $h < 0$  
- $h > 0$

**Direction of Opening and Shape of Parabola**

- $a > 0$
- $a < 0$
- $|a| < 1$

### Teaching Tip
To help students remember that as $|a|$ increases, the graph gets narrower and not wider, discuss the fact that a greater multiplier for the quantity $(x - h)^2$ will make the corresponding $y$ value greater as well. Point out that greater values of $y$ result in a steeper (and thus narrower) graph.

### Intervention
Encourage students to ask questions about any aspects they may find confusing that are covered in the Concept Summary chart on this page. Ask them to write and use their own summary on an index card. Explain to students that a thorough understanding of these concepts will save them time since this knowledge will enable them to sketch approximate graphs quickly.
Write quadratic functions in vertex form

In-Class Examples

2 Write \( y = x^2 + 2x + 4 \) in vertex form. Then analyze the function. \( y = (x + 1)^2 + 3; \) vertex: \((-1, 3)\); axis of symmetry: \( x = -1 \); opens up; The graph has the same shape as the graph of \( y = x^2 \), but it is translated 1 unit left and 3 units up.

3 Write \( y = -2x^2 - 4x + 2 \) in vertex form. Then analyze and graph the function. \( y = -2(x + 1)^2 + 4; \) vertex: \((-1, 4)\); axis of symmetry: \( x = -1 \); opens down; The graph is narrower than the graph of \( y = x^2 \), and it is translated 1 unit left and 4 units up.

Study Tip

Check

As an additional check, graph the function in Example 2 to verify the location of its vertex and axis of symmetry.

Example 2
Write \( y = x^2 + bx + c \) in Vertex Form

Write \( y = x^2 + 8x - 5 \) in vertex form. Then analyze the function.

Original equation
\[ y = x^2 + 8x - 5 \]

Notice that \( x^2 + 8x - 5 \) is not a perfect square.

Complete the square by adding \( \left(\frac{8}{2}\right)^2 = 16 \) as a perfect square.

Balance this addition by subtracting 16.

Write \( x^2 + 8x + 16 \) as a perfect square.

This function can be rewritten as \( y = (x + 4)^2 - 21 \). Written in this way, you can see that \( h = -4 \) and \( k = -21 \).

The vertex is at \((-4, -21)\), and the axis of symmetry is \( x = -4 \). Since \( a = 1 \), the graph opens up and has the same shape as the graph of \( y = x^2 \), but it is translated 4 units left and 21 units down.

CHECK

You can check the vertex and axis of symmetry using the formula
\[ x = -\frac{b}{2a} \].

In the original equation, \( a = 1 \) and \( b = 8 \), so the axis of symmetry is \( x = -\frac{8}{2(1)} = -4 \) or \(-4\). Thus, the \( x \)-coordinate of the vertex is \(-4\), and the \( y \)-coordinate of the vertex is \( y = (-4)^2 + 8(-4) - 5 = -21 \).

When writing a quadratic function in which the coefficient of the quadratic term is not 1 in vertex form, the first step is to factor out that coefficient from the quadratic and linear terms. Then you can complete the square and write in vertex form.

Example 3
Write \( y = ax^2 + bx + c \) in Vertex Form, \( a \neq 1 \)

Write \( y = -3x^2 + 6x - 1 \) in vertex form. Then analyze and graph the function.

Original equation
\[ y = -3x^2 + 6x - 1 \]

Group \( ax^2 + bx \) and factor, dividing by \( a \).

Complete the square by adding 1 inside the parentheses. Notice that this is an overall addition of \(-3(1)\). Balance this addition by subtracting \(-3(1)\).

Write \( x^2 - 2x + 1 \) as a perfect square.

The vertex form of this function is \( y = -3(x - 1)^2 + 2 \). So, \( h = 1 \) and \( k = 2 \).

The vertex is at \((1, 2)\), and the axis of symmetry is \( x = 1 \). Since \( a = -3 \), the graph opens downward and is narrower than the graph of \( y = x^2 \). It is also translated 1 unit right and 2 units up.

Now graph the function. Two points on the graph to the right of \( x = 1 \) are \((1.5, 1.25)\) and \((2, -1)\). Use symmetry to complete the graph.

Differentiated Instruction

Naturalist

Have students observe or research some natural events that can be modeled by parabolas, such as the fountain’s water stream discussed in Exercise 14 on p. 326. Students should report their observations and findings to the class. If students are able to determine a quadratic function that models the event, they should present the function and explain how the characteristics of the equation can be used to analyze its graph.
If the vertex and one other point on the graph of a parabola are known, you can write the equation of the parabola in vertex form.

**Example 4 Write an Equation Given Points**

Write an equation for the parabola whose vertex is at (−1, 4) and passes through (2, 1).

The vertex of the parabola is at (−1, 4), so $h = −1$ and $k = 4$. Since (2, 1) is a point on the graph of the parabola, let $x = 2$ and $y = 1$. Substitute these values into the vertex form of the equation and solve for $a$.

\[
y = a(x - h)^2 + k \quad \text{Vertex form}
\]

\[
1 = a(2 - (-1))^2 + 4 \quad \text{Substitute 1 for } y, 2 \text{ for } x, -1 \text{ for } h, \text{ and 4 for } k.
\]

\[
1 = a(9) + 4 \quad \text{Simplify.}
\]

\[
-3 = 9a
\]

\[
-\frac{1}{3} = a \quad \text{Divide each side by 9.}
\]

The equation of the parabola in vertex form is $y = -\frac{1}{3}(x + 1)^2 + 4$.

**CHECK** A graph of $y = -\frac{1}{3}(x + 1)^2 + 4$ verifies that the parabola passes through the point at (2, 1).

---

**Check for Understanding**

**Concept Check**

1.d. $y = 2(x - 2)^2 + 3$
1.e. Sample answer: $y = 4(x + 1)^2 + 3$
1.f. Sample answer: $y = (x + 1)^2 + 3$
3. Sample answer: $y = 2(x - 2)^2 - 1$

1. Write a quadratic equation that transforms the graph of $y = 2(x + 1)^2 + 3$ so that it is:
   a. 2 units up. $y = 2(x + 1)^2 + 5$
   b. 3 units down. $y = 2(x + 1)^2$
   c. 2 units to the left. $y = 2(x + 3)^2 + 3$
   d. 3 units to the right.
   e. narrower.
   f. wider.
   g. opening in the opposite direction. $y = -2(x + 1)^2 + 3$

2. Explain how you can find an equation of a parabola using its vertex and one other point on its graph. See margin.

3. OPEN ENDED Write the equation of a parabola with a vertex of (2, −1).

4. **FIND THE ERROR** Jenny and Ruben are writing $y = x^2 - 2x + 5$ in vertex form.

   Jenny
   \[
   y = x^2 - 2x + 5
   \]
   \[
   y = (x^2 - 2x + 1) + 5 - 1
   \]
   \[
   y = (x - 1)^2 + 4
   \]

   Ruben
   \[
   y = x^2 - 2x + 5
   \]
   \[
   y = (x^2 - 2x + 1) + 5 + 1
   \]
   \[
   y = (x - 1)^2 + 6
   \]

Who is correct? Explain your reasoning.

**Guided Practice**

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. 5–7. See margin.

5. $y = 5(x + 3)^2 - 1$
6. $y = x^2 + 8x - 3$
7. $y = -3x^2 - 18x + 11$

Answers

2. Substitute the $x$-coordinate of the vertex for $h$ and the $y$-coordinate of the vertex for $k$ in the equation $y = a(x - h)^2 + k$. Then substitute the $x$-coordinate of the other point for $x$ and the $y$-coordinate for $y$ into this equation and solve for $a$. Replace $a$ with this value in the equation you wrote with $h$ and $k$.

5. $(-3, -1); x = -3$; up
6. $y = (x + 4)^2 - 19$, $(-4, -19)$; $x = -4$; up
7. $y = -3(x + 3)^2 + 38; (-3, 38)$; $x = -3$; down
About the Exercises...

Organization by Objective
- Analyze Quadratic Functions: 15, 16, 27–30, 37, 38, 47, 51, 52
- Write Quadratic Functions in Vertex Form: 19–26, 31–36, 39–46, 48–50

Odd/Even Assignments
Exercises 15–46 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 15–33 odd, 39–45 odd, 53–71
Average: 15–47 odd, 48–50, 53–71
Advanced: 16–46 even, 48–67 (optional: 68–71)
All: Practice Quiz 2 (1–10)

Answers
8. 
9. 
10. 

