12-3 Study Guide and Intervention

Surface Areas of Prisms

Lateral Areas of Prisms Here are some characteristics of prisms.

- The bases are parallel and congruent.
- The lateral faces are the faces that are not bases.
- The lateral faces intersect at lateral edges, which are parallel.
- The altitude of a prism is a segment that is perpendicular to the bases with an endpoint in each base.
- For a right prism, the lateral edges are perpendicular to the bases. Otherwise, the prism is oblique.

| Lateral Area of a Prism | If a prism has a lateral area of \( L \) square units, a height of \( h \) units, and each base has a perimeter of \( P \) units, then \( L = Ph \). |

Example Find the lateral area of the regular pentagonal prism above if each base has a perimeter of 75 centimeters and the altitude is 10 centimeters.

\[
L = Ph \\
= 75(10) \\
= 750 \\
\text{Multiply.}
\]

The lateral area is 750 square centimeters.

Exercise

Find the lateral area of each prism.

1. \( 120 \text{ m} \)

2. \( 400 \text{ in}^2 \)

3. \( 540 \text{ in}^2 \)

4. \( 588 \text{ cm}^2 \)

5. \( 192 \text{ in}^2 \)

6. \( 384 \text{ m}^2 \)
Surface Areas of Prisms  The surface area of a prism is the lateral area of the prism plus the areas of the bases.

<table>
<thead>
<tr>
<th>Surface Area of a Prism</th>
<th>If the total surface area of a prism is $T$ square units, its height is $h$ units, and each base has an area of $B$ square units and a perimeter of $P$ units, then $T = L + 2B$.</th>
</tr>
</thead>
</table>

**Example**  Find the surface area of the triangular prism above.

Find the lateral area of the prism.

$L = Ph$

$L = (18)(10)$

$L = 180$ cm$^2$

Multiply.

Find the area of each base. Use the Pythagorean Theorem to find the height of the triangular base.

$h^2 + 3^2 = 6^2$

$h^2 = 27$

$h = 3\sqrt{3}$

Take the square root of each side.

$B = \frac{1}{2} \times \text{base} \times \text{height}$  

$= \frac{1}{2} \times 6 \times 3\sqrt{3}$  

$= 9\sqrt{3}$ cm$^2$

Area of a triangle

$= 15.6$ cm$^2$

The total area is the lateral area plus the area of the two bases.

$T = 180 + 2(15.6)$  

$= 211.2$ cm$^2$

Simplify.

**Exercises**

Find the surface area of each prism. Round to the nearest tenth if necessary.

1. 

2. 

3. 

4. 

5. 

6. 

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Glencoe Geometry
12-4 Study Guide and Intervention
Surface Areas of Cylinders

Lateral Areas of Cylinders A cylinder is a solid whose bases are congruent circles that lie in parallel planes. The axis of a cylinder is the segment whose endpoints are the centers of these circles. For a right cylinder, the axis and the altitude of the cylinder are equal. The lateral area of a right cylinder is the circumference of the cylinder multiplied by the height.

| Lateral Area of a Cylinder | If a cylinder has a lateral area of $L$ square units, a height of $h$ units, and the bases have radii of $r$ units, then $L = 2\pi rh$. |

Example Find the lateral area of the cylinder above if the radius of the base is 6 centimeters and the height is 14 centimeters.

$L = 2\pi rh$  
$L = 2\pi (6)(14)$  
$L = 527.8$  

The lateral area is about 527.8 square centimeters.

Exercises

Find the lateral area of each cylinder. Round to the nearest tenth.

1.  
2.  
3.  
4.  
5.  
6.  

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12-4 Study Guide and Intervention (continued)

Surface Areas of Cylinders

Surface Areas of Cylinders The surface area of a cylinder is the lateral area of the cylinder plus the areas of the bases.

| Surface Area of a Cylinder | If a cylinder has a surface area of $T$ square units, a height of $h$ units, and the bases have radii of $r$ units, then $T = 2\pi rh + 2\pi r^2$. |

**Example**

Find the surface area of the cylinder.

Find the lateral area of the cylinder. If the diameter is 12 centimeters, then the radius is 6 centimeters.

$L = Ph$

| Lateral area of a cylinder | $P = 2\pi r$
| $= (2\pi r)h$ | $= 2\pi (6)(14)$ | $= 527.8$ |
| $r = 6$, $h = 14$ | Simplify. |

Find the area of each base.

$B = \pi r^2$

| Area of a circle | $= \pi (6)^2$ |
| $r = 6$ | $= 113.1$ |

Simplify.

The total area is the lateral area plus the area of the two bases.

$T = 527.8 + 113.1 + 113.1$ or 754 square centimeters.

