Reading to Learn Mathematics

**Vocabulary Builder**

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 1. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

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Name Points, Lines, and Planes  

In geometry, a **point** is a location, a **line** contains points, and a **plane** is a flat surface that contains points and lines. If points are on the same line, they are **collinear**. If points on are the same plane, they are **coplanar**.

*Example*  
Use the figure to name each of the following.

a. a line containing point **A**  
The line can be named as \(\ell\). Also, any two of the three points on the line can be used to name it.

\[ \overline{AB}, \overline{AC}, \text{ or } \overline{BC} \]

b. a plane containing point **D**  
The plane can be named as plane \(\mathcal{N}\) or can be named using three noncollinear points in the plane, such as plane \(\overline{ABD}\), plane \(\overline{ACD}\), and so on.

*Exercises*  
Refer to the figure.

1. Name a line that contains point **A**.

2. What is another name for line \(\ell\)?

3. Name a point not on \(\overline{AC}\).

4. Name the intersection of \(\overline{AC}\) and \(\overline{DB}\).

5. Name a point not on line \(\ell\) or line \(m\).

Draw and label a plane \(Q\) for each relationship.

6. \(\overline{AB}\) is in plane \(Q\).

7. \(\overline{ST}\) intersects \(\overline{AB}\) at \(P\).

8. Point \(X\) is collinear with points \(A\) and \(P\).

9. Point \(Y\) is not collinear with points \(T\) and \(P\).

10. Line \(\ell\) contains points \(X\) and \(Y\).
Points, Lines, and Planes in Space  Space is a boundless, three-dimensional set of all points. It contains lines and planes.

**Example**

a. How many planes appear in the figure?

There are three planes: plane $\mathcal{N}$, plane $O$, and plane $P$.

b. Are points $A$, $B$, and $D$ coplanar?

Yes. They are contained in plane $O$.

**Exercises**

Refer to the figure.

1. Name a line that is not contained in plane $\mathcal{N}$.

2. Name a plane that contains point $B$.

3. Name three collinear points.

Refer to the figure.

4. How many planes are shown in the figure?


6. Name a point coplanar with $D$, $C$, and $E$.

Draw and label a figure for each relationship.

7. Planes $\mathcal{M}$ and $\mathcal{N}$ intersect in $\overline{HJ}$.

8. Line $r$ is in plane $\mathcal{N}$, line $s$ is in plane $\mathcal{M}$, and lines $r$ and $s$ intersect at point $J$.

9. Line $t$ contains point $H$ and line $t$ does not lie in plane $\mathcal{M}$ or plane $\mathcal{N}$.
Skills Practice
Points, Lines, and Planes

Refer to the figure.

1. Name a line that contains point \( D \).

2. Name a point contained in line \( n \).

3. What is another name for line \( p \)?

4. Name the plane containing lines \( n \) and \( p \).

Draw and label a figure for each relationship.

5. Point \( K \) lies on \( \overline{RT} \).

6. Plane \( J \) contains line \( s \).

7. \( \overline{YP} \) lies in plane \( \mathcal{B} \) and contains point \( C \), but does not contain point \( H \).

8. Lines \( q \) and \( f \) intersect at point \( Z \) in plane \( \mathcal{U} \).

Refer to the figure.

9. How many planes are shown in the figure?

10. How many of the planes contain points \( F \) and \( E \)?

11. Name four points that are coplanar.

12. Are points \( A \), \( B \), and \( C \) coplanar? Explain.
1-1 Practice

Points, Lines, and Planes

Refer to the figure.

1. Name a line that contains points $T$ and $P$.

2. Name a line that intersects the plane containing points $Q$, $N$, and $P$.

3. Name the plane that contains $TN$ and $QR$.

Draw and label a figure for each relationship.

4. $\overline{AK}$ and $\overline{CG}$ intersect at point $M$ in plane $T$.

5. A line contains $L(-4, -4)$ and $M(2, 3)$. Line $q$ is in the same coordinate plane but does not intersect $LM$. Line $q$ contains point $N$.

Refer to the figure.

6. How many planes are shown in the figure?

7. Name three collinear points.


VISUALIZATION Name the geometric term(s) modeled by each object.

9. stop

10. tip of pin

11. strings

12. a car antenna

13. a library card
Reading to Learn Mathematics

Points, Lines, and Planes

Pre-Activity  Why do chairs sometimes wobble?

Read the introduction to Lesson 1-1 at the top of page 6 in your textbook.

• Find three pencils of different lengths and hold them upright on your desk so that the three pencil points do not lie along a single line. Can you place a flat sheet of paper or cardboard so that it touches all three pencil points?
• How many ways can you do this if you keep the pencil points in the same position?
• How will your answer change if there are four pencil points?

Reading the Lesson

1. Complete each sentence.
   a. Points that lie on the same lie are called ________________ points.
   b. Points that do not lie in the same plane are called ________________ points.
   c. There is exactly one ________________ through any two points.
   d. There is exactly one ________________ through any three noncollinear points.

2. Refer to the figure at the right. Indicate whether each statement is true or false.
   a. Points A, B, and C are collinear.
   b. The intersection of plane ABC and line m is point P.
   c. Line ℓ and line m do not intersect.
   d. Points A, P, and B can be used to name plane ℋ.
   e. Line ℓ lies in plane ACB.

3. Complete the figure at the right to show the following relationship: Lines ℓ, m, and n are coplanar and lie in plane Q. Lines ℓ and m intersect at point P. Line n intersects line m at R, but does not intersect line ℓ.

Helping You Remember

4. Recall or look in a dictionary to find the meaning of the prefix co-. What does this prefix mean? How can it help you remember the meaning of collinear?
Points and Lines on a Matrix

A matrix is a rectangular array of rows and columns. Points and lines on a matrix are not defined in the same way as in Euclidean geometry. A point on a matrix is a dot, which can be small or large. A line on a matrix is a path of dots that “line up.” Between two points on a line there may or may not be other points. Three examples of lines are shown at the upper right. The broad line can be thought of as a single line or as two narrow lines side by side.

Dot-matrix printers for computers used dots to form characters. The dots are often called pixels. The matrix at the right shows how a dot-matrix printer might print the letter P.

Draw points on each matrix to create the given figures.

1. Draw two intersecting lines that have four points in common.

2. Draw two lines that cross but have no common points.

3. Make the number 0 (zero) so that it extends to the top and bottom sides of the matrix.

4. Make the capital letter O so that it extends to each side of the matrix.

5. Using separate grid paper, make dot designs for several other letters. Which were the easiest and which were the most difficult?
Measure Line Segments A part of a line between two endpoints is called a line segment. The lengths of $MN$ and $RS$ are written as $MN$ and $RS$. When you measure a segment, the precision of the measurement is half of the smallest unit on the ruler.

**Example 1** Find the length of $MN$.

![Measurement of a line segment in centimeters and millimeters]

The long marks are centimeters, and the shorter marks are millimeters. The length of $MN$ is 3.4 centimeters. The measurement is accurate to within 0.5 millimeter, so $MN$ is between 3.35 centimeters and 3.45 centimeters long.

**Example 2** Find the length of $RS$.

![Measurement of a line segment in inches and quarter inches]

The long marks are inches and the short marks are quarter inches. The length of $RS$ is about $1\frac{3}{4}$ inches. The measurement is accurate to within one half of a quarter inch, or $\frac{1}{8}$ inch, so $RS$ is between $1\frac{5}{8}$ inches and $1\frac{7}{8}$ inches long.

**Exercises**

Find the length of each line segment or object.

1. $A$ $B$

![Measurement of a line segment in centimeters]

2. $S$ $T$

![Measurement of a line segment in inches]

3. 

![Measurement of a pencil]

4. 

![Measurement of a line segment in centimeters]

Find the precision for each measurement.

5. 10 in. 
6. 32 mm 
7. 44 cm 
8. 2 ft 
9. 3.5 mm 
10. $2\frac{1}{2}$ yd
1-2 Study Guide and Intervention (continued)

**Linear Measure and Precision**

**Calculate Measures** On \( PQ \), to say that point \( M \) is between points \( P \) and \( Q \) means \( P, Q, \) and \( M \) are collinear and \( PM + MQ = PQ \).

On \( AC, AB = BC = 3 \text{ cm} \). We can say that the segments are **congruent**, or \( AB \cong BC \). Slashes on the figure indicate which segments are congruent.

**Example 1** Find \( EF \).

Calculate \( EF \) by adding \( ED \) and \( DF \).

\[
ED + DF = EF \\
1.2 + 1.9 = EF \\
3.1 = EF
\]

Therefore, \( EF \) is 3.1 centimeters long.

**Example 2** Find \( x \) and \( AC \).

\( B \) is between \( A \) and \( C \).

\[
AB + BC = AC \\
x + 2x = 2x + 5 \\
3x = 2x + 5 \\
x = 5 \\
AC = 2x + 5 = 2(5) + 5 = 15
\]

**Exercises**

Find the measurement of each segment. Assume that the art is not drawn to scale.

1. \( RT \)

2. \( BC \)

3. \( XZ \)

4. \( WX \)

Find \( x \) and \( RS \) if \( S \) is between \( R \) and \( T \).

5. \( RS = 5x, ST = 3x, \) and \( RT = 48 \).

6. \( RS = 2x, ST = 5x + 4, \) and \( RT = 32 \).

7. \( RS = 6x, ST = 12, \) and \( RT = 72 \).

8. \( RS = 4x, RS \cong ST, \) and \( RT = 24 \).

Use the figures to determine whether each pair of segments is congruent.

9. \( AB \) and \( CD \)

10. \( XY \) and \( YZ \)
Skills Practice

Linear Measure and Precision

Find the length of each line segment or object.

1. [Measurement of a line segment in centimeters]

2. [Measurement of a line segment in inches]

Find the precision for each measurement.

3. 40 feet

4. 12 centimeters

5. $9\frac{1}{2}$ inches

Find the measurement of each segment.

6. $\overline{NQ}$

7. $\overline{AC}$

8. $\overline{GH}$

Find the value of the variable and $YZ$ if $Y$ is between $X$ and $Z$.