Graph each function. 8–10. See margin.
8. \( y = 3(x + 3)^2 \)
9. \( y = \frac{1}{3}(x - 1)^2 + 3 \)
10. \( y = -2x^2 + 16x - 31 \)

Write an equation for the parabola with the given vertex that passes through the given point.
11. vertex: (2, 0) point: (1, 4)
12. vertex: (-3, 6) point: (-5, 2)
13. vertex: (-2, -3) point: (-4, -5)
\( y = 4(x - 2)^2 \)
\( y = -(x + 3)^2 + 6 \)
\( y = -\frac{1}{2}(x + 2)^2 - 3 \)

Application
14. **FOUNTAINS** The height of a fountain’s water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet. If the water lands 3 feet away from the jet, find a quadratic function that models the height of the water at any given distance \( d \) feet from the jet.

\( h(d) = -2d^2 + 4d + 6 \)

Practice and Apply

**Homework Help**

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<td>28.</td>
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<td>35.</td>
<td>( y = -\frac{1}{2}x^2 + 5x - \frac{27}{2} )</td>
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<tr>
<td>36.</td>
<td>( y = \frac{1}{3}x^2 - 4x + 15 )</td>
</tr>
</tbody>
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37. Write one sentence that compares the graphs of \( y = 0.2(x + 3)^2 + 1 \) and \( y = 0.4(x + 3)^2 + 1 \). Sample answer: the graph of \( y = 0.4(x + 3)^2 + 1 \) is narrower than the graph of \( y = 0.2(x + 3)^2 + 1 \).

38. Compare the graphs of \( y = 2(x - 5)^2 + 4 \) and \( y = 2(x - 4)^2 - 1 \).

Write an equation for the parabola with the given vertex that passes through the given point.
39. vertex: (6, 1) point: (5, 10)
40. vertex: (-4, 3) point: (-3, 6)
41. vertex: (3, 0) point: (6, -6)
42. vertex: (5, 4) point: (3, -8)
43. vertex: (0, 5) point: (3, 9)
44. vertex: (-3, -2) point: (-1, 8)

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45. Write an equation for a parabola whose vertex is at the origin and passes through \((2, -8)\).\[ y = -2x^2 \]

46. Write an equation for a parabola with vertex at \((-3, -4)\) and y-intercept 8.\[ y = \frac{1}{3}(x + 3)^2 - 4 \]

47. AEROSPACE NASA’s KC135A aircraft flies in parabolic arcs to simulate the weightlessness experienced by astronauts in space. The height \(h\) of the aircraft (in feet) \(t\) seconds after it begins its parabolic flight can be modeled by the equation \(h(t) = -9.09(t - 32.5)^2 + 3400\). What is the maximum height of the aircraft during this maneuver and when does it occur? 34,000 feet; 32.5 s after the aircraft begins its parabolic flight

DIVING For Exercises 48–50, use the following information.
The distance of a diver above the water \(d(t)\) (in feet) \(t\) seconds after diving off a platform is modeled by the equation \(d(t) = -16t^2 + 8t + 30\).

48. Find the time it will take for the diver to hit the water. about 1.6 s

49. Write an equation that models the diver’s distance above the water if the platform were 20 feet higher. \(d(t) = -16t^2 + 8t + 50\)

50. Find the time it would take for the diver to hit the water from this new height. about 2.0 s

LAWN CARE For Exercises 51 and 52, use the following information.
The path of water from a sprinkler can be modeled by a quadratic function.
The three functions below model paths for three different angles of the water.

Angle A: \(y = -0.28(x - 3.09)^2 + 3.37\)

Angle B: \(y = -0.14(x - 3.57)^2 + 2.39\)

Angle C: \(y = -0.09(x - 3.22)^2 + 1.53\)

51. Which sprinkler angle will send water the highest? Explain your reasoning.
52. Which sprinkler angle will send water the furthest? Explain your reasoning.

53. CRITICAL THINKING Given \(y = ax^2 + bx + c\) with \(a \neq 0\), derive the equation for the axis of symmetry by completing the square and rewriting the equation in the form \(y = a(x - h)^2 + k\). See pp. 343A–343F.

54. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See pp. 343A–343F.

How can the graph \(y = x^2\) be used to graph any quadratic function? Include the following in your answer:
• a description of the effects produced by changing \(a, b,\) and \(k\) in the equation \(y = ax^2 + bx + c\), and
• a comparison of the graph of \(y = x^2\) and the graph of \(y = ax^2 + bx + c\) using values of your own choosing for \(a, b,\) and \(k\).

55. If \(f(x) = x^2 - 5x\) and \(f(n) = -4\), then which of the following could be \(n\)?

\[ \boxed{D} \quad 1 \]

56. The vertex of the graph of \(y = 2(x - 6)^2 + 3\) is located at which of the following points? B

\[ \boxed{B} \quad (2, 3) \]

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---

**Patterns with Differences and Sums of Squares**

Some whole numbers can be written as the difference of two squares and some cannot. Formulas can be developed to describe the sets of numbers, respectively.

If possible, write each number as the difference of two squares.

\[ \begin{align*}
13 & = 2^2 - 1^2 \\
14 & = 3^2 - 2^2 \\
15 & = 4^2 - 3^2 \\
16 & = 5^2 - 4^2 \\
17 & = 6^2 - 5^2 \\
18 & = 7^2 - 6^2 \\
19 & = 8^2 - 7^2 \\
20 & = 9^2 - 8^2
\end{align*} \]

Even numbers can be written as \(2n\), where \(n\) is one of the numbers 1, 2, 4, and so on. Odd numbers can be written in the form \(4n + 1\). Use these forms and formulas to write the appropriate number as the difference of two squares.

---

**Read to Learn Mathematics, p. 347**

**ELL**

**Pre-Activity** How can the graph of \(y = x^2\) be used to graph any quadratic function? Read the introduction to Lesson 6-6 at the top of page 332 in your textbook.

What does adding a positive number \(c\) to the graph of \(y = x^2\) do to the graph? It moves the graph up.

What does subtracting a positive number \(c\) to the graph of \(y = x^2\) do to the graph? It moves the graph down.

**Reading the Lesson**

1. Complete the following information about the graph of \(y = ax^2 + bx + c\):
   - What are the coordinates of the vertex? \((h, k)\)
   - What is the equation of the axis of symmetry? \(x = h\)
   - In which direction does the graph open? \(a > 0\); up; \(a < 0\); down
   - What do you know about the graph if \(a > 0\)? It is wider than the graph of \(y = x^2\).

2. Match each graph with the description of the constant in the equation in vertex form:
   - \(a > 0\): \(a \cdot k < k + b > 0\)
   - \(a < 0\): \(a \cdot k < k + b < 0\)

---

**Helping You Remember**

When graphing quadratic functions such as \(y = ax^2 + bx + c\), many students make several common mistakes, which represent a translation of the graph \(y = x^2\) to the left and right, which represents a translation of the graph to the right. What is the origin of this translation? The translation is to the right. What is the vertex? The vertex is \((h, k)\).

Sample answer: In functions like \(y = (x - 4)^2\), the plus sign puts the graph “ahead” so that the vertex comes “sooner” than the origin and the translation is to the left. In functions like \(y = (x + 4)^2\), the minus sign puts the graph “behind” so that the vertex comes “later” than the origin and the translation is to the left.

---

**Write an equation for a parabola with vertex at \((1.2, 2.7)\); x-intercepts are \(-0.5, 4.2\); and is wider than the graph of \(y = x^2\).**

\[ y = a(x - 1.2)^2 + k \]

Write an equation for a parabola with vertex at \((0, 2)\); x-intercepts are \(-4, 2)\; and is wider than the graph of \(y = x^2\).**

## Enrichment, p. 348

Patterns with Differences and Sums of Squares

Some whole numbers can be written as the difference of two squares and some cannot. Formulas can be developed to describe the sets of numbers, respectively.

If possible, write each number as the difference of two squares.

\[ \begin{align*}
13 & = 2^2 - 1^2 \\
14 & = 3^2 - 2^2 \\
15 & = 4^2 - 3^2 \\
16 & = 5^2 - 4^2 \\
17 & = 6^2 - 5^2 \\
18 & = 7^2 - 6^2 \\
19 & = 8^2 - 7^2 \\
20 & = 9^2 - 8^2
\end{align*} \]

Even numbers can be written as \(2n\), where \(n\) is one of the numbers 1, 2, 4, and so on. Odd numbers can be written in the form \(4n + 1\). Use these forms and formulas to write the appropriate number as the difference of two squares.
Maintain Your Skills

Mixed Review

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.  