**Exercises**

Find the surface area of each cylinder. Round to the nearest tenth.

1. \[ \text{10 in.} \quad \text{12 in.} \]

\[ 603.2 \text{ in}^2 \]

2. \[ 2 \text{ m} \]

\[ 50.3 \text{ m}^2 \]

3. \[ 3 \text{ yd} \quad 2 \text{ yd} \]

\[ 94.2 \text{ yd}^2 \]

4. \[ 8 \text{ in.} \quad 12 \text{ in.} \]

\[ 1005.3 \text{ in}^2 \]

5. \[ 2 \text{ m} \quad 15 \text{ m} \]

\[ 213.6 \text{ m}^2 \]

6. \[ 8 \text{ in.} \quad 20 \text{ in.} \]

\[ 1407.4 \text{ in}^2 \]
12-5 Study Guide and Intervention

Surface Areas of Pyramids

Lateral Areas of Regular Pyramids  Here are some properties of pyramids.
- The base is a polygon.
- All of the faces, except the base, intersect in a common point known as the vertex.
- The faces that intersect at the vertex, which are called lateral faces, are triangles.

For a regular pyramid, the base is a regular polygon and the slant height is the height of each lateral face.

| Lateral Area of a | If a regular pyramid has a lateral area of \( L \) square units, a slant height of \( \ell \) units, and its base has a perimeter of \( P \) units, then \( L = \frac{1}{2}Pl \). |

**Example**  The roof of a barn is a regular octagonal pyramid. The base of the pyramid has sides of 12 feet, and the slant height of the roof is 15 feet. Find the lateral area of the roof.
The perimeter of the base is 8(12) or 96 feet.

\[
L = \frac{1}{2}Pl = \frac{1}{2}(96)(15) = \frac{1}{2}(1440) = 720
\]

Multiply.
The lateral area is 720 square feet.

**Exercises**  Find the lateral area of each regular pyramid. Round to the nearest tenth if necessary.

1. \[
\frac{1}{2}(24)(15) = 180 \text{ cm}^2
\]

2. \[
\frac{1}{2}(14)(10) = 70 \text{ ft}^2
\]

3. \[
\frac{1}{2}(100)(42) = 2100 \text{ m}^2
\]

4. \[
\frac{1}{2}(24)(37.3) = 62.4 \text{ ft}^2
\]

5. \[
\frac{1}{2}(90)(4.5) = 201.5 \text{ in}^2
\]

6. \[
\frac{1}{2}(72)(6) = 216 \text{ yd}^2
\]
12-5 Study Guide and Intervention (continued)

Surface Areas of Pyramids

The surface area of a regular pyramid is the lateral area plus the area of the base.

| Surface Area of a Regular Pyramid | If a regular pyramid has a surface area of $T$ square units, a slant height of $l$ units, and its base has a perimeter of $P$ units and an area of $B$ square units, then $T = \frac{1}{2}Pl + B$. |

**Example**

For the regular square pyramid above, find the surface area to the nearest tenth if each side of the base is 12 centimeters and the height of the pyramid is 8 centimeters.

Look at the pyramid above. The slant height is the hypotenuse of a right triangle. One leg of that triangle is the height of the pyramid, and the other leg is half the length of a side of the base. Use the Pythagorean Theorem to find the slant height $l$.

$$l^2 = 6^2 + 8^2 \quad \text{Pythagorean Theorem}$$
$$= 36 + 64$$
$$= 100 \quad \text{Simplify.}$$
$$l = 10 \quad \text{Take the square root of each side.}$$

$$T = \frac{1}{2}Pl + B \quad \text{Surface area of a pyramid}$$
$$= \frac{1}{2}(4)(12)(10) + 12^2 \quad P = (4)(12), \quad l = 10, \quad B = 12^2$$
$$= 384 \quad \text{Simplify.}$$

The surface area is 384 square centimeters.

**Exercises**

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1. \[ \text{base} = 9.7 \text{ cm} \] \[ \text{slant} = 45.6 \text{ cm} \] \[ \text{total} = 547.4 \text{ cm}^2 \]

2. \[ \text{base} = 25.6 \text{ ft} \] \[ \text{total} = 618 \text{ ft}^2 \]

3. \[ \text{base} = 13.3 \text{ cm} \] \[ \text{total} = 397.1 \text{ cm}^2 \]

4. \[ \text{base} = 136.5 \text{ in.} \] \[ \text{total} = 456.8 \text{ in}^2 \]

5. \[ \text{base} = 12 \text{ cm} \] \[ \text{total} = 360 \text{ cm}^2 \]

6. \[ \text{base} = 100 \text{ yd} \] \[ \text{total} = 340 \text{ yd}^2 \]
12-6 Study Guide and Intervention

Surface Areas of Cones

Lateral Areas of Cones  Cones have the following properties.
- A cone has one circular base and one vertex.
- The segment whose endpoints are the vertex and the center of the base is the axis of the cone.
- The segment that has one endpoint at the vertex, is perpendicular to the base, and has its other endpoint on the base is the altitude of the cone.
- For a right cone the axis is also the altitude, and any segment from the circumference of the base to the vertex is the slant height \( \ell \). If a cone is not a right cone, it is oblique.