9. $XY = 5p$, $YZ = p$, and $XY = 25$

10. $XY = 12$, $YZ = 2g$, and $XZ = 28$

11. $XY = 4m$, $YZ = 3m$, and $XZ = 42$

12. $XY = 2c + 1$, $YZ = 6c$, and $XZ = 81$

Use the figures to determine whether each pair of segments is congruent.

13. $\overline{BE}$, $\overline{CD}$

14. $\overline{MP}$, $\overline{NP}$

15. $\overline{WX}$, $\overline{WZ}$
Find the length of each line segment or object.

1. \(EF\)

2. \(\star\star\star\star\star\star\star\star\star\)

Find the precision for each measurement.

3. 120 meters

4. \(7\frac{1}{4}\) inches

5. 30.0 millimeters

Find the measurement of each segment.

6. \(PS\)

7. \(AD\)

8. \(WX\)

Find the value of the variable and \(KL\) if \(K\) is between \(J\) and \(L\).

9. \(JK = 6r, KL = 3r, and JL = 27\)

10. \(JK = 2s, KL = s + 2, and JL = 5s - 10\)

Use the figures to determine whether each pair of segments is congruent.

11. \(\overline{TU}, \overline{SW}\)

12. \(\overline{AD}, \overline{BC}\)

13. \(\overline{GF}, \overline{FE}\)

14. **CARPENTRY** Jorge used the figure at the right to make a pattern for a mosaic he plans to inlay on a tabletop. Name all of the congruent segments in the figure.
Pre-Activity  Why are units of measure important?
Read the introduction to Lesson 1-2 at the top of page 13 in your textbook.
• The basic unit of length in the metric system is the meter. How many meters are there in one kilometer?
• Do you think it would be easier to learn the relationships between the different units of length in the customary system (used in the United States) or in the metric system? Explain your answer.

Reading the Lesson
1. Explain the difference between a line and a line segment and why one of these can be measured, while the other cannot.

2. What is the smallest length marked on a 12-inch ruler?
What is the smallest length marked on a centimeter ruler?

3. Find the precision of each measurement.
   a. 15 cm
   b. 15.0 cm

4. Refer to the figure at the right. Which one of the following statements is true? Explain your answer.
   \[ \overline{AB} = \overline{CD} \quad \overline{AB} \equiv \overline{CD} \]

5. Suppose that S is a point on \( \overline{VW} \) and S is not the same point as V or W. Tell whether each of the following statements is always, sometimes, or never true.
   a. \( VS = SW \)
   b. S is between V and W.
   c. \( VS + VW = SW \)

Helping You Remember
6. A good way to remember terms used in mathematics is to relate them to everyday words you know. Give three words that are used outside of mathematics that can help you remember that there are 100 centimeters in a meter.
Points Equidistant from Segments

The distance from a point to a segment is zero if the point is on the segment. Otherwise, it is the length of the shortest segment from the point to the segment.

A figure is a **locus** if it is the set of all points that satisfy a set of conditions. The locus of all points that are $\frac{1}{4}$ inch from the segment $AB$ is shown by two dashed segments with semicircles at both ends.

1. Suppose $A$, $B$, $C$, and $D$ are four different points, and consider the locus of all points $x$ units from $AB$ and $x$ units from $CD$. Use any unit you find convenient. The locus can take different forms. Sketch at least three possibilities. List some of the things that seem to affect the form of the locus.

2. Conduct your own investigation of the locus of points **equidistant** from two segments. Describe your results on a separate sheet of paper.
Distance Between Two Points

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<td>[ AB =</td>
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**Example 1**

Find \( AB \).

\[
AB = |(-4) - 2| \\
= |-6| \\
= 6
\]

**Example 2**

Find the distance between \( A(-2, -1) \) and \( B(1, 3) \).

Distance Formula:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
AB = \sqrt{(1 - (-2))^2 + (3 - (-1))^2} \\
= \sqrt{25} \\
= 5
\]

**Exercises**

Use the number line to find each measure.

1. \( BD \)
2. \( DG \)
3. \( AF \)
4. \( EF \)
5. \( BG \)
6. \( AG \)
7. \( BE \)
8. \( DE \)

Use the Pythagorean Theorem to find the distance between each pair of points.

9. \( A(0, 0), B(6, 8) \)
10. \( R(-2, 3), S(3, 15) \)
11. \( M(1, -2), N(9, 13) \)
12. \( E(-12, 2), F(-9, 6) \)

Use the Distance Formula to find the distance between each pair of points.

13. \( A(0, 0), B(15, 20) \)
14. \( O(-12, 0), P(-8, 3) \)
15. \( C(11, -12), D(6, 2) \)
16. \( E(-2, 10), F(-4, 3) \)
Midpoint of a Segment

| Midpoint on a Number Line | If the coordinates of the endpoints of a segment are \( a \) and \( b \), then the coordinate of the midpoint of the segment is \( \frac{a + b}{2} \).
| Midpoint on a Coordinate Plane | If a segment has endpoints with coordinates \( (x_1, y_1) \) and \( (x_2, y_2) \), then the coordinates of the midpoint of the segment are \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \). |

**Example 1** Find the coordinate of the midpoint of \( \overline{PQ} \).

\[
P \quad Q
\]
\[-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \]

The coordinates of \( P \) and \( Q \) are \(-3\) and \(1\).

If \( M \) is the midpoint of \( \overline{PQ} \), then the coordinate of \( M \) is \( \frac{-3 + 1}{2} = \frac{-2}{2} = -1 \) or \(-0.5\).

**Example 2** \( M \) is the midpoint of \( \overline{PQ} \) for \( P(-2, 4) \) and \( Q(4, 1) \). Find the coordinates of \( M \).

\[
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-2 + 4}{2}, \frac{4 + 1}{2} \right) = \left( \frac{2}{2}, \frac{5}{2} \right) = (1, 2.5)
\]

**Exercises**

Use the number line to find the coordinate of the midpoint of each segment.

1. \( \overline{CE} \)  
2. \( \overline{DG} \)

3. \( \overline{AF} \)  
4. \( \overline{EG} \)

5. \( \overline{AB} \)  
6. \( \overline{BG} \)

7. \( \overline{BD} \)  
8. \( \overline{DE} \)

Find the coordinates of the midpoint of a segment having the given endpoints.

9. \( A(0, 0), \overline{B(12, 8)} \)  
10. \( R(-12, 8), \overline{S(6, 12)} \)

11. \( M(11, -2), \overline{N(-9, 13)} \)  
12. \( E(-2, 6), \overline{F(-9, 3)} \)

13. \( S(10, -22), \overline{T(9, 10)} \)  
14. \( M(-11, 2), \overline{N(-19, 6)} \)
Skills Practice

Distance and Midpoints

Use the number line to find each measure.

1. LN  
2. JL  
3. KN  
4. MN

Use the Pythagorean Theorem to find the distance between each pair of points.

5.  
6.  
7. K(2, 3), F(4, 4)  
8. C(−3, −1), Q(−2, 3)

Use the Distance Formula to find the distance between each pair of points.

9. Y(2, 0), P(2, 6)  
10. W(−2, 2), R(5, 2)  
11. A(−7, −3), B(5, 2)  
12. C(−3, 1), Q(2, 6)

Use the number line to find the coordinate of the midpoint of each segment.

13. \(DE\)  
14. \(BC\)  
15. \(BD\)  
16. \(AD\)

Find the coordinates of the midpoint of a segment having the given endpoints.

17. T(3, 1), U(5, 3)  
18. J(−4, 2), F(5, −2)

Find the coordinates of the missing endpoint given that \(P\) is the midpoint of \(\overline{NQ}\).

19. \(N(2, 0), P(5, 2)\)  
20. \(N(5, 4), P(6, 3)\)  
21. \(Q(3, 9), P(−1, 5)\)
1-3 Practice

Distance and Midpoints

Use the number line to find each measure.

1. $VW$  
2. $TV$  
3. $ST$  
4. $SV$

Use the Pythagorean Theorem to find the distance between each pair of points.

5. 

6.

Use the Distance Formula to find the distance between each pair of points.

7. $L(-7, 0), Y(5, 9)$  
8. $U(1, 3), B(4, 6)$

Use the number line to find the coordinate of the midpoint of each segment.

9. $RT$  
10. $QR$  
11. $ST$  
12. $PR$

Find the coordinates of the midpoint of a segment having the given endpoints.

13. $K(-9, 3), H(5, 7)$  
14. $W(-12, -7), T(-8, -4)$

Find the coordinates of the missing endpoint given that $E$ is the midpoint of $DF$.

15. $F(5, 8), E(4, 3)$  
16. $F(2, 9), E(-1, 6)$  
17. $D(-3, -8), E(1, -2)$

18. PERIMETER The coordinates of the vertices of a quadrilateral are $R(-1, 3), S(3, 3), T(5, -1)$, and $U(-2, -1)$. Find the perimeter of the quadrilateral. Round to the nearest tenth.
1-3 Reading to Learn Mathematics

Distance and Midpoints

Pre-Activity  How can you find the distance between two points without a ruler?
Read the introduction to Lesson 1-3 at the top of page 21 in your textbook.

- Look at the triangle in the introduction to this lesson. What is the special name for AB in this triangle?
- Find AB in this figure. Write your answer both as a radical and as a decimal number rounded to the nearest tenth.

Reading the Lesson

1. Match each formula or expression in the first column with one of the names in the second column.
   a. \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)   i. Pythagorean Theorem
   b. \( \frac{a + b}{2} \)   ii. Distance Formula in the Coordinate Plane
   c. \( XY = |a - b| \)   iii. Midpoint of a Segment in the Coordinate Plane
   d. \( c^2 = a^2 + b^2 \)   iv. Distance Formula on a Number Line
   e. \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \)   v. Midpoint of a Segment on a Number Line

2. Fill in the steps to calculate the distance between the points \( M(4, -3) \) and \( N(-2, 7) \).
   Let \( (x_1, y_1) = (4, -3) \). Then \( (x_2, y_2) = (\_\_\_, \_\_\_) \).
   \[ d = \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2} \]
   \[ MN = \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2} \]
   \[ MN = \sqrt{(\_\_\_)^2 + (\_\_\_)^2} \]
   \[ MN = \sqrt{\_\_\_ + \_\_\_} \]
   \[ MN = \_\_\_ \]
   Find a decimal approximation for \( MN \) to the nearest hundredth.