57. \(3x^2 - 6x + 2 = 0\)  58. \(4x^2 + 7x = 11\)  59. \(2x^2 - 5x + 6 = 0\)

12; 2 irrational  225; 2 rational  

Solve each equation by completing the square.  

60. \(x^2 + 10x + 17 = 0\)  61. \(x^2 - 6x + 18 = 0\)  62. \(4x^2 + 8x = 9\)

\(-5 \pm 2\sqrt{2}\)  \(3 \pm 3i\)  \(-2 \pm \sqrt{13}\)  

Find each quotient.  

63. \((3t^2 - 2t - 3) / (t - 1)\)  64. \((t^3 - 3t + 2) / (t + 2)\)

65. \((n^4 - 8n^3 + 54n + 105) / (n - 5)\)  66. \((y^4 + 3y^3 + y - 1) / (y + 3)\)

67. EDUCATION  
The graph shows the number of U.S. students in study-abroad programs.  

Prerequisite Skill  
Determine whether the given value satisfies the inequality.  

[To review inequalities, see Lesson 1-6.]

68. \(-2x^2 + 3 < 0; x = 5\) yes  69. \(4x^2 + 2x - 3 \geq 0; x = -1\) no  

70. \(4x^2 - 4x + 1 \leq 0; x = 2\) yes  71. \(6x^2 + 3x > 8; x = 0\) no


USA TODAY Snapshots®

More Americans study abroad

The number of U.S. college students in study-abroad programs rose 11.4% in the year ending June 1997 (latest available) to about 150,000. Annual numbers:


Note: Includes any student getting credit at a university while abroad.

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Graphing and Solving Quadratic Inequalities

**What You’ll Learn**
- Graph quadratic inequalities in two variables.
- Solve quadratic inequalities in one variable.

**How You Can Use This**
Can you find the time a trampolinist spends above a certain height?

Trampolining was first featured as an Olympic sport at the 2000 Olympics in Sydney, Australia. The competitors performed two routines consisting of 10 different skills. Suppose the height \( h(t) \) in feet of a trampolinist above the ground during one bounce is modeled by the quadratic function \( h(t) = -16t^2 + 42t + 3.75 \). We can solve a quadratic inequality to determine how long this performer is more than a certain distance above the ground.

**Mathematical Background**
Notes are available for this lesson on p. 284D.

**Building on Prior Knowledge**
In Lesson 6-6, students analyzed and graphed equations. In this lesson, students use the same techniques to graph and solve inequalities.

**Graphing Quadratic Inequalities**
You can graph quadratic inequalities in two variables using the same techniques you used to graph linear inequalities in two variables.

**Step 1**
Graph the related quadratic equation, \( y = ax^2 + bx + c \). Decide if the parabola should be solid or dashed.

**Step 2**
Test a point \((x_1, y_1)\) inside the parabola. Check to see if this point is a solution of the inequality.

**Step 3**
If \((x_1, y_1)\) is a solution, shade the region inside the parabola. If \((x_1, y_1)\) is not a solution, shade the region outside the parabola.

**Example 1**
Graph a Quadratic Inequality

Graph \( y > -x^2 - 6x - 7 \).

**Step 1**
Graph the related quadratic equation, \( y = -x^2 - 6x - 7 \).

Since the inequality symbol is >, the parabola should be dashed.

(continued on the next page)

Lesson 6-7 Graphing and Solving Quadratic Inequalities 329
Graph $y > x^2 - 3x + 2$.

Teaching Tip For In-Class Example 1, encourage students to write the calculations as in Step 2 of Example 1 when testing a point inside the parabola. Emphasize that they must write a question mark over each inequality sign after substituting the coordinates of the point for the variables in the inequality.

Solve Quadratic Inequalities

In-Class Example

1. Solve $x^2 - 4x + 3 > 0$ by graphing.

Step 2 Test a point inside the parabola, such as $(-3, 0)$.

$y > -x^2 - 6x - 7$

$0 \not> -(-3)^2 - 6(-3) - 7$

$0 \not> -9 + 18 - 7$

$0 \not> 2 \times$

So, $(-3, 0)$ is not a solution of the inequality.

Step 3 Shade the region outside the parabola.

Solve Quadratic Inequalities

To solve $ax^2 + bx + c < 0$, graph $y = ax^2 + bx + c$. Identify the $x$ values for which the graph lies below the $x$-axis.

For $\leq$, include the $x$-intercepts in the solution.

To solve $ax^2 + bx + c > 0$, graph $y = ax^2 + bx + c$. Identify the $x$ values for which the graph lies above the $x$-axis.

For $\geq$, include the $x$-intercepts in the solution.

Example 2 Solve $ax^2 + bx + c > 0$

Solve $x^2 + 2x - 3 > 0$ by graphing.

The solution consists of the $x$ values for which the graph of the related quadratic function lies above the $x$-axis. Begin by finding the roots of the related equation.

$x^2 + 2x - 3 = 0$ Related equation

$(x + 3)(x - 1) = 0$ Factor.

$x + 3 = 0$ or $x - 1 = 0$ Zero Product Property

$x = -3$ or $x = 1$ Solve each equation.

Sketch the graph of a parabola that has $x$-intercepts at $-3$ and $1$. The graph should open up since $a > 0$.

The graph lies above the $x$-axis to the left of $x = -3$ and to the right of $x = 1$. Therefore, the solution set is $\{x | x < -3$ or $x > 1\}$. 

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Example 3 Solve $ax^2 + bx + c \leq 0$

Solve $0 \geq 3x^2 - 7x - 1$ by graphing.

This inequality can be rewritten as $3x^2 - 7x - 1 \leq 0$. The solution consists of the $x$ values for which the graph of the related quadratic function lies on and below the $x$-axis. Begin by finding the roots of the related equation.

$$3x^2 - 7x - 1 = 0$$

Related equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the Quadratic Formula.

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2(3)}$$

Replace $a$ with $3$, $b$ with $-7$, and $c$ with $-1$.

$$x = \frac{7 \pm \sqrt{49 + 12}}{6}$$

Simplify and write as two equations.

$$x = \frac{7 + \sqrt{61}}{6}$$

$$x = \frac{7 - \sqrt{61}}{6}$$

Simplify.

Sketch the graph of a parabola that has $x$-intercepts of $2.47$ and $-0.14$. The graph should open up since $a > 0$.

The graph lies on and below the $x$-axis at $x = -0.14$ and $x = 2.47$ and between these two values. Therefore, the solution set of the inequality is approximately $\{x \mid -0.14 \leq x \leq 2.47\}$.

**CHECK** Test one value of $x$ less than $-0.14$, one between $-0.14$ and $2.47$, and one greater than $2.47$ in the original inequality.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$3x^2 - 7x - 1$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$3(-1)^2 - 7(-1) - 1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$0$</td>
<td>$3x^2 - 7x - 1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$3$</td>
<td>$3(3)^2 - 7(3) - 1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Real-world problems that involve vertical motion can often be solved by using a quadratic inequality.

Example 4 Write an Inequality

**FOOTBALL** The height of a punted football can be modeled by the function $H(x) = -4.9x^2 + 20x + 1$, where the height $H(x)$ is given in meters and the time $x$ is in seconds. At what time in its flight is the ball within 5 meters of the ground?

The function $H(x)$ describes the height of the football. Therefore, you want to find the values of $x$ for which $H(x) \leq 5$.

$$H(x) \leq 5$$

Original inequality

$$-4.9x^2 + 20x + 1 \leq 5$$

$$-4.9x^2 + 20x - 4 \leq 0$$

Subtract 5 from each side.

Graph the related function $y = -4.9x^2 + 20x - 4$ using a graphing calculator. The zeros of the function are about 0.21 and 3.87, and the graph lies below the $x$-axis when $x < 0.21$ or $x > 3.87$.

Thus, the ball is within 5 meters of the ground for the first 0.21 second of its flight and again after 3.87 seconds until the ball hits the ground at 4.13 seconds.

www.algebra2.com/extra_examples

Lesson 6-7 Graphing and Solving Quadratic Inequalities 331

**In-Class Examples**

3 Solve $0 \leq -2x^2 - 6x + 1$ by graphing.

$$y = -2x^2 - 6x + 1$$

$$\{x \mid -3.16 \leq x \leq 0.16\}$$

**Teaching Tip** When discussing Example 4, be aware that some students may not be familiar with all of the aspects of the game of football. Ask students who are familiar with the terms in this example and the margin note to explain them to the class.