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>If a cone has a lateral area of ( L ) square units, a slant height of ( \ell ) units, and the radius of the base is ( r ) units, then ( L = \pi r \ell ).</th>
</tr>
</thead>
</table>

**Example**  
Find the lateral area of a cone with slant height of 10 centimeters and a base with a radius of 6 centimeters.

\[
L = \pi r \ell \\
= \pi (6)(10) \\
= 188.5 \text{ cm}^2
\]

The lateral area is about 188.5 square centimeters.

**Exercises**  
Find lateral area of each circular cone. Round to the nearest tenth.

1. \( 47.1 \text{ cm}^2 \)
2. \( 424.1 \text{ cm}^2 \)
3. \( 391.8 \text{ m}^2 \)
4. \( 816.8 \text{ mm}^2 \)
5. \( 204.2 \text{ in}^2 \)
6. \( 284.3 \text{ yd}^2 \)
12-6 Study Guide and Intervention (continued)

Surface Areas of Cones

The surface area of a cone is the lateral area of the cone plus the area of the base.

| Surface Area of a Right Cone | If a cone has a surface area of $T$ square units, a slant height of $\ell$ units, and the radius of the base is $r$ units, then $T = \pi \ell + \pi r^2$. |

**Example**

For the cone above, find the surface area to the nearest tenth if the radius is 6 centimeters and the height is 8 centimeters.

The slant height is the hypotenuse of a right triangle with legs of length 6 and 8. Use the Pythagorean Theorem.

\[
\ell^2 = 6^2 + 8^2
\]

Pythagorean Theorem

\[
\ell^2 = 100
\]

Simplify.

\[
\ell = 10
\]

Take the square root of each side.

\[
T = \pi r \ell + \pi r^2
\]

Surface area of a cone

\[
= \pi(6)(10) + \pi \cdot 6^2
\]

$r = 6$, $\ell = 10$

Simplify.

\[
\approx 301.6
\]

The surface area is about 301.6 square centimeters.

**Exercises**

Find the surface area of each cone. Round to the nearest tenth.

1. [Diagram]

2. [Diagram]

3. [Diagram]

4. [Diagram]

5. [Diagram]

6. [Diagram]
12-7 Study Guide and Intervention (continued)

Surface Areas of Spheres

Surface Areas of Spheres You can think of the surface area of a sphere as the total area of all of the nonoverlapping strips it would take to cover the sphere. If \( r \) is the radius of the sphere, then the area of a great circle of the sphere is \( \pi r^2 \). The total surface area of the sphere is four times the area of a great circle.

| Surface Area of a Sphere | If a sphere has a surface area of \( T \) square units and a radius of \( r \) units, then \( T = 4\pi r^2 \). |

Example

Find the surface area of a sphere to the nearest tenth if the radius of the sphere is 6 centimeters.

\[
T = 4\pi r^2 \\
= 4\pi \cdot 6^2 \\
= 452.4 \quad \text{Simplify.}
\]

The surface area is 452.4 square centimeters.

Exercises

Find the surface area of each sphere with the given radius or diameter to the nearest tenth.

1. \( r = 8 \text{ cm} \) \( 804.2 \text{ cm}^2 \)
2. \( r = 2\sqrt{2} \text{ ft} \) \( 100.5 \text{ ft}^2 \)
3. \( r = \pi \text{ cm} \) \( 124 \text{ cm}^2 \)
4. \( d = 10 \text{ in.} \) \( 314.2 \text{ in}^2 \)
5. \( d = 6\pi \text{ m} \) \( 1110.2 \text{ m}^2 \)
6. \( d = 16 \text{ yd} \) \( 804.2 \text{ yd}^2 \)

7. Find the surface area of a hemisphere with radius 12 centimeters.

\[
\frac{1}{2} \text{ of sphere plus great circle} = 1357 \text{ cm}^2
\]

8. Find the surface area of a hemisphere with diameter \( \pi \) centimeters.

\[
\frac{1}{2} \text{ of sphere plus great circle} = 23.2 \text{ cm}^2
\]

9. Find the radius of a sphere if the surface area of a hemisphere is 192\( \pi \) square centimeters.

\( 8 \text{ cm} \)