Helping You Remember

3. A good way to remember a new formula in mathematics is to relate it to one you already know. If you forget the Distance Formula, how can you use the Pythagorean Theorem to find the distance \( d \) between two points on a coordinate plane?
Lengths on a Grid

Evenly-spaced horizontal and vertical lines form a grid. You can easily find segment lengths on a grid if the endpoints are grid-line intersections. For horizontal or vertical segments, simply count squares. For diagonal segments, use the Pythagorean Theorem (proven in Chapter 7). This theorem states that in any right triangle, if the length of the longest side (the side opposite the right angle) is $c$ and the two shorter sides have lengths $a$ and $b$, then $c^2 = a^2 + b^2$.

**Example** Find the measure of $EF$ on the grid at the right. Locate a right triangle with $EF$ as its longest side.

$$EF = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.4 \text{ units}$$

Find each measure to the nearest tenth of a unit.

1. $IJ$  
2. $MN$  
3. $RS$  
4. $QS$
5. $IK$  
6. $JK$  
7. $LM$  
8. $LN$

Use the grid above. Find the perimeter of each triangle to the nearest tenth of a unit.

9. $\triangle ABC$  
10. $\triangle QRS$  
11. $\triangle DEF$  
12. $\triangle LMN$

13. Of all the segments shown on the grid, which is longest? What is its length?

14. On the grid, 1 unit = 0.5 cm. How can the answers above be used to find the measures in centimeters?

15. Use your answer from exercise 8 to calculate the length of segment $LN$ in centimeters. Check by measuring with a centimeter ruler.

16. Use a centimeter ruler to find the perimeter of triangle $IJK$ to the nearest tenth of a centimeter.
### Measure Angles

If two noncollinear rays have a common endpoint, they form an **angle**. The rays are the **sides** of the angle. The common endpoint is the **vertex**. The angle at the right can be named as $\angle A$, $\angle BAC$, $\angle CAB$, or $\angle 1$.

A **right angle** is an angle whose measure is 90. An **acute angle** has measure less than 90. An **obtuse angle** has measure greater than 90 but less than 180.

#### Example 1

a. Name all angles that have $R$ as a vertex.

Three angles are $\angle 1$, $\angle 2$, and $\angle 3$. For other angles, use three letters to name them: $\angle SRQ$, $\angle PRT$, and $\angle SRT$.

b. Name the sides of $\angle 1$.

$RS$, $RP$

#### Example 2

Measure each angle and classify it as **right**, **acute**, or **obtuse**.

a. $\angle ABD$

Using a protractor, $m\angle ABD = 50$.

50 < 90, so $\angle ABD$ is an acute angle.

b. $\angle DBC$

Using a protractor, $m\angle DBC = 115$.

180 > 115 > 90, so $\angle DBC$ is an obtuse angle.

c. $\angle EBC$

Using a protractor, $m\angle EBC = 90$.

$\angle EBC$ is a right angle.

### Exercises

Refer to the figure.

1. Name the vertex of $\angle 4$.

2. Name the sides of $\angle BDC$.

3. Write another name for $\angle DBC$.

Measure each angle in the figure and classify it as **right**, **acute**, or **obtuse**.

4. $\angle MPR$

5. $\angle RPN$

6. $\angle NPS$
Congruent Angles  Angles that have the same measure are congruent angles. A ray that divides an angle into two congruent angles is called an angle bisector. In the figure, \( \overparen{PN} \) is the angle bisector of \( \angle MPR \). Point \( N \) lies in the interior of \( \angle MPR \) and \( \angle MPN \cong \angle NPR \).

**Example**  Refer to the figure above. If \( m\angle MPN = 2x + 14 \) and \( m\angle NPR = x + 34 \), find \( x \) and find \( m\angle MPR \).

Since \( \overparen{PN} \) bisects \( \angle MPR \), \( \angle MPN \equiv \angle NPR \), or \( m\angle MPN = m\angle NPR \).

\[
\begin{align*}
2x + 14 &= x + 34 \\
2x + 14 - x &= x + 34 - x \\
x + 14 &= 34 - 14 \\
x &= 20
\end{align*}
\]

**Exercises**  

\( \overparen{QS} \) bisects \( \angle PQT \), and \( \overparen{QP} \) and \( \overparen{QR} \) are opposite rays.

1. If \( m\angle PQT = 60 \) and \( m\angle PQS = 4x + 14 \), find the value of \( x \).

2. If \( m\angle PQS = 3x + 13 \) and \( m\angle SQT = 6x - 2 \), find \( m\angle PQT \).

\( \overparen{BA} \) and \( \overparen{BC} \) are opposite rays, \( \overparen{BF} \) bisects \( \angle CBE \), and \( \overparen{BD} \) bisects \( \angle ABE \).

3. If \( m\angle EBF = 6x + 4 \) and \( m\angle CBF = 7x - 2 \), find \( m\angle EBC \).

4. If \( m\angle 1 = 4x + 10 \) and \( m\angle 2 = 5x \), find \( m\angle 2 \).

5. If \( m\angle 2 = 6y + 2 \) and \( m\angle 1 = 8y - 14 \), find \( m\angle ABE \).

6. Is \( \angle DBF \) a right angle? Explain.
Skills Practice
Angle Measure

For Exercises 1–12, use the figure at the right.

Name the vertex of each angle.

1. \(\angle 4\)  
2. \(\angle 1\)
3. \(\angle 2\)  
4. \(\angle 5\)

Name the sides of each angle.

5. \(\angle 4\)  
6. \(\angle 5\)
7. \(\angle STV\)  
8. \(\angle 1\)

Write another name for each angle.

9. \(\angle 3\)  
10. \(\angle 4\)
11. \(\angle WTS\)  
12. \(\angle 2\)

Measure each angle and classify it as right, acute, or obtuse.

13. \(\angle NMP\)  
14. \(\angle OMN\)
15. \(\angle QMN\)  
16. \(\angle QMO\)

ALGEBRA In the figure, \(\overline{BA}\) and \(\overline{BC}\) are opposite rays, \(\overline{BD}\) bisects \(\angle EBC\), and \(\overline{BF}\) bisects \(\angle ABE\).

17. If \(m\angle EBD = 4x + 16\) and \(m\angle DBC = 6x + 4\), find \(m\angle EBD\).

18. If \(m\angle ABF = 7x - 8\) and \(m\angle EBF = 5x + 10\), find \(m\angle EBF\).
For Exercises 1–10, use the figure at the right.

Name the vertex of each angle.

1. \( \angle 5 \)
2. \( \angle 3 \)
3. \( \angle 8 \)
4. \( \angle NMP \)

Name the sides of each angle.

5. \( \angle 6 \)
6. \( \angle 2 \)
7. \( \angle MOP \)
8. \( \angle OMN \)

Write another name for each angle.

9. \( \angle QPR \)
10. \( \angle 1 \)

Measure each angle and classify it as right, acute, or obtuse.

11. \( \angle UZW \)
12. \( \angle YZW \)
13. \( \angle TZW \)
14. \( \angle UZT \)

ALGEBRA In the figure, \( \overline{CB} \) and \( \overline{CD} \) are opposite rays, \( \overline{CE} \) bisects \( \angle DCF \), and \( \overline{CG} \) bisects \( \angle FCB \).

15. If \( m \angle DCE = 4x + 15 \) and \( m \angle ECF = 6x - 5 \), find \( m \angle DCE \).

16. If \( m \angle FCG = 9x + 3 \) and \( m \angle GCB = 13x - 9 \), find \( m \angle GCB \).

17. TRAFFIC SIGNS The diagram shows a sign used to warn drivers of a school zone or crossing. Measure and classify each numbered angle.
1-4 Reading to Learn Mathematics

Angle Measure

Pre-Activity How big is a degree?
Read the introduction to Lesson 1-4 at the top of page 29 in your textbook.
- A semicircle is half a circle. How many degrees are there in a semicircle?
- How many degrees are there in a quarter circle?

Reading the Lesson

1. Match each description in the first column with one of the terms in the second column. Some terms in the second column may be used more than once or not at all.

a. a figure made up of two noncollinear rays with a common endpoint
b. angles whose degree measures are less than 90
c. angles that have the same measure
d. angles whose degree measures are between 90 and 180
e. a tool used to measure angles
f. the common endpoint of the rays that form an angle
g. a ray that divides an angle into two congruent angles

1. vertex
2. angle bisector
3. opposite rays
4. angle
5. obtuse angles
6. congruent angles
7. right angles
8. acute angles
9. compass
10. protractor

2. Use the figure to name each of the following.

a. a right angle
b. an obtuse angle
c. an acute angle
d. a point in the interior of \( \angle EBC \)
e. a point in the exterior of \( \angle EBA \)
f. the angle bisector of \( \angle EBC \)
g. a point on \( \angle CBE \)
h. the sides of \( \angle ABF \)
i. a pair of opposite rays
j. the common vertex of all angles shown in the figure
k. a pair of congruent angles
l. the angle with the greatest measure

Helping You Remember

3. A good way to remember related geometric ideas is to compare them and see how they are alike and how they are different. Give some similarities and differences between congruent segments and congruent angles.
Angle Relationships

Angles are measured in degrees (°). Each degree of an angle is divided into 60 minutes (’), and each minute of an angle is divided into 60 seconds (").

\[
\begin{align*}
60' &= 1° \\
60" &= 1'
\end{align*}
\]

\[
\begin{align*}
67\frac{1}{2}' &= 67°30' \\
70.4° &= 70°24' \\
90° &= 89°60'
\end{align*}
\]

Two angles are complementary if the sum of their measures is 90°. Find the complement of each of the following angles.