4 SPORTS The height of a ball above the ground after it is thrown upwards at 40 feet per second can be modeled by the function $h(x) = 40x - 16x^2$, where the height $h(x)$ is given in feet and the time $x$ is in seconds. At what time in its flight is the ball within 15 feet of the ground? The ball is within 15 feet of the ground for the first 0.46 second of its flight and again after 2.04 seconds until the ball hits the ground at 2.5 seconds.
You can also solve quadratic inequalities algebraically.

**Example 5** Solve a Quadratic Inequality

Solve \( x^2 + x > 6 \) algebraically.

First solve the related quadratic equation \( x^2 + x = 6 \).

\[
\begin{align*}
\text{Related quadratic equation} & : x^2 + x = 6 \\
\text{Subtract 6} & : x^2 + x - 6 = 0 \\
\text{Factor.} & : (x + 3)(x - 2) = 0 \\
\text{Zero Product Property} & : x + 3 = 0 \quad \text{or} \quad x - 2 = 0 \\
\text{Solve each equation.} & : x = -3 \quad \text{or} \quad x = 2
\end{align*}
\]

Plot -3 and 2 on a number line. Use circles since these values are not solutions of the original inequality. Notice that the number line is now separated into three intervals.

Test a value in each interval to see if it satisfies the original inequality.

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<thead>
<tr>
<th>( x &lt; -3 )</th>
<th>(-3 &lt; x &lt; 2 )</th>
<th>( x &gt; 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test ( x = -4 ).</td>
<td>Test ( x = 0 ).</td>
<td>Test ( x = 4 ).</td>
</tr>
<tr>
<td>( x^2 + x &gt; 6 )</td>
<td>( x^2 + x &gt; 6 )</td>
<td>( x^2 + x &gt; 6 )</td>
</tr>
<tr>
<td>( (-4)^2 + (-4) - 6 )</td>
<td>( 0^2 + 0 - 6 )</td>
<td>( 2^2 + 4 - 6 )</td>
</tr>
<tr>
<td>( 16 - 4 - 6 )</td>
<td>( 0 - 6 )</td>
<td>( 4 - 4 - 6 )</td>
</tr>
<tr>
<td>( 12 &gt; 6 )</td>
<td>( 0 &gt; 6 \times )</td>
<td>( 20 &gt; 6 \times )</td>
</tr>
</tbody>
</table>

The solution set is \( \{ x \mid x < -3 \text{ or } x > 2 \} \). This is shown on the number line below.

---

**Concept Check**

1. Determine which inequality, \( y \geq (x - 3)^2 - 1 \) or \( y \leq (x - 3)^2 - 1 \), describes the graph at the right. \( y \geq (x - 3)^2 - 1 \)

2. **OPEN ENDED** List three points you might test to find the solution of \((x + 3)(x - 5) < 0\). Sample answer: one number less than -3, one number between -3 and 5, and one number greater than 5

3. Examine the graph of \( y = x^2 - 4x - 5 \) at the right. a. \( x = -1, 5 \) b. \( x \leq -1 \text{ or } x \geq 5 \)

   a. What are the solutions of \( 0 = x^2 - 4x - 5 \)?

   b. What are the solutions of \( x^2 - 4x - 5 = 0 \)?

   c. What are the solutions of \( x^2 - 4x - 5 \leq 0 \)?

   \(-1 \leq x \leq 5\)
Guided Practice

Graph each inequality. 4–7. See margin.

4. \( y \geq x^2 - 10x + 25 \)  
5. \( y < x^2 - 16 \)  
6. \( y > -2x^2 - 4x + 3 \)  
7. \( y \leq -x^2 + 5x + 6 \)

8. Use the graph of the related function of \(-x^2 + 6x - 5 < 0\), which is shown at the right, to write the solutions of the inequality. 
\( x < 1 \) or \( x > 5 \)

Solve each inequality algebraically.

9. \( x^2 - 6x - 7 < 0 \) \( \{x \mid -1 < x < 7\} \)
10. \( x^2 - x - 12 > 0 \) \( \{x \mid x < -3 \text{ or } x > 4\} \)
11. \( x^2 < 10x - 25 \) \( \emptyset \)
12. \( x^2 \leq 3 \) \( \{x \mid -\sqrt{3} \leq x \leq \sqrt{3}\} \)

Application

13. BASEBALL A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height \( h(t) \) of the ball in meters \( t \) seconds after being hit is modeled by \( h(t) = -4.9t^2 + 30t + 1.4 \). How long does a player on the opposing team have to catch the ball if he catches it 1.7 meters above the ground? about 6.1 s

* indicates increased difficulty

Practice and Apply

Homework Help

For Exercises | See Examples
--- | ---
14–25 | 1
26–29 | 2, 3
30–42 | 2, 3, 5
43–48 | 4

Extra Practice

See page 841.

Graph each inequality. 14–25. See pp. 343A–343F.

14. \( y \geq x^2 + 3x - 18 \)  
15. \( y < -x^2 + 7x + 8 \)  
16. \( y \leq x^2 + 4x + 4 \)  
17. \( y \leq x^2 + 4x \)  
18. \( y > x^2 - 36 \)  
19. \( y > x^2 + 6x + 5 \)  
20. \( y \leq -x^2 - 3x + 10 \)  
21. \( y \geq -x^2 - 7x + 10 \)  
22. \( y > -x^2 + 10x - 23 \)  
23. \( y < -x^2 + 13x - 36 \)  
24. \( y < 2x^2 + 3x - 5 \)  
25. \( y \geq 2x^2 + x - 3 \)

Use the graph of its related function to write the solutions of each inequality.

26. \(-x^2 + 10x - 25 \geq 0 \) \( 5 \)
27. \( x^2 - 4x - 12 \leq 0 \) \( -2 \leq x \leq 6 \)

28. \( x^2 - 9 > 0 \) \( x < -3 \text{ or } x > 3 \)
29. \(-x^2 - 10x - 21 \leq 0 \) \( x < -7 \text{ or } x \geq -3 \)

Answers
Solve each inequality algebraically.

30. \( x^2 - 3x - 18 > 0 \) \( \{x | x < -3 \text{ or } x > 6\} \)
31. \( x^3 + 3x - 28 < 0 \) \( \{x | -7 < x < 4\} \)
32. \( x^2 - 4x + 5 < 0 \) \( \{x | -\infty < x < 0, x > 5\} \)
33. \( x^2 + 2x + \frac{2}{3} \geq 0 \) \( \{x | x \geq -1 \text{ or } x < -2\} \)
34. \( -x^2 - x + 12 > 0 \) \( \{x | -4 < x < 3\} \)
35. \( x^2 - 8x + 15 < 0 \) \( \{x | 3 < x < 5\} \)
36. \( 9x^2 - 6x + 1 < 0 \) \( \{x | x = \frac{1}{3}\} \)
37. \( 4x^2 + 20x + 25 \geq 0 \) all reals
38. \( x^2 + 12x < -36 \) \( \emptyset \)
39. \( x^2 - 14x + 49 \geq 0 \) \( \{x | x = 7\} \)
40. \( 18x - x^2 < 81 \) all reals
41. \( 16x^2 + 9 < 24x \) \( \emptyset \)

42. Solve \((x - 1)(x + 4)(x - 3) > 0\). \( \{x | -4 < x < 1 \text{ or } x > 3\} \)

43. LANDSCAPING

Kimu wants to plant a garden and surround it with decorative stones. She has enough stones to enclose a rectangular garden with a perimeter of 68 feet, but she wants the garden to cover no more than 240 square feet. What could the width of her garden be? 0 to 10 ft or 24 to 34 ft

44. BUSINESS

A mall owner has determined that the relationship between monthly rent charged for store space \( r \) (in dollars per square foot) and monthly profit \( P(r) \) (in thousands of dollars) can be approximated by the function

\[ P(r) = -8.1r^2 + 46.9r - 38.2 \]

Solve each quadratic equation or inequality.

Explain what each answer tells about the relationship between monthly rent and profit for this mall.

a. \(-8.1r^2 + 46.9r - 38.2 = 0\)
b. \(-8.1r^2 + 46.9r - 38.2 > 0\)
c. \(-8.1r^2 + 46.9r - 38.2 < 0\)
d. \(-8.1r^2 + 46.9r - 38.2 \leq 0\)

45. GEOMETRY

A rectangle is 6 centimeters longer than it is wide. Find the possible dimensions if the area of the rectangle is more than 216 square centimeters. The width should be greater than 12 cm and the length should be greater than 18 cm

46. FUND-RAISING

For Exercises 46–48, use the following information.