1. 35°15’
2. 27°16’
3. 15°54’

4. 29°18’22”
5. 34°29’45”
6. 87°2’3”

Two angles are supplementary if the sum of their measures is 180°. Find the supplement of each of the following angles.

7. 120°18’
8. 84°12’
9. 110°2’

10. 45°16’24”
11. 39°21’54”
12. 129°18’36”

13. 98°52’59”
14. 9°2’32”
15. 1°2’3”
Pairs of Angles  Adjacent angles are angles in the same plane that have a common vertex and a common side, but no common interior points. Vertical angles are two nonadjacent angles formed by two intersecting lines. A pair of adjacent angles whose noncommon sides are opposite rays is called a linear pair.

Example  Identify each pair of angles as adjacent angles, vertical angles, and/or as a linear pair.

a. \( \angle SRT \) and \( \angle TRU \) have a common vertex and a common side, but no common interior points. They are adjacent angles.

b. \( \angle 1 \) and \( \angle 3 \) are nonadjacent angles formed by two intersecting lines. They are vertical angles. \( \angle 2 \) and \( \angle 4 \) are also vertical angles.

c. \( \angle 6 \) and \( \angle 5 \) are adjacent angles whose noncommon sides are opposite rays. The angles form a linear pair.

d. \( \angle A \) and \( \angle B \) are two angles whose measures have a sum of 90. They are complementary. \( \angle F \) and \( \angle G \) are two angles whose measures have a sum of 180. They are supplementary.

Exercises

Identify each pair of angles as adjacent, vertical, and/or as a linear pair.

1. \( \angle 1 \) and \( \angle 2 \)
2. \( \angle 1 \) and \( \angle 6 \)
3. \( \angle 1 \) and \( \angle 5 \)
4. \( \angle 3 \) and \( \angle 2 \)

For Exercises 5–7, refer to the figure at the right.

5. Identify two obtuse vertical angles.
6. Identify two acute adjacent angles.
7. Identify an angle supplementary to \( \angle TNU \).
8. Find the measures of two complementary angles if the difference in their measures is 18.
### Example

Find \(x\) so that \(DZ \perp PZ\).

If \(DZ \perp PZ\), then \(\angle DZP = 90\).

\[
\begin{align*}
\angle DZQ + \angle QZP &= \angle DZP \\
(9x + 5) + (3x + 1) &= 90 \\
12x + 6 &= 90 \\
12x &= 84 \\
x &= 7
\end{align*}
\]

### Exercises

1. Find \(x\) and \(y\) so that \(NR \perp MQ\).

2. Find \(m\angle MSN\).

3. \(m\angle EBF = 3x + 10\), \(m\angle DBE = x\), and \(BD \perp BF\). Find \(x\).

4. If \(m\angle EBF = 7y - 3\) and \(m\angle FBC = 3y + 3\), find \(y\) so that \(EB \perp BC\).

5. Find \(x\), \(m\angle PQS\), and \(m\angle SQR\).

6. Find \(y\), \(m\angle RPT\), and \(m\angle TPW\).
For Exercises 1–6, use the figure at the right and a protractor.

1. Name two acute vertical angles.

2. Name two obtuse vertical angles.

3. Name a linear pair.

4. Name two acute adjacent angles.

5. Name an angle complementary to $\angle EKH$.

6. Name an angle supplementary to $\angle FKG$.

7. Find the measures of an angle and its complement if one angle measures 18 degrees more than the other.

8. The measure of the supplement of an angle is 36 less than the measure of the angle. Find the measures of the angles.

ALGEBRA For Exercises 9–10, use the figure at the right.

9. If $m\angle RTS = 8x + 18$, find $x$ so that $\overline{TR} \perp \overline{TS}$.

10. If $m\angle PTQ = 3y - 10$ and $m\angle QTR = y$, find $y$ so that $\angle PTR$ is a right angle.

Determine whether each statement can be assumed from the figure. Explain.

11. $\angle WZU$ is a right angle.

12. $\angle YZU$ and $\angle UZV$ are supplementary.

13. $\angle VZU$ is adjacent to $\angle YZX$. 
1-5 Practice

Angle Relationships

For Exercises 1–4, use the figure at the right and a protractor.

1. Name two obtuse vertical angles.

2. Name a linear pair whose vertex is $B$.

3. Name an angle not adjacent to but complementary to $\angle FGC$.

4. Name an angle adjacent and supplementary to $\angle DCB$.

5. Two angles are complementary. The measure of one angle is 21 more than twice the measure of the other angle. Find the measures of the angles.

6. If a supplement of an angle has a measure 78 less than the measure of the angle, what are the measures of the angles?

ALGEBRA For Exercises 7–8, use the figure at the right.

7. If $m\angle FGE = 5x + 10$, find $x$ so that $\overline{FC} \perp \overline{AE}$.

8. If $m\angle BGC = 16x - 4$ and $m\angle CGD = 2x + 13$, find $x$ so that $\angle BGD$ is a right angle.

Determine whether each statement can be assumed from the figure. Explain.

9. $\angle NQO$ and $\angle OQP$ are complementary.

10. $\angle SRQ$ and $\angle QRP$ is a linear pair.

11. $\angle MQN$ and $\angle MQR$ are vertical angles.

12. STREET MAPS Darren sketched a map of the cross streets nearest to his home for his friend Miguel. Describe two different angle relationships between the streets.
1-5 Reading to Learn Mathematics

Angle Relationships

Pre-Activity What kinds of angles are formed when streets intersect?

Read the introduction to Lesson 1-5 at the top of page 37 in your textbook.

• How many separate angles are formed if three lines intersect at a common point? (Do not use an angle whose interior includes part of another angle.)

• How many separate angles are formed if \(n\) lines intersect at a common point? (Do not count an angle whose interior includes part of another angle.)

Reading the Lesson

1. Name each of the following in the figure at the right.
   a. two pairs of congruent angles
   b. a pair of acute vertical angles
   c. a pair of obtuse vertical angles
   d. four pairs of adjacent angles
   e. two pairs of vertical angles
   f. four linear pairs
   g. four pairs of supplementary angles

2. Tell whether each statement is always, sometimes, or never true.
   a. If two angles are adjacent angles, they form a linear pair.
   b. If two angles form a linear pair, they are complementary.
   c. If two angles are supplementary, they are congruent.
   d. If two angles are complementary, they are adjacent.
   e. When two perpendicular lines intersect, four congruent angles are formed.
   f. Vertical angles are supplementary.
   g. Vertical angles are complementary.
   h. The two angles in a linear pair are both acute.
   i. If two angles form a linear pair, one is acute and the other is obtuse.

3. Complete each sentence.
   a. If two angles are supplementary and \(x\) is the measure of one of the angles, then the measure of the other angle is \(\_{\ }\_{\ }\_{\ }\_{\ }\_{\ }\).
   b. If two angles are complementary and \(x\) is the measure of one of the angles, then the measure of the other angle is \(\_{\ }\_{\ }\_{\ }\_{\ }\_{\ }\).

Helping You Remember

4. Look up the nonmathematical meaning of supplementary in your dictionary. How can this definition help you to remember the meaning of supplementary angles?
**Curve Stitching**

The star design at the right was created by a method known as **curve stitching**. Although the design appears to contain curves, it is made up entirely of line segments.

To begin the star design, draw a 60° angle. Mark eight equally-spaced points on each ray, and number the points as shown below. Then connect pairs of points that have the same number.

To make a complete star, make the same design in six 60° angles that have a common central vertex.

1. Complete the section of the star design above by connecting pairs of points that have the same number.

2. Complete the following design.

3. Create your own design. You may use several angles, and the angles may overlap.
Polygons  A polygon is a closed figure formed by a finite number of coplanar line segments. The sides that have a common endpoint must be noncollinear and each side intersects exactly two other sides at their endpoints. A polygon is named according to its number of sides. A regular polygon has congruent sides and congruent angles. A polygon can be concave or convex.

Example  Name each polygon by its number of sides. Then classify it as concave or convex and regular or irregular.

a.  

The polygon has 4 sides, so it is a quadrilateral. It is concave because part of DE or EF lies in the interior of the figure. Because it is concave, it cannot have all its angles congruent and so it is irregular.

b.  

The figure is not closed, so it is not a polygon.

c.  

The polygon has 5 sides, so it is a pentagon. It is convex. All sides are congruent and all angles are congruent, so it is a regular pentagon.

d.  

The figure has 8 congruent sides and 8 congruent angles. It is convex and is a regular octagon.

Exercises  Name each polygon by its number of sides. Then classify it as concave or convex and regular or irregular.

1.  

2.  

3.  

4.  

5.  

6.
**Perimeter** The perimeter of a polygon is the sum of the lengths of all the sides of the polygon. There are special formulas for the perimeter of a square or a rectangle.

**Example** Write an expression or formula for the perimeter of each polygon.

Find the perimeter.

a. \[ P = a + b + c \]
   \[ = 3 + 4 + 5 \]
   \[ = 12 \text{ in.} \]

b. \[ P = 4s \]
   \[ = 4(5) \]
   \[ = 20 \text{ cm} \]

c. \[ P = 2\ell + 2w \]
   \[ = 2(3) + 2(2) \]
   \[ = 10 \text{ ft} \]

**Exercises**

Find the perimeter of each figure.

1. 
   ![Triangle with sides 2.5 cm, 3 cm, and 3.5 cm]

2. 
   ![Square with side 5.5 ft]

3. 
   ![Pentagon with sides 19 yd, 27 yd, 12 yd, 24 yd, and 14 yd]

4. 
   ![Star polygon with sides]

Find the length of each side of the polygon for the given perimeter.

5. \( P = 96 \)
   ![Rectangle with sides 2x and x]

6. \( P = 48 \)
   ![Parallelogram with sides x, x, x, and x - 2]
Name each polygon by its number of sides and then classify it as convex or concave and regular or irregular.

1. 
2. 
3. 
4. 
5. 
6. 

Find the perimeter of each figure.

7. 
8. 
9. 

COORDINATE GEOMETRY Find the perimeter of each polygon.

10. triangle ABC with vertices A(3, 5), B(3, 1), and C(0, 1)

11. quadrilateral QRST with vertices Q(−3, 2), R(1, 2), S(1, −4), and T(−3, −4)

12. quadrilateral LMNO with vertices L(−1, 4), M(3, 4), N(2, 1), and O(−2, 1)

ALGEBRA Find the length of each side of the polygon for the given perimeter.

13. \( P = 104 \) millimeters
14. \( P = 84 \) kilometers
15. \( P = 88 \) feet
1-6 Practice

Polygons

Name each polygon by its number of sides and then classify it as convex or concave and regular or irregular.

1.  

2.  

3.  

Find the perimeter of each figure.

4.  

5.  

6.  

COORDINATE GEOMETRY Find the perimeter of each polygon.

7. quadrilateral OPQR with vertices O(−3, 2), P(1, 5), Q(6, 4), and R(5, −2)

8. pentagon STUVW with vertices S(0, 0), T(3, −2), U(2, −5), V(−2, −5), and W(−3, −2)

ALGEBRA Find the length of each side of the polygon for the given perimeter.

9. \( P = 26 \) inches

10. \( P = 39 \) centimeters

11. \( P = 89 \) feet

SEWING For Exercises 12–13, use the following information.

Jasmine plans to sew fringe around the scarf shown in the diagram.

12. How many inches of fringe does she need to purchase?

13. If Jasmine doubles the width of the scarf, how many inches of fringe will she need?
Pre-Activity  How are polygons related to toys?

Read the introduction to Lesson 1-6 at the top of page 45 in your textbook.

Name four different shapes that can each be formed by four sticks connected to form a closed figure. Assume you have sticks with a good variety of lengths.

Reading the Lesson

1. Tell why each figure is not a polygon.
   a. 
   b. 
   c. 

2. Name each polygon by its number of sides. Then classify it as convex or concave and regular or not regular.
   a. 
   b. 
   c. 

3. What is another name for a regular quadrilateral?

4. Match each polygon in the first column with the formula in the second column that can be used to find its perimeter. (s represents the length of each side of a regular polygon.)
   a. regular dodecagon
   b. square
   c. regular hexagon
   d. rectangle
   e. regular octagon
   f. triangle
   i. $P = 8s$
   ii. $P = 6s$
   iii. $P = a + b + c$
   iv. $P = 12s$
   v. $P = 2l + 2w$
   vi. $P = 4s$

Helping You Remember

5. One way to remember the meaning of a term is to explain it to another person. How would you explain to a friend what a regular polygon is?
**Perimeter and Area of Irregular Shapes**

Two formulas that are used frequently in mathematics are perimeter and area of a rectangle.

**Perimeter:** \( P = 2l + 2w \)

**Area:** \( A = \ell w \), where \( \ell \) is the length and \( w \) is the width

However, many figures are combinations of two or more rectangles creating **irregular shapes**. To find the area of an irregular shape, it helps to separate the shape into rectangles, calculate the formula for each rectangle, then find the sum of the areas.

**Example**

Find the area of the figure at the right.

Separate the figure into two rectangles.

\[
A = \ell w
\]

\[
A_1 = 9 \cdot 2 \quad A_2 = 3 \cdot 3
\]

\[
= 18 \quad = 9
\]

\[
18 + 9 = 27
\]

The area of the irregular shape is 27 m\(^2\).

**Find the area and perimeter of each irregular shape.**

1. 2. 3. 4.

For Exercises 5–8, find the perimeter of the figures in Exercises 1–4.

5. 6. 7. 8.

9. Describe the steps you used to find the perimeter in Exercise 1.
**Name Points, Lines, and Planes**

In geometry, a **point** is a location, a **line** contains points, and a **plane** is a flat surface that contains points and lines. If points are on the same line, they are **collinear**. If points on are the same plane, they are **coplanar**.

**Example**

Use the figure to name each of the following.

a. a line containing point \(A\)  
The line can be named as \(\ell\). Also, any two of the three points on the line can be used to name it.  
\(AB, AC, \text{ or } BC\)

b. a plane containing point \(D\)  
The plane can be named as plane \(\mathcal{D}\) or can be named using three noncollinear points in the plane, such as plane \(ABD\), plane \(ACD\), and so on.

**Exercises**

Refer to the figure.

1. Name a line that contains point \(A\). \(AB, AC, BC, \text{ or } \ell\)

2. What is another name for line \(m\)? \(BD\)

3. Name a point not on \(AC\). \(D\) or \(E\)

4. Name the intersection of \(AC\) and \(DB\). \(B\)

5. Name a point not on line \(\ell\) or line \(m\). \(E\)

Draw and label a plane \(Q\) for each relationship.

6. \(\overline{AB}\) is in plane \(Q\).

7. \(ST\) intersects \(AB\) at \(P\).

8. Point \(X\) is collinear with points \(A\) and \(P\).

9. Point \(Y\) is not collinear with points \(T\) and \(P\).

10. Line \(\ell\) contains points \(X\) and \(Y\).

**Answers for Exercises 6–10**

---

**Points, Lines, and Planes in Space**

Space is a boundless, three-dimensional set of all points. It contains lines and planes.

**Example**

a. How many planes appear in the figure?  
There are three planes: plane \(\mathcal{N}\), plane \(\mathcal{O}\), and plane \(\mathcal{T}\).

b. Are points \(A, B, \text{ and } D\) coplanar?  
Yes. They are contained in plane \(\mathcal{O}\).

**Exercises**

Refer to the figure.

1. Name a line that is not contained in plane \(\mathcal{N}\) \(\overline{AB}\)

2. Name a plane that contains point \(B\). \(\text{plane } \mathcal{N}, \text{plane } ABC, \text{plane } ABD, \text{plane } EBC, \text{plane } EBD\)

3. Name three collinear points. \(A, B, E\)

4. How many planes are shown in the figure? \(6\)

5. Are points \(B, E, G, \text{ and } H\) coplanar? Explain.  
No; \(B, G, \text{ and } H\) lie in plane \(\mathcal{BGH}\), but \(E\) does not.

6. Name a point coplanar with \(D, C, \text{ and } E\). \(F\) or \(J\)

**Draw and label a figure for each relationship.**

7. Planes \(\mathcal{M}\) and \(\mathcal{N}\) intersect in \(\overline{HJ}\).

8. Line \(r\) is in plane \(\mathcal{N}\), line \(s\) is in plane \(\mathcal{M}\), and lines \(r\) and \(s\) intersect at point \(J\).

9. Line \(t\) contains point \(H\) and line \(t\) does not lie in plane \(\mathcal{M}\) or plane \(\mathcal{N}\).

**Answers for Exercises 7–9**
Refer to the figure.

1. Name a line that contains point D.
   \( p \) or \( CD \)

2. Name a point contained in line \( n \).
   \( A \) or \( B \)

3. What is another name for line \( p \)?
   \( CD \) or \( DC \)

4. Name the plane containing lines \( n \) and \( p \).
   Sample answer: plane \( G \)

Draw and label a figure for each relationship. Sample answers are given.

5. Point \( K \) lies on \( RT \).

6. Plane \( J \) contains line \( s \).

7. \( TP \) lies in plane \( B \) and contains point \( C \), but does not contain point \( H \).

8. Lines \( q \) and \( f \) intersect at point \( Z \) in plane \( U \).

Refer to the figure.

9. How many planes are shown in the figure? \( 5 \)

10. How many of the planes contain points \( F \) and \( E \)? \( 2 \)

11. Name four points that are coplanar.
    \( A, B, E, F \) or \( B, C, D, E \) or \( A, C, D, F \)

12. Are points \( A, B, \) and \( C \) coplanar? Explain.
    Yes; points \( A, B, \) and \( C \) lie in plane \( W \).

VISUALIZATION Name the geometric term(s) modeled by each object.

9. \( \) plane and line

10. tip of pin

11. lines

12. a car antenna

13. a library card

14. plane
1-1 Reading to Learn Mathematics
Points, Lines, and Planes

Pre-Activity Why do chairs sometimes wobble?
Read the introduction to Lesson 1-1 at the top of page 6 in your textbook.
- Find three pencils of different lengths and hold them upright on your
desk so that the three pencil points do not lie along a single line. Can you
place a flat sheet of paper or cardboard so that it touches all three pencil
points? **yes**
- How many ways can you do this if you keep the pencil points in the same
position? **one**
- How will your answer change if there are four pencil points? **Sample
answer: It may not be possible to place the paper to touch
all four points.**

Reading the Lesson
1. Complete each sentence.
   a. Points that lie on the same lie are called **collinear** points.
   b. Points that do not lie in the same plane are called **noncoplanar** points.
   c. There is exactly one **line** through any two points.
   d. There is exactly one **plane** through any three noncollinear points.
2. Refer to the figure at the right. Indicate whether each
statement is true or false.
   a. Points A, B, and C are collinear. **false**
   b. The intersection of plane ABC and line m is point P. **true**
   c. Line ℓ and line m do not intersect. **false**
   d. Points A, P, and B can be used to name plane ?UV. **false**
   e. Line ℓ lies in plane ABC. **true**
3. Complete the figure at the right to show the following
relationship: Lines ℓ, m, and n are coplanar and lie in
plane Q. Lines ℓ and ℓ intersect at point P. Line n
intersects line m at R, but does not intersect line ℓ.

Helping You Remember
4. Recall or look in a dictionary to find the meaning of the prefix co-. What does this prefix
mean? How can it help you remember the meaning of collinear?
**Sample answer: The prefix co- means together. The word collinear contains the word line, so collinear means together on a line.**
1-2 Study Guide and Intervention
Linear Measure and Precision

Measure Line Segments  A part of a line between two endpoints is called a line segment. The lengths of $MN$ and $RS$ are written as $MN$ and $RS$. When you measure a segment, the precision of the measurement is half of the smallest unit on the ruler.

**Example 1**

Find the length of $MN$.

![Ruler showing length of MN](image)

The long marks are centimeters and the shorter marks are millimeters. The length of $MN$ is 3.4 centimeters. The measurement is accurate to within 0.5 millimeter, so $MN$ is between 3.35 centimeters and 3.45 centimeters long.