The girls’ softball team is sponsoring a fund-raising trip to see a professional baseball game. They charter a 60-passenger bus for $525. In order to make a profit, they will charge $15 per person if all seats on the bus are sold, but for each empty seat, they will increase the price by $1.50 per person.

Write a quadratic function giving the softball team’s profit \( P(n) \) from this fund-raiser as a function of the number of passengers \( n \).

What is the minimum number of passengers needed in order for the softball team not to lose money?

What is the maximum profit the team can make with this fund-raiser, and how many passengers will take to achieve this maximum?

$1312.50; 35 passengers

48. CRITICAL THINKING

Graph the intersection of the graphs of \( y \leq -x^2 + 4 \) and

\( y > x - 2 \). See margin.

49. WRITING IN MATH

Answer the question that was posed at the beginning of the lesson. See margin.

How can you find the time a trampolinist spends above a certain height?

Include the following in your answer:

• a quadratic inequality that describes the time the performer spends more than 10 feet above the ground, and

• two approaches to solving this quadratic inequality.
51. Which is a reasonable estimate of the area under the curve from \( x = 0 \) to \( x = 18 \)?
   - A. 29 square units
   - B. 58 square units
   - C. 116 square units
   - D. 232 square units

52. If \((x + 1)(x - 2)\) is positive, then
   - A. \( x < -1 \) or \( x > 2 \).
   - B. \( x > -2 \) or \( x < 1 \).
   - C. \(-1 < x < 2 \).
   - D. \(-2 < x < 1 \).

**Extending the Lesson**

SOLVE ABSOLUTE VALUE INEQUALITIES BY GRAPHING

Similar to quadratic inequalities, you can solve absolute value inequalities by graphing.

Graph the related absolute value function for each inequality using a graphing calculator. For \( > \) and \( \geq \), identify the \( x \) values, if any, for which the graph lies below the \( x \)-axis. For \( < \) and \( \leq \), identify the \( x \) values, if any, for which the graph lies above the \( x \)-axis.

53. \(|x - 2| > 0 \) \( \{ \text{all reals, } x \neq 2 \} \)
54. \(|x - 7| < 0 \) \( \{ x \mid -7 < x < 7 \} \)
55. \(-|x + 3| + 6 < 0 \) \( \{ x \mid x < -9 \text{ or } x > 3 \} \)
56. \(2|x + 3| - 1 \geq 0 \) \( \{ x \mid x \leq -3.5 \text{ or } x \geq -2.5 \} \)
57. \(|5x + 4| - 2 \leq 0 \) \( \{ x \mid -1.2 \leq x \leq -0.4 \} \)

**Maintain Your Skills**

**Mixed Review**

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. \[(Lesson 6-6)\]

59. \( y = (x - 1)^2 + 8 \) \( \{0, 8\}, x = 1; \text{ up} \)
60. \( y = -2(x - 4)^2 \); \( \{4, 0\}, x = 4; \text{ down} \)
61. \( y = \frac{1}{2}x^2 + 6x + 18 \)
62. \( -5 \pm \sqrt{3} \)

63. \( -5 \frac{2}{3} \)

64. \( \frac{3}{2} \)

65. \( -3 \sqrt{6} \)

66. \( -3 \sqrt{4} \)

67. \( -3 \sqrt{2} \)

68. \( -3 \sqrt{1} \)

Find each product, if possible. \[(Lesson 6-3)\]

69. \( [6 - 3]\) \( [2 - 5]\) \( [21 - 48]\) \( [6 - 13]\)
70. \( [2 - 6]\) \( [3 - 9]\) \( [0 - 54]\)
71. \( [3 - 6]\) \( [2 - 4]\) \( [6 - 22]\)

**LAW ENFORCEMENT** Thirty-four states classify drivers having at least a 0.1 blood alcohol content (BAC) as intoxicated. An infrared device measures a person's BAC through an analysis of his or her breath. A certain detector measures BAC to within 0.002. If a person’s actual blood alcohol content is 0.08, write and solve an absolute value equation to describe the range of BACs that might register on this device. \[(Lesson 1-6)\]

\( |x - 0.08| \leq 0.002 \) or \( 0.078 \leq x \leq 0.082 \)

50. Answers should include the following.

- \(-16t^2 + 42t + 3.75 > 10\)
- One method of solving this inequality is to graph the related quadratic function \( h(t) = -16t^2 + 42t + 3.75 - 10 \). The interval(s) at which the graph is above the \( x \)-axis represents the times when the tramplinist is above 10 feet. A second method of solving this inequality would be find the roots of the related quadratic equation \(-16t^2 + 42t + 3.75 - 10 = 0\) and then test points in the three intervals determined by these roots to see if they satisfy the inequality. The interval(s) at which the inequality is satisfied represent the times when the tramplinist is above 10 feet.

**Answers**

44a. 0.98, 4.81; The owner will break even if he charges $0.98 or $4.81 per square foot.
44b. 0.98 < \( r \) < 4.81; The owner will make a profit if the rent is between $0.98 and $4.81.
44c. 1.34 < \( r \) < 4.45; If rent is set between $1.34 and $4.45 per sq ft, the profit will be greater than $10,000.
44d. \( r < 1.34 \) or \( r > 4.45 \); If rent is set between $0 and $1.34 or above $4.45 per sq ft, the profit will be less than $10,000.
Chapter 6 Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 6 includes a page reference where each term was introduced.
- Assessment A vocabulary test/review for Chapter 6 is available on p. 368 of the Chapter 6 Resource Masters.

Lesson-by-Lesson Review

For each lesson,
- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students’ mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

6-1 Graphing Quadratic Functions

Concept Summary

The graph of \( y = ax^2 + bx + c \), \( a \neq 0 \),
- opens up, and the function has a minimum value when \( a > 0 \), and
- opens down, and the function has a maximum value when \( a < 0 \).

Example

Find the maximum or minimum value of \( f(x) = -x^2 + 4x - 12 \).

Since \( a < 0 \), the graph opens down and the function has a maximum value. The maximum value of the function is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-4}{2(-1)} = 2 \). Find the \( y \)-coordinate by evaluating the function for \( x = 2 \).

\[
\begin{align*}
f(x) &= -x^2 + 4x - 12 \\
f(2) &= -(2)^2 + 4(2) - 12 \\
&= -8 - 12 \\
&= -20
\end{align*}
\]

Therefore, the maximum value of the function is \(-20\).
Exercises  Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.
b. Make a table of values that includes the vertex.
c. Use this information to graph the function. (See Example 2 on pages 287 and 288.)

9. \( f(x) = x^2 + 6x + 20 \)  
10. \( f(x) = x^2 - 2x - 15 \)  
11. \( f(x) = x^2 - 8x + 7 \)  
12. \( f(x) = -2x^2 + 12x - 9 \)  
13. \( f(x) = -x^2 - 4x - 3 \)  
14. \( f(x) = 3x^2 + 9x + 6 \)

9–14. See margin.

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function. (See Example 3 on pages 288 and 289.)

15. \( f(x) = 4x^2 - 3x - 5 \)  
16. \( f(x) = -3x^2 + 2x - 2 \)  
17. \( f(x) = -2x^2 + 7 \)

Exercises  Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (See Examples 1–3 on pages 294 and 295.)

18. \( x^2 - 36 = 0 \)  
19. \( -x^2 - 3x + 10 = 0 \)  
20. \( 2x^2 + x - 3 = 0 \)  
21. between \(-3\) and \(-2\); between \(-36\) and \(-37\)

6–2  Solving Quadratic Equations by Graphing

Concept Summary
- The solutions, or roots, of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the x-intercepts of its graph.
- A quadratic equation can have one real solution, two real solutions, or no real solution.

One Real Solution  |  Two Real Solutions  |  No Real Solution
---|---|---
[Graphs]

Example  Solve \( 2x^2 - 5x + 2 = 0 \) by graphing.

The equation of the axis of symmetry is \( x = \frac{-5}{2(2)} \) or \( x = \frac{-5}{4} \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>5/2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
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</tbody>
</table>

The zeros of the related function are \( \frac{1}{2} \) and 2. Therefore, the solutions of the equation are \( \frac{1}{2} \) and 2.

Exercises  Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (See Examples 1–3 on pages 294 and 295.)