**Example 2**

Find the length of $RS$.

![Ruler showing length of RS](image)

The long marks are inches and the short marks are quarter inches. The length of $RS$ is about 1 3/4 inches. The measurement is accurate to within one half of a quarter inch, or $\frac{1}{2}$ inch, so $RS$ is between $1\frac{5}{8}$ inches and $1\frac{7}{8}$ inches long.

**Exercises**

Find the length of each line segment or object.

1. $A$ is 2.5 cm
2. $S$ is $1\frac{1}{4}$ in.
3. $2\frac{1}{4}$ in.
4. 1.7 cm

Find the precision for each measurement.

5. 10 in.
6. 32 mm
7. 44 cm
8. 2 ft or 6 in.
9. 3.5 mm
10. $2\frac{1}{2}$ yd

11. $\frac{1}{2}$ in.
12. 0.5 mm
13. 0.5 cm
14. 0.05 mm
15. $\frac{1}{4}$ yd or 9 in.

1-2 Study Guide and Intervention (continued)
Linear Measure and Precision

Calculate Measures  On $PQ$, to say that point $M$ is between points $P$ and $Q$ means $P$, $Q$, and $M$ are collinear and $PM + MQ = PQ$.

On $AC$, $AB = BC = 3$ cm. We can say that the segments are congruent, or $AB \cong BC$. Slashes on the figure indicate which segments are congruent.

**Example 1**

Find $EF$.

![Figure with EF](image)

Calculate $EF$ by adding $ED$ and $DF$.

$ED + DF = EF$

$1.2 + 1.9 = EF$

$3.1 = EF$

Therefore, $EF$ is $3.1$ centimeters long.

**Example 2**

Find $x$ and $AC$.

![Figure with AC and x](image)

$B$ is between $A$ and $C$.

$AB + BC = AC$

$x + 2x = 2x + 5$

$3x = 2x + 5$

$x = 5$

$AC = 2x + 5 = 2(5) + 5 = 15$

**Exercises**

Find the measurement of each segment. Assume that the art is not drawn to scale.

1. $RT = 2.0$ cm, $ST = 2.5$ cm
2. $BC = \frac{8}{1}$ in.
3. $XY = \frac{3}{4}$ in.
4. $WX = \frac{6}{1}$ cm

Find $x$ and $RS$ if $S$ is between $R$ and $T$.

5. $RS = 5x$, $ST = 3x$, and $RT = 48$.
6. $RS = 2x$, $ST = 5x + 4$, and $RT = 32$.
7. $RS = 6x$, $ST = 12$, and $RT = 72$.
8. $RS = 4x$, $RS = ST$, and $RT = 24$.

Use the figures to determine whether each pair of segments is congruent.

9. $\overline{AB}$ and $\overline{CD}$ yes
10. $\overline{XY}$ and $\overline{YZ}$ no

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Find the length of each line segment or object.

1. EF
2. GH
3. JK
4. LM
5. NO
6. PQ
7. RS
8. UV
9. WX
10. YZ

Find the precision for each measurement.

1. 2 mm
2. 0.5 cm
3. 3.0 centimeters
4. 4.7 feet
5. 5.0 millimeters
6. 6.1 inches
7. 0.5 inch
8. 8.0 millimeters
9. 9.1 inches
10. 10.1 centimeters

Find the value of the variable and KL if KJ and JL.

9. KL = 500, KJ = 50, and JL = 100
10. KL = 750, KJ = 75, and JL = 150

Use the figures to determine whether each pair of segments is congruent.

11. JK = 10, JL = 12
12. AB = BC
13. GP \parallel FE
14. BC \parallel FE, AB \parallel CD = DE = FA

Use the figures to determine whether each pair of segments is congruent.

11. JK = 10, JL = 12
12. AB = BC
13. GP \parallel FE
14. BC \parallel FE, AB \parallel CD = DE = FA

Carpentry

Jorge used the figure at the right to make a pattern for a mosaic he plans to inlay on a tabletop. Name all of the congruent segments in the figure.
**Pre-Activity**

Why are units of measure important?

Read the introduction to Lesson 1-2 at the top of page 13 in your textbook.

- The basic unit of length in the metric system is the meter. How many meters are there in one kilometer? **1000**
- Do you think it would be easier to learn the relationships between the different units of length in the customary system (used in the United States) or in the metric system? Explain your answer. **Sample answer:** The metric system is easier because you can change between the different units by just moving the decimal point.

**Reading the Lesson**

1. Explain the difference between a **line** and a **line segment** and why one of these can be measured, while the other cannot. **Sample answer:** A line is infinite. Since it has no endpoints, a line does not have a definite length and cannot be measured. A line segment has two endpoints, so it has a definite length and can be measured.

2. What is the smallest length marked on a 12-inch ruler? **Sample answer:** $\frac{1}{16}$ in.

3. Find the precision of each measurement.
   a. 15 cm $0.5$ cm
   b. 15.0 cm $0.05$ cm

4. Refer to the figure at the right. Which one of the following statements is true? Explain your answer. **Sample answer:** $\overline{AB} = \overline{CD}$; $\overline{AB} \neq \overline{CD}$

   - **$\overline{AB} = \overline{CD}$:** Sample answer: The two segments are congruent because they have the same measure or length. They are not equal because they are not the same segment.

5. Suppose that S is a point on $\overline{VW}$ and S is not the same point as V or W. Tell whether each of the following statements is always, sometimes, or never true.
   a. $\overline{VS} = \overline{SW}$ **sometimes**
   b. S is between V and W. **always**
   c. $\overline{VS} + \overline{VW} = \overline{SW}$ **never**

**Helping You Remember**

6. A good way to remember terms used in mathematics is to relate them to everyday words you know. Give three words that are used outside of mathematics that can help you remember that there are 100 centimeters in a meter. **Sample answer:** cent, century, centennial

---

**Points Equidistant from Segments**

The distance from a point to a segment is zero if the point is on the segment. Otherwise, it is the length of the shortest segment from the point to the segment.

A figure is a **locus** if it is the set of all points that satisfy a set of conditions. The locus of all points that are $\frac{1}{4}$ inch from the segment $\overline{AB}$ is shown by two dashed segments with semicircles at both ends.

1. Suppose $A, B, C,$ and $D$ are four different points, and consider the locus of all points $x$ units from $\overline{AB}$ and $x$ units from $\overline{CD}$. Use any unit you find convenient. The locus can take different forms. Sketch at least three possibilities. **List some of the things that seem to affect the form of the locus. Sample answers are shown.**

2. Conduct your own investigation of the locus of points equidistant from two segments. Describe your results on a separate sheet of paper. **See students’ work.**
Study Guide and Intervention
Distance and Midpoints

**Distance Between Two Points**

<table>
<thead>
<tr>
<th>Distance on a Number Line</th>
<th>Distance in the Coordinate Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td><strong>B</strong></td>
</tr>
<tr>
<td><img src="image1" alt="Number Line" /></td>
<td><img src="image2" alt="Coordinate Plane" /></td>
</tr>
</tbody>
</table>

**Example 1** Find AB.

- A(-2, -1) and B(1, 3).
- AB = |-4 - 2| = 6

**Example 2** Find the distance between A(-3, 2) and B(6, 8).

- \(AB^2 = (AC)^2 + (BC)^2\)
- \(AB = \sqrt{185}\)
- AB ≈ 13.59 units

**Example 3** Find the coordinate of the midpoint of PQ.

- PQ is the midpoint of AB.
- The coordinates of A and B are -3 and 1.
- If M is the midpoint of PQ, then the coordinate of M is \(-\frac{3 + 1}{2} = -2\) or -1.

**Exercises**

1. **BD** 8
2. **DG** 9
3. **AF** 12
4. **EF** 3
5. **BG** 15
6. **AG** 17
7. **BE** 7
8. **DE** 1

Use the number line to find the midpoint of each segment.

1. **CE** -1
2. **DG** 4
3. **AF** -3
4. **EG** 5
5. **AB** -8
6. **BG** 1
7. **BD** -3
8. **DE** 1

Find the coordinates of the midpoint of a segment having the given endpoints.

9. A(0, 0), B(6, 8) (6, 4)
10. R(-2, 3), S(3, 15) 13
11. M(1, -2), N(9, 13) 17
12. E(-12, 2), F(-9, 6) 5

Use the Distance Formula to find the distance between each pair of points.

13. A(0, 0), B(15, 20) 25
14. O(-12, 0), P(-8, 3) 5
15. C(11, -12), D(6, 2) \(\sqrt{221} \approx 14.9\)
16. E(-2, 10), F(-4, 3) \(\sqrt{53} \approx 7.3\)
1-3 Practice (Average)  

Distance and Midpoints

Use the number line to find each measure.

1. LW 4  
2. TV 5  
3. ST 3  
4. SV 8

Use the Pythagorean Theorem to find the distance between each pair of points.

5. \( \sqrt{50} \approx 7.1 \)  
6. \( \sqrt{113} \approx 10.6 \)

Use the Distance Formula to find the distance between each pair of points.

7. \( \sqrt{208} \approx 14.4 \)  
8. \( \sqrt{108} \approx 10.4 \)

Find the coordinates of the midpoint of each segment.

13. \( \frac{1}{2} \)  
14. \( \frac{1}{2} \)

Find the coordinates of the midpoint of a segment having the given endpoints.

17. \( (3, 3) \), \( (5, 5) \)  
18. \( (2, 2) \), \( (4, 4) \)

Find the coordinates of the missing endpoint given that \( E \) is the midpoint of \( \overline{DF} \).

15. \( (5, 5) \), \( (4, 4) \)  
16. \( (2, 2) \), \( (1, 1) \)

Find the coordinates of the missing endpoint given that \( E \) is the midpoint of \( \overline{NQ} \).

19. \( (5, 5) \), \( (4, 4) \)  
20. \( (2, 2) \), \( (1, 1) \)

Find the coordinates of the midpoint of each segment.

11. \( \frac{1}{2} \)  
12. \( \frac{1}{2} \)

Find the coordinates of a segment having the given endpoints.

13. \( (3, 3) \), \( (5, 5) \)  
14. \( (2, 2) \), \( (1, 1) \)

Find the coordinates of the missing endpoint given that \( E \) is the midpoint of \( \overline{DG} \).

16. \( (2, 2) \), \( (1, 1) \)

Find the perimeter of the quadrilateral. Round to the nearest tenth.

19.6 units
Reading the Lesson

1. Match each formula or expression in the first column with one of the names in the second column.
   a. \[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]
   b. \[\frac{a + b}{2}\]
   c. \[XY = |a - b|\]
   d. \[c^2 = a^2 + b^2\]
   e. \[\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\]

   i. Pythagorean Theorem
   ii. Distance Formula in the Coordinate Plane
   iii. Distance Formula on a Number Line
   iv. Midpoint of a Segment in the Coordinate Plane
   v. Midpoint of a Segment on a Number Line

2. Fill in the steps to calculate the distance between the points \((4, -3)\) and \((-2, 7)\).
   Let \((x_1, y_1) = (4, -3)\) and \((x_2, y_2) = (-2, 7)\).
   \[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]
   \[= \sqrt{(-2 - 4)^2 + (7 - (-3))^2}\]
   \[= \sqrt{(-6)^2 + 10^2}\]
   \[= \sqrt{36 + 100}\]
   \[= \sqrt{136}\]

Find a decimal approximation for \(MN\) to the nearest hundredth. \(11.66\)

Helping You Remember

3. A good way to remember a new formula in mathematics is to relate it to one you already know. If you forget the Distance Formula, how can you use the Pythagorean Theorem to find the distance \(d\) between two points on a coordinate plane? Sample answer: If the segment determined by the points is neither horizontal nor vertical, draw a right triangle that has the segment as its hypotenuse. The horizontal side will have length \(|x_2 - x_1|\) and the vertical side will have length \(|y_2 - y_1|\). By the Pythagorean Theorem, \[d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.\]
**Angle Measure**

Measure Angles: If two noncollinear rays have a common endpoint, they form an **angle**. The rays are the **sides** of the angle. The common endpoint is the **vertex**. The angle at the right can be named as $\angle A$, $\angle BAC$, $\angle CAB$, or $\angle 1$.

- A **right angle** is an angle whose measure is 90°. An **acute angle** has measure less than 90°. An **obtuse angle** has measure greater than 90° but less than 180°.

**Example 1**

Refer to the figure.

- Name all angles that have $R$ as a vertex.
  - Three angles are $\angle 1$, $\angle 2$, and $\angle 3$. For other angles, use three letters to name them: $\angle SRQ$, $\angle PRT$, and $\angle SRT$.
- Name the sides of $\angle 1$. $RS$, $RP$

**Example 2**

Measure each angle and classify it as right, acute, or obtuse.

- $\angle ABD$ Using a protractor, $\angle ABD = 50°$. $50° < 90°$, so $\angle ABD$ is an acute angle.
- $\angle DBC$ Using a protractor, $\angle DBC = 115°$. $180° > 115° > 90°$, so $\angle DBC$ is an obtuse angle.
- $\angle EBC$ Using a protractor, $\angle EBC = 90°$. $\angle EBC$ is a right angle.

**Exercises**

Refer to the figure.

1. Name the vertex of $\angle 4$. $B$
2. Name the sides of $\angle BDC$. $DB$, $DC$
3. Write another name for $\angle BDC$. $\angle 3$ or $\angle CBD$

Measure each angle in the figure and classify it as right, acute, or obtuse.

- $\angle MPR$ $120°$; obtuse
- $\angle RPN$ $90°$; right
- $\angle NSR$ $45°$; acute

**Congruent Angles**

Angles that have the same measure are **congruent angles**. A ray that divides an angle into two congruent angles is called an **angle bisector**.

**Example**

Refer to the figure. If $m\angle MPN = 2x + 14$ and $m\angle NPR = x + 34$, find $x$ and find $m\angle MPB$.

Since $PN$ bisects $\angle MPN$, $\angle MPN = \angle NPR$, or $m\angle MPN = m\angle NPR$.

- $2x + 14 = x + 34$  
  $m\angle NPR = (2x + 14) + (x + 34)$
- $2x + 14 - x = x + 34 - x$  
  $= 54 + 54$  
  $x + 14 = 34$  
  $= 108$  
  $x + 14 - 14 = 34 - 14$  
  $x = 20$

**Exercises**

- $QS$ bisects $\angle PQT$, and $QF$ and $QR$ are opposite rays.
  1. If $m\angle PQT = 60°$ and $m\angle PQS = 4x + 14$, find the value of $x$.
     
     $4$
  2. If $m\angle PQS = 3x + 13$ and $m\angle SQT = 6x - 2$, find $m\angle PQT$.
     
     $56$

- $BA$ and $BC$ are opposite rays, $BF$ bisects $\angle CBE$, and $BD$ bisects $\angle ABE$.
  3. If $m\angle BAF = 6x + 4$ and $m\angle CBF = 7x - 2$, find $m\angle EBC$.
     
     $80$
  4. If $m\angle 1 = 4x + 10$ and $m\angle 2 = 5x$, find $m\angle 2$.
     
     $50$
  5. If $m\angle 2 = 6y + 2$ and $m\angle 1 = 8y - 14$, find $m\angle ABE$.
     
     $100$
  6. Is $\angle DBF$ a right angle? Explain.
     
     Yes; since $BD$ and $BF$ are bisectors, $m\angle 2 + m\angle 3$ must equal half the total angle measure, and half of 180° is 90°.
For Exercises 1–12, use the figure at the right.

Name the vertex of each angle.

1. ∠4 2. ∠1 3. ∠2 4. ∠5

Name the sides of each angle.

5. ∠4 6. ∠5
7. ∠STV

Write another name for each angle.

9. ∠3 10. ∠4

Measure each angle and classify it as right, acute, or obtuse.

11. ∠STW, ∠5

ALGEBRA In the figure, \( \overline{BA} \) and \( \overline{BC} \) are opposite rays, \( \overline{BD} \) bisects \( ∠EBC \), and \( \overline{BF} \) bisects \( ∠ABE \).

17. If \( m \angle EBD = 4x + 16 \) and \( m \angle DBC = 6x + 4 \), find \( m \angle EBD \).

18. If \( m \angle ABF = 7x - 8 \) and \( m \angle EBF = 5x + 10 \), find \( m \angle EBF \).

ALGEBRA In the figure, \( \overline{CD} \) and \( \overline{CD} \) are opposite rays, \( \overline{CE} \) bisects \( ∠DCF \), and \( \overline{CD} \) bisects \( ∠FCB \).

15. If \( m \angle DCE = 4x + 15 \) and \( m \angle ECF = 6x - 5 \), find \( m \angle DCE \).

16. If \( m \angle FCG = 9x + 3 \) and \( m \angle GCB = 13x - 9 \), find \( m \angle GCB \).

17. TRAFFIC SIGNS The diagram shows a sign used to warn drivers of a school zone or crossing. Measure and classify each numbered angle.

\( m \angle 1 = 90 \), right angle; \( m \angle 2 = 130 \), obtuse
1-4 Reading to Learn Mathematics

Angle Measure

Pre-Activity How big is a degree?

Read the introduction to Lesson 1-4 at the top of page 29 in your textbook.

- A semicircle is half a circle. How many degrees are there in a semicircle? 180
- How many degrees are there in a quarter circle? 90

Reading the Lesson

1. Match each description in the first column with one of the terms in the second column.
   Some terms in the second column may be used more than once or not at all.
   a. a figure made up of two noncollinear rays with a common endpoint 4
   b. angles whose degree measures are less than 90° 8
   c. angles that have the same measure 6
   d. angles whose degree measures are between 90° and 180° 5
   e. a tool used to measure angles 10
   f. the common endpoint of the rays that form an angle 1
   g. a ray that divides an angle into two congruent angles 2
   h. an obtuse angle 6
   i. an acute angle 6
   j. a right angle 5
   k. a pair of congruent angles 2
   l. the angle with the greatest measure 5

2. Use the figure to name each of the following.
   a. a right angle \( \angle ABE \) or \( \angle EBG \)
   b. an obtuse angle \( \angle ABF \) or \( \angle ABC \)
   c. an acute angle \( \angle EBF, \angle FBC, \angle CBG, \angle EBC, \) or \( \angle FBG \)
   d. a point in the interior of \( \angle EBC \) F
   e. a point in the exterior of \( \angle EBA \) F, C, or G
   f. the angle bisector of \( \angle EBC \) BF
   g. a point on \( \angle CBE \) C, B, or E
   h. the sides of \( \angle ABE \) BA and BE
   i. a pair of opposite rays \( \overline{BA} \) and \( \overline{BG} \)
   j. the common vertex of all angles shown in the figure B
   k. a pair of congruent angles \( \angle EBF \) and \( \angle FBC \), or \( \angle ABE \) and \( \angle EBG \)
   l. the angle with the greatest measure \( \angle ABE \)

Helping You Remember

3. A good way to remember related geometric ideas is to compare them and see how they are alike and how they are different. Give some similarities and differences between congruent segments and congruent angles.

Sample answer: Congruent segments and congruent angles are alike because they both involve a pair of figures with the same measure. They are different because congruent segments have the same length, which can be measured in units such as inches or centimeters, while congruent angles have the same degree measure.

Answers

1. 35°15’ 2. 27°16’ 3. 15°54’
   54°45’ 62°44’ 74°06’

2. 4. 29°18’22’’ 5. 34°29’45’’ 6. 87°2’3’’
   60°41’38’’ 55°30’15’’ 2°57’57’’

Two angles are supplementary if the sum of their measures is 180°. Find the supplement of each of the following angles.