19. \( 2, -5 \)  
20. \( 1, -\frac{3}{2} \)  
21. \( \frac{1}{2}(x + 3)^2 - 5 = 0 \)  
22. between \(-36\) and \(-37\)

11. 7; \( x = 4; 4 \)

11b. \( x / f(x) \)

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<tr>
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<th>( f(x) )</th>
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<tbody>
<tr>
<td>2</td>
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<td>5</td>
<td>-8</td>
</tr>
<tr>
<td>6</td>
<td>-5</td>
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</tbody>
</table>

12a. \(-9; x = 3; 3\)  
12b. \( x / f(x) \)

<table>
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<tr>
<td>1</td>
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<tr>
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<tr>
<td>5</td>
<td>1</td>
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12c. \( f(x) = x^2 - 8x + 7 \)

(continued on the next page)
Answers

13a. \(-3; x = -2; -2\)

13b. 

<table>
<thead>
<tr>
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<tbody>
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13c. 

<table>
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14a. \(x = -\frac{3}{2}, -\frac{3}{2}\)

14b. 

<table>
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14c. 

<table>
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6-3 Solving Quadratic Equations by Factoring

Concept Summary

- Zero Product Property: For any real numbers \(a\) and \(b\), if \(ab = 0\), then either \(a = 0\), \(b = 0\), or both \(a\) and \(b = 0\).

Example

Solve \(x^2 + 9x + 20 = 0\) by factoring.

Original equation:

\(x^2 + 9x + 20 = 0\)

Factor the trinomial:

\((x + 4)(x + 5) = 0\)

Zero Product Property:

\(x = -4\) or \(x = -5\)

The solution set is \([-5, -4]\).

Exercises

Solve each equation by factoring. (See Examples 1–3 on pages 301 and 302.)

24. \(x^2 - 4x - 32 = 0\)
25. \(3x^2 + 6x + 3 = 0\)
26. \(5y^2 = 80\)
27. \(2x^2 + 18x - 44 = 0\)
28. \(25x^2 - 30x = -9\)
29. \(6x^2 + 7x = 3\)
30. \(-4, -25\)
31. \(10, -7\)
32. \(1, 2\)
33. \(3x^2 - 7x + 2 = 0\)

6-4 Completing the Square

Concept Summary

- To complete the square for any quadratic expression \(x^2 + bx\):
  
  Step 1 Find one half of \(b\), the coefficient of \(x\).
  
  Step 2 Square the result in Step 1.
  
  Step 3 Add the result of Step 2 to \(x^2 + bx\).
  
  \(x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2\)

Example

Solve \(x^2 + 10x - 39 = 0\) by completing the square.

Original equation:

\(x^2 + 10x - 39 = 0\)

Rewrite as two equations.

\(x^2 + 10x = 39\)

\(x^2 + 10x + 25 = 39 + 25\)

Write the left side as a perfect square by factoring.

\((x + 5)^2 = 64\)

Square Root Property:

\(x + 5 = 8\) or \(x + 5 = -8\)

Rewrite as two equations.

\(x = 3\) or \(x = -13\)

The solution set is \((-13, 3)\).

Exercises

Find the value of \(c\) that makes each trinomial a perfect square. Then write the trinomial as a perfect square. (See Example 3 on page 307.)

33. \(x^2 + 34x + c\)
34. \(x^2 - 11x + c\)
35. \(x^2 + \frac{7}{2}x + c\)
36. \(2x^2 - 7x - 15 = 0\)
37. \(2n^2 - 12n - 22 = 0\)
38. \(2x^2 - 5x + 7 = 3\)
6-5 The Quadratic Formula and the Discriminant

**Concept Summary**
- Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a \neq 0$

Solve $x^2 - 5x - 66 = 0$ by using the Quadratic Formula.

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-66)}}{2(1)}$

$x = \frac{5 \pm 17}{2}$

$x = \frac{5 + 17}{2}$ or $x = \frac{5 - 17}{2}$ Write as two equations.

$x = 11$ $x = -6$ The solution set is $\{11, -6\}$.

**Exercises** Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

(See Examples 1–4 on pages 314–316.) 39–41. See margin.

39. $x^2 + 2x + 7 = 0$ 40. $-2x^2 + 12x - 5 = 0$ 41. $3x^2 + 7x - 2 = 0$

6-6 Analyzing Graphs of Quadratic Functions

**Concept Summary**
- As the values of $h$ and $k$ change, the graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated
- $|h|$ units left if $h$ is negative or $|h|$ units right if $h$ is positive.
- $|k|$ units up if $k$ is positive or $|k|$ units down if $k$ is negative.
- Consider the equation $y = a(x - h)^2 + k$.
- If $a > 0$, the graph opens up; if $a < 0$ the graph opens down.
- If $|a| > 1$, the graph is narrower than the graph of $y = x^2$.
- If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

**Example** Write the quadratic function $y = 3x^2 + 42x + 142$ in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

$y = 3x^2 + 42x + 142$ Original equation
$y = 3(x^2 + 14x) + 142$ Group $ax^2 + bx$ and factor, dividing by $a$.
$y = 3(x^2 + 14x + 49) + 142 - 3(49)$ Complete the square by adding $\frac{14^2}{2}$. Balance this with a subtraction of $3(49)$.
$y = 3(x + 7)^2 - 5$ Write $x^2 + 14x + 7$ as a perfect square.

So, $a = 3$, $h = -7$, and $k = -5$. The vertex is at $(-7, -5)$, and the axis of symmetry is $x = -7$. Since $a$ is positive, the graph opens up.
Exercises  Write each equation in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. (See Examples 1 and 3 on pages 322 and 324.) 42–44. See margin.

42. \(y = -6(x + 2)^2 + 3\)  
43. \(y = 5x^2 + 35x + 58\)  
44. \(y = -\frac{3}{4}x^2 + 8x\)

Graph each function. (See Examples 1–3 on pages 322 and 324.) 45–47. See margin.

45. \(y = (x - 2)^2 - 2\)  
46. \(y = 2x^2 + 8x + 10\)  
47. \(y = -9x^2 - 18x - 6\)

Write an equation for the parabola with the given vertex that passes through the given point. (See Example 4 on page 325.)

48. vertex: \((4, 1)\) point: \((2, 13)\)  
49. vertex: \((-2, 3)\) point: \((2, 13)\)  
50. vertex: \((-3, -5)\) point: \((0, -4)\)

\[y = 3(x - 4)^2 + 1\]  
\[y = \frac{1}{2}(x + 2)^2 + 3\]  
\[y = -(x + 3)^2 - 5\]

6–7 Graphing and Solving Quadratic Inequalities

Concept Summary
- Graph quadratic inequalities in two variables as follows.

1. Graph the related quadratic equation, \(y = ax^2 + bx + c\). Decide if the parabola should be solid or dashed.
2. Test a point \((x_1, y_1)\) inside the parabola. Check to see if this point is a solution of the inequality.
3. If \((x_1, y_1)\) is a solution, shade the region inside the parabola. If \((x_1, y_1)\) is not a solution, shade the region outside the parabola.

- To solve a quadratic inequality in one variable, graph the related quadratic function. Identify the \(x\) values for which the graph lies below the \(x\)-axis for \(<\) and \(\leq\). Identify the \(x\) values for which the graph lies above the \(x\)-axis for \(>\) and \(\geq\).

Example
Solve \(x^2 + 3x - 10 < 0\) by graphing.

Find the roots of the related equation.

\(0 = x^2 + 3x - 10\)  
\(0 = (x + 5)(x - 2)\)  
\(x + 5 = 0\) or \(x - 2 = 0\)  
Zero Product Property

\(x = -5\)  
\(x = 2\)  
Solve each equation.

Sketch the graph of the parabola that has \(x\)-intercepts at \(-5\) and \(2\). The graph should open up since \(a > 0\). The graph lies below the \(x\)-axis between \(x = -5\) and \(x = 2\). Therefore, the solution set is \([-5 < x < 2]\).

Exercises  Graph each inequality. (See Example 1 on pages 329 and 330.) 51–53. See margin.

51. \(y > x^2 - 5x + 15\)  
52. \(y \leq 4x^2 - 36x + 17\)  
53. \(y \geq -x^2 + 7x - 11\)

Solve each inequality. (See Examples 2, 3, and 5 on pages 330–332.) 54–59. See pp. 343A–343F.

54. \(6x^2 + 5x > 4\)  
55. \(8x + x^2 \geq -16\)  
56. \(2x^2 + 5x < 12\)

57. \(2x^2 - 5x > 3\)  
58. \(4x^2 - 9 \leq -4x\)  
59. \(3x^2 - 5 > 6x\)
Choose the word or term that best completes each statement.

1. The \( y \)-coordinate of the vertex of the graph of \( y = ax^2 + bx + c \) is the (maximum, minimum) value obtained by the function when \( a \) is positive.

2. (The Square Root Property, Completing the square) can be used to solve any quadratic equation.

Complete parts a–c for each quadratic function.

a. Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

10. \( f(x) = x^2 - 2x + 5 \)  
   11. \( f(x) = -3x^2 + 8x \)  
   12. \( f(x) = -2x^2 - 7x - 1 \)

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

13. \( f(x) = x^2 + 6x + 9 \)  
   14. \( f(x) = 3x^2 - 12x - 24 \)  
   15. \( f(x) = -x^2 + 4x \)

Write a quadratic equation with roots \(-4\) and \(5\). Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a \), \( b \), and \( c \) are integers. \( x^2 - x - 20 = 0 \)

Solve each equation using the method of your choice. Find exact solutions.

10. \( x^2 + x - 42 = 0 \)  
   11. \( -1.6x^2 - 3.2x + 18 = 0 \)  
   12. \( 15x^2 + 16x - 7 = 0 \)

13. \( x^2 + 8x - 48 = 0 \)  
   14. \( x^2 + 12x + 11 = 0 \)  
   15. \( x^2 - 9x - 19 = 0 \)

16. \( 3x^2 + 7x - 31 = 0 \)  
   17. \( 10x^2 + 3x = 1 \)  
   18. \( -11x^2 - 174x + 221 = 0 \)

19. **BALLOONING** At a hot-air balloon festival, you throw a weighted marker straight down from an altitude of 250 feet toward a bull’s eye below. The initial velocity of the marker when it leaves your hand is 28 feet per second. Find how long it will take the marker to hit the target by solving the equation \(-16t^2 - 28t + 250 = 0 \). about \( 3.17 \) s

Write each equation in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

20. \( y = (x + 2)^2 - 3 \)  
   21. \( y = x^2 + 10x + 27 \)  
   22. \( y = -9x^2 + 54x - 8 \)

Graph each inequality.

23. \( y \leq x^2 + 6x - 7 \)  
   24. \( y > -2x^2 + 9 \)  
   25. \( y \geq -\frac{1}{2}x^2 - 3x + 1 \)

Solve each inequality.

26. \( x - 5(5x + 7) < 0 \)  
   27. \( 3x^2 \geq 16 \)  
   28. \( -5x^2 + x + 2 < 0 \)

29. **PETS** A rectangular turtle pen is 6 feet long by 4 feet wide. The pen is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new pen? \( 8 \) ft by \( 6 \) ft

30. **STANDARDIZED TEST PRACTICE** Which of the following is the sum of both solutions of the equation \( x^2 + 8x - 48 = 0 \)?

\[ \begin{align*}  
\text{(A) } & -16 \\  
\text{(B) } & -8 \\  
\text{(C) } & -4 \\  
\text{(D) } & 12 
\end{align*} \]

**Vocabulary and Concepts**

- **Choose the word or term that best completes each statement.**
  1. The \( y \)-coordinate of the vertex of the graph of \( y = ax^2 + bx + c \) is the (maximum, minimum) value obtained by the function when \( a \) is positive.
  2. (The Square Root Property, Completing the square) can be used to solve any quadratic equation.

**Skills and Applications**

- **Complete parts a–c for each quadratic function.**
  - a. Find the \( y \)-intercept, the equation of the axis of symmetry, and the \( x \)-coordinate of the vertex.
  - b. Make a table of values that includes the vertex.
  - c. Use this information to graph the function.

- **Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.**
  6. \( f(x) = x^2 + 6x + 9 \) min.; 0  
  7. \( f(x) = 3x^2 - 12x - 24 \) min.; -36  
  8. \( f(x) = -x^2 + 4x \) max.; 4

- **Write a quadratic equation with roots \(-4\) and \(5\). Write the equation in the form \( ax^2 + bx + c = 0 \), where \( a \), \( b \), and \( c \) are integers. \( x^2 - x - 20 = 0 \)

- **Solve each equation using the method of your choice. Find exact solutions.**
  10. \( x^2 + x - 42 = 0 \)  
  11. \( -1.6x^2 - 3.2x + 18 = 0 \)  
  12. \( 15x^2 + 16x - 7 = 0 \)

- **Graph each inequality.**
  23. \( y \leq x^2 + 6x - 7 \)  
  24. \( y > -2x^2 + 9 \)  
  25. \( y \geq -\frac{1}{2}x^2 - 3x + 1 \)

- **Solve each inequality.**
  26. \( x - 5(5x + 7) < 0 \)  
  27. \( 3x^2 \geq 16 \)  
  28. \( -5x^2 + x + 2 < 0 \)

**Portfolio Suggestion**

**Introduction** In this chapter quadratic equations have been graphed and solved using many different methods, often following a process that involved numerous steps.

**Ask Students** Select an item from this chapter that shows your best work, including a graph, and place it in your portfolio. Explain why you believe it to be your best work and how you came to choose this particular piece of work.
These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the Chapter 6 Resource Masters.

**Part 1 Multiple Choice**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In a class of 30 students, half are girls and 24 ride the bus to school. If 4 of the girls do not ride the bus to school, how many boys ride the bus to school? 
   - A 2 
   - B 11 
   - C 13 
   - D 15

2. In the figure below, the measures of $\angle m + \angle n + \angle p = ?$ 
   - A 90 
   - B 180 
   - C 270 
   - D 360

3. Of the points $(−4, −2), (1, −3), (−1, 3), (3, 1),$ and $(−2, 1),$ which three lie on the same side of the line $y = x − 0?$ 
   - A $(−4, −2), (1, −3), (−2, 1)$ 
   - B $(−4, −2), (1, −3), (3, 1)$ 
   - C $(−4, −2), (−1, 3), (−2, 1)$ 
   - D $(1, −3), (−1, 3), (3, 1)$

4. If $k$ is an integer, then which of the following must also be integers? 
   - A I only 
   - B II only 
   - C I and II 
   - D II and III

\[
\begin{array}{llll}
\text{I.} & \frac{5k + 5}{5k} & \text{II.} & \frac{5k + 5}{k + 1} \\
\text{III.} & \frac{5k^2 + k}{5k} \\
\end{array}
\]

5. Which of the following is a factor of $x^2 − 7x − 8?$ 
   - A $x + 2$ 
   - B $x − 1$ 
   - C $x − 4$ 
   - D $x − 8$

6. If $x > 0,$ then $\frac{\sqrt{16x^2 + 64x + 64}}{x + 2} = ?$ 
   - A 2 
   - B 4 
   - C 8 
   - D 16

7. If $x$ and $y$ are both greater than zero and $4x^3 + 4y^3 − 3xy − 3 = 0,$ then what is the value of $x$ in terms of $y?$ 
   - A $\frac{3}{x}$ 
   - B $\frac{3}{4y}$ 
   - C $\frac{11}{4x}$ 
   - D $\frac{11}{4y}$

8. For all positive integers $n,$ $\sqrt[n]{n} = 3\sqrt{n}.$ Which of the following equals 12? 
   - A $\frac{4}{12}$ 
   - B $\frac{8}{12}$ 
   - C $\frac{16}{12}$ 
   - D $\frac{32}{12}$

9. Which number is the sum of both solutions of the equation $x^2 − 3x − 18 = 0?$ 
   - A $−6$ 
   - B $−3$ 
   - C $3$ 
   - D $6$

10. One of the roots of the polynomial $6x^2 + kx + 20 = 0$ is $\frac{5}{2}.$ What is the value of $k?$ 
    - A $−23$ 
    - B $\frac{4}{3}$ 
    - C $23$ 
    - D $\frac{7}{3}$

**Additional Practice**

See pp. 373–374 in the Chapter 6 Resource Masters for additional standardized test practice.

---

**Test-Taking Tip**

Questions 8, 11, 13, 16, 21, and 27  Be sure to use the information that describes the variables in any standardized test item. For example, if an item says that $x > 0,$ check to be sure that your solution for $x$ is not a negative number.

**TestCheck and Worksheet Builder**

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

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Log On for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com
21. If \( s \) is a positive integer, what is the value of \( s \)?

22. In \( \triangle ABC \), side \( AB \) has length 8, and side \( BC \) has length 4.

23. What is the length of side \( AC \)?

24. If \( x^2 = 36 \) and \( y^2 = 9 \), what is the greatest possible value of \( (x - y)^2 \)?

25. If \( x^2 - y^2 = 42 \) and \( x + y = 6 \), what is the value of \( x - y \)?

26. By what amount does the sum of the roots exceed the product of the roots of the equation \((x - 7)(x + 3) = 0\)?

27. If \( x^2 + 12x + 36 = 0 \), what is the value of \( x \)?

28. The measure of side \( x \) is 71°. Which is greater, the measure of side \( y \) or the measure of side \( x \)?

www.algebra2.com/standardized_test
22a. $-5; x = 2; 2$

22b. 

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</tr>
<tr>
<td>4</td>
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</table>

22c. $f(x) = x^2 - 4x - 5$

22d. $(2, -9)$

23a. $36; x = -6; -6$

23b. 

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<td>-5</td>
<td>1</td>
</tr>
<tr>
<td>-4</td>
<td>4</td>
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</tbody>
</table>

23c. $f(x) = x^2 + 12x + 36$

24a. $-1; x = -1; -1$

24b. 

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<td>8</td>
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</table>

24c. $f(x) = 3x^2 + 6x - 1$

25a. $-3; x = 2, 2$

25b. 

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<td>3</td>
</tr>
<tr>
<td>4</td>
<td>-3</td>
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</tbody>
</table>

25c. $f(x) = -2x^2 + 8x - 3$

26a. $0; x = -\frac{2}{3}; -\frac{2}{3}$

26b. 

<table>
<thead>
<tr>
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<th>f(x)</th>
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<tbody>
<tr>
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26c. $f(x) = -3x^2 - 4x$

27a. $0; x = -\frac{5}{4}; -\frac{5}{4}$

27b. 

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27c. $f(x) = 2x^2 + 5x$

28a. $-1; x = 0; 0$

28b. 

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<td>-\frac{1}{2}</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
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</table>

28c. $f(x) = 0.5x^2 - 1$

29a. $0; x = -6; -6$

29b. 

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<td>8.75</td>
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<tr>
<td>-4</td>
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29c. $f(x) = -0.25x^2 - 3x$

30a. $\frac{9}{2}; x = -3, -3$

30b. 

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<th>f(x)</th>
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<tbody>
<tr>
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<td>-3</td>
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<td>-2</td>
<td>0.5</td>
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<tr>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

30c. $f(x) = \frac{1}{2}x^2 + 3x + \frac{5}{2}$

31a. $-\frac{8}{9}; x = \frac{1}{3}; \frac{1}{3}$

31b. 

<table>
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<th>f(x)</th>
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<tbody>
<tr>
<td>-1</td>
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<td>-\frac{8}{9}</td>
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<tr>
<td>\frac{1}{3}</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-\frac{5}{9}</td>
</tr>
<tr>
<td>2</td>
<td>\frac{17}{9}</td>
</tr>
</tbody>
</table>

31c. $f(x) = x^2 - \frac{2}{3}x - \frac{1}{3}$
39. Let \( x \) be the first number. Then, \( 7 - x \) is the other number.
\[
x(7 - x) = 14
\]
\[
-x^2 + 7x - 14 = 0
\]
Since the graph of the related function does not intersect the \( x \)-axis, this equation has no real solutions. Therefore no such numbers exist.

40. Let \( x \) be the first number. Then, \( -9 - x \) is the other number.
\[
x(-9 - x) = 24
\]
\[
-x^2 - 9x - 24 = 0
\]
Since the graph of the related function does not intersect the \( x \)-axis, this equation has no real solutions. Therefore no such numbers exist.

48. Answers should include the following.

- Locate the positive \( x \)-intercept at about 3.4. This represents the time when the height of the ride is 0. Thus, if the ride were allowed to fall to the ground, it would take about 3.4 seconds.

Page 300, Follow-Up of Lesson 6-2
Graphing Calculator Investigation
1. linear: \( y = 4.343x - 89.669; \) quadratic: \( y = 0.044x^2 - 0.003x + 0.218 \)

59. \[ \begin{align*}
x + y &= 9 \\
y - x &= 4 \\
x + y &= 6
\end{align*} \]

Page 301, Preview of Lesson 6-6
Graphing Calculator Investigation
4. Both graphs have the same shape, but the graph of \( y = x^2 + 2.5 \) is 2.5 units above the graph of \( y = x^2 \).
5. Both graphs have the same shape, but the graph of \(y = -x^2\) opens downward while the graph of \(y = x^2 - 9\) opens upward and is 9 units lower than the graph of \(y = x^2\).

6. The graph of \(y = 3x^2\) is narrower than the graph of \(y = x^2\).

7. The graph of \(y = -6x^2\) opens downward and is narrower than the graph of \(y = x^2\).

8. The graphs have the same shape, but the graph of \(y = (x + 3)^2\) is 3 units to the left of the graph of \(y = x^2\).

9. The graphs have the same shape and open downward, but the graph of \(y = -\frac{1}{3}x^2 + 2\) is 2 units above the graph of \(y = -\frac{1}{3}x^2\).

10. The graphs have the same shape, but the graph of \(y = (x - 7)^2\) is 7 units to the right of the graph of \(y = x^2\).

11. The graph of \(y = 3(x + 4)^2 - 7\) is 4 units to the left, 7 units below, and narrower than the graph of \(y = x^2\).

12. The graph of \(y = -\frac{1}{4}x^2 + 1\) opens downward, is wider than and 1 unit above the graph of \(y = -\frac{1}{4}x^2\).

13. The graphs have the same shape, but the graph of \(y = (x + 3)^2 + 5\) is 7 units above the graph of \(y = (x + 3)^2 - 2\).

14. The graph of \(y = 6(x + 2)^2 - 1\) is narrower than the graph of \(y = 3(x + 2)^2 - 1\).

15. The graph of \(y = \frac{1}{4}(x - 2)^2 - 1\) is wider than the graph of \(y = 4(x - 2)^2 - 3\), and its vertex is 2 units above the vertex of \(y = 4(x - 2)^2 - 3\).
53. \( y = ax^2 + bx + c \) 
   \( y = a(x^2 + \frac{b}{a}x) + c \) 
   \( y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b}{2a}\right)^2 \) 
   \( y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \) 
   The axis of symmetry is \( x = h \) or \( -\frac{b}{2a} \).

54. All quadratic equations are a transformation of the parent graph \( y = x^2 \). By identifying these transformations when a quadratic function is written in vertex form, you can redraw the graph of \( y = x^2 \). Answers should include the following.
   - In the equation \( y = a(x - h)^2 + k \), \( h \) translated the graph of \( y = x^2 \) \( h \) units to the right when \( h \) is positive and \( h \) units to the left when \( h \) is negative. The graph of \( y = x^2 \) translated \( k \) units up when \( k \) is positive and \( k \) units down when \( k \) is negative. When \( a \) is positive, the graph opens upward, and when \( a \) is negative, the graph opens downward. If the absolute value of \( a \) is less than 1, the graph will be narrower than the graph of \( y = x^2 \), and if the absolute value of \( a \) is greater than 1, the graph will be wider than the graph of \( y = x^2 \).
   - Sample answer: \( y = 2(x + 2)^2 - 3 \) is the graph of \( y = x^2 \) translated 2 units left and 3 units down. The graph opens upward, but is narrower that the graph of \( y = x^2 \).
Page 341, Chapter 6 Practice Test

3a. \((0, 5); x = 1; 1\)

3b.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

3c. \(f(x) = x^2 - 2x + 5\)

4a. \((0, 0); x = \frac{4}{3}, \frac{4}{3}\)

4b.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>(\frac{4}{3})</td>
<td>(\frac{16}{3})</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

5a. \((0, -1); x = -\frac{7}{4}, -\frac{7}{4}\)

5b.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>-(\frac{7}{4})</td>
<td>-(\frac{41}{8})</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Chapter 6 Additional Answers

20. \((-2, -3); x = -2; \text{up}\)

21. \(y = (x + 5)^2 + 2; (-5, 2); x = -5; \text{up}\)

22. \(y = -9(x - 3)^2 + 73; (3, 73); x = 3; \text{down}\)

23. \(y = x^2 + 6x - 7\)

24. \(y = -2x^2 + 9\)

25. \(y = -\frac{1}{2}x^2 - 3x + 1\)

26. \(x| x \leq -\frac{4\sqrt{3}}{3} \text{ or } x \geq \frac{4\sqrt{3}}{3}\)

28. \(x| x < -\frac{1 - \sqrt{41}}{10} \text{ or } x > \frac{1 + \sqrt{41}}{10}\)