7. 120°18’ 8. 84°12’ 9. 110°2’
   59°42’ 95°48’ 69°58’

10. 45°16’24’’ 11. 39°21’54’’ 12. 129°18’36’’
    134°43’36’’ 140°38’6’’ 50°41’24’’

13. 98°52’59’’ 14. 9°2’32’’ 15. 1°2’3’’
    81°7’1’’ 170°57’28’’ 178°57’57’’
Pairs of Angles

Adjacent angles are angles in the same plane that have a common vertex and a common side, but no common interior points. Vertical angles are two nonadjacent angles formed by two intersecting lines. A pair of adjacent angles whose noncommon sides are opposite rays is called a linear pair.

Example

Identify each pair of angles as adjacent angles, vertical angles, and/or as a linear pair.

a. ∠SRT and ∠TRU have a common vertex and a common side, but no common interior points. They are adjacent angles.

b. ∠1 and ∠3 are nonadjacent angles formed by two intersecting lines. They are vertical angles. ∠2 and ∠4 are also vertical angles.

c. ∠6 and ∠5 are adjacent angles whose noncommon sides are opposite rays. The angles form a linear pair.

d. ∠A and ∠B are two angles whose measures have a sum of 90. They are complementary. ∠F and ∠G are two angles whose measures have a sum of 180. They are supplementary.

Exercises

Identify each pair of angles as adjacent, vertical, and/or as a linear pair.

1. ∠1 and ∠2 adjacent

2. ∠1 and ∠6 linear pair; adjacent

3. ∠1 and ∠5 vertical adjacent

4. ∠3 and ∠2 adjacent

For Exercises 5–7, refer to the figure at the right.

5. Identify two obtuse vertical angles. ∠RNT and ∠SNU

6. Identify two acute adjacent angles. ∠RNV and ∠VNT or ∠NTV and ∠TNV

7. Identify an angle supplementary to ∠TNU. ∠UNS or ∠TNR

8. Find the measures of two complementary angles if the difference in their measures is 18. 36 and 54

Example

Find x so that DZ ⊥ PQ.

If DZ ⊥ PQ, then m∠DZP = 90.

m∠DZQ + m∠DQP = m∠DZP

(9x + 5) + (3x + 1) = 90

Substitution

12x + 6 = 90

Simplify

12x = 84

Subtract 6 from each side.

x = 7

Divide each side by 12.

Exercises

1. Find x and y so that NR ⊥ MQ. x = 15, y = 8

2. Find m∠MSN. 90

3. m∠EBF = 3x + 10, m∠DBE = x, and BD ⊥ BF. Find x. x = 20

4. If m∠EBF = 7x – 3 and m∠FBC = 3y + 3, find y so that EB ⊥ BC. 9

5. Find x, m∠PQS, and m∠SQR. x = 8, m∠PQS = 24, m∠SQR = 66

6. Find y, m∠RPT, and m∠TPW. y = 15, m∠RPT = 55, m∠TPW = 35
1-5 1-5 Practice (Average)

Angle Relationships

For Exercises 1–6, use the figure at the right and a protractor.

1. Name two acute vertical angles. \( \angle EKH, \angle FKG \)

2. Name two obtuse vertical angles. \( \angle EKF, \angle HKG \)

3. Name a linear pair. Sample answer: \( \angle EKH, \angle EKF \)

4. Name two acute adjacent angles. \( \angle FKG, \angle GKJ \)

5. Name an angle complementary to \( \angle EKH \).

6. Name an angle supplementary to \( \angle FKG \).

For Exercises 7–8, use the figure at the right.

7. If \( m\angle PTO = 3y - 10 \) and \( m\angle QTR = y \), find \( y \) so that \( \angle PTR \) is a right angle. \( 25\)°

8. If \( m\angle PGE = 5x + 10 \), find \( x \) so that \( \overline{FC} \perp \overline{AE} \). \( 16\)

Determine whether each statement can be assumed from the figure. Explain.

9. \( \angle WZU \) is a right angle.

   Yes; it is marked with a right angle symbol.

10. \( \angle YZU \) and \( \angle UZV \) are supplementary.

    Yes; the sum of their measures is 180 since the angles form a linear pair.

11. \( \angle VZU \) is adjacent to \( \angle YZX \).

    No; the angles do not share a common side.

12. STREET MAPS Darren sketched a map of the cross streets nearest to his home for his friend Miguel. Describe two different angles formed by Bacon and Main into pairs of complementary angles.

    Sample answer: Beacon \perp Main; Olive divides two of the angles formed by Bacon and Main into pairs of complementary angles.

ALGEBRA For Exercises 9–10, use the figure at the right.

9. If \( m\angle RTS = 8x + 18 \), find \( x \) so that \( \overline{TR} \perp \overline{TS} \). \( 9\)

10. If \( m\angle PTQ = 3y - 10 \) and \( m\angle QTR = y \), find \( y \) so that \( \angle PTR \) is a right angle. \( 25\)

For Exercises 7–8, use the figure at the right.

7. If \( m\angle FGE = 5x + 10 \), find \( x \) so that \( \overline{FC} \perp \overline{AE} \). \( 16\)

8. If \( m\angle BOC = 16x - 4 \) and \( m\angle CGD = 2x + 13 \), find \( x \) so that \( \angle BGD \) is a right angle. \( 4.5\)

Determine whether each statement can be assumed from the figure. Explain.

9. \( \angle NQO \) and \( \angle OQP \) are complementary.

   No; \( m\angle NQP \) is not known to be 90.

10. \( \angle SRQ \) and \( \angle QRP \) is a linear pair. Yes; they are adjacent angles whose noncommon sides are opposite rays.

11. \( \angle MQN \) and \( \angle MQR \) are vertical angles.

    No; the angles are adjacent.

12. STREET MAPS Darren sketched a map of the cross streets nearest to his home for his friend Miguel. Describe two different angles formed by Bacon and Main into pairs of complementary angles.

    Sample answer: Beacon \perp Main; Olive divides two of the angles formed by Bacon and Main into pairs of complementary angles.
Enrichment

Curve Stitching

The star design at the right was created by a method known as curve stitching. Although the design appears to contain curves, it is made up entirely of line segments.

To begin the star design, draw a 60° angle. Mark eight equally-spaced points on each ray, and number the points as shown below. Then connect pairs of points that have the same number.

To make a complete star, make the same design in six 60° angles that have a common central vertex.

1. Complete the section of the star design above by connecting pairs of points that have the same number.

2. Complete the following design.

3. Create your own design. You may use several angles, and the angles may overlap.

Helping You Remember

4. Look up the nonmathematical meaning of supplementary in your dictionary. How can this definition help you to remember the meaning of supplementary angles? Sample answer: Supplementary means something added to complete a thing. An angle and its supplement can be joined to obtain a linear pair.
**Polygons**

A polygon is a closed figure formed by a finite number of coplanar line segments. The sides that have a common endpoint must be noncollinear and each side intersects exactly two other sides at their endpoints. A polygon is named according to its number of sides. A regular polygon has congruent sides and congruent angles. A polygon can be concave or convex.

**Example**

Name each polygon by its number of sides. Then classify it as concave or convex and regular or irregular.

a. The polygon has 4 sides, so it is a quadrilateral. It is concave because part of $DE$ or $EF$ lies in the interior of the figure. Because it is concave, it cannot have all its angles congruent and so it is irregular.

b. The figure is not closed, so it is not a polygon.

c. The polygon has 5 sides, so it is a pentagon. It is convex. All sides are congruent and all angles are congruent, so it is a regular pentagon.

d. The figure has 8 congruent sides and 8 congruent angles. It is convex and is a regular octagon.

**Exercises**

Name each polygon by its number of sides. Then classify it as concave or convex and regular or irregular.

1. hexagon; convex; regular
2. quadrilateral; convex; irregular
3. pentagon; concave; irregular
4. triangle; convex; irregular
5. pentagon; concave; irregular
6. octagon; concave; irregular

**Perimeter**

The perimeter of a polygon is the sum of the lengths of all the sides of the polygon. There are special formulas for the perimeter of a square or a rectangle.

**Example**

Write an expression or formula for the perimeter of each polygon.

Find the perimeter.

a. $P = a + b + c$
   
   $= 3 + 4 + 5$
   
   $= 12$ in.

b. $P = 4s$
   
   $= 4(5)$
   
   $= 20$ cm

c. $P = 2l + 2w$
   
   $= 2(3) + 2(2)$
   
   $= 10$ ft

**Exercises**

Find the perimeter of each figure.

1. 2. 3.

   9 cm 22 ft 96 yd

4. 5. 6.

   $14$ yd 3 cm 27 yd

   $16$ 32 8, 10, 20

Find the length of each side of the polygon for the given perimeter.

5. $P = 96$
   
   16, 32

6. $P = 48$
   
   8, 10, 20
1-6 Skills Practice

1. 2. 3.

1. quadrilateral; convex; irregular;
2. triangle; convex; regular;
3. pentagon; concave; irregular;
4. heptagon; convex; irregular;
5. quadrilateral; convex; irregular;
6. dodecagon; concave; irregular

Find the perimeter of each figure.

7. 8. 9.

7. 98 yd
8. 20 m
9. 32 in.

COORDINATE GEOMETRY Find the perimeter of each polygon.

10. triangle ABC with vertices A(3, 5), B(3, 1), and C(0, 1)

11. quadrilateral QRST with vertices Q(−3, 2), R(1, 2), S(1, −4), and T(−3, −4)

12. quadrilateral LMNO with vertices L(−1, 4), M(3, 4), N(2, 1), and O(−2, 1)

ALGEBRA Find the length of each side of the polygon for the given perimeter.

13. $P = 104$ millimeters
14. $P = 84$ kilometers
15. $P = 88$ feet

13. All are 13 mm.
14. All are 28 km.
15. 9 ft, 9 ft, 35 ft, 35 ft

Answers (Lesson 1-6)
Reading to Learn Mathematics

Polynomials

Pre-Activity How are polygons related to toys?

Read the introduction to Lesson 1-6 at the top of page 45 in your textbook.

Name four different shapes that can each be formed by four sticks connected to form a closed figure. Assume you have sticks with a good variety of lengths.

Sample answer: square, rectangle, parallelogram, trapezoid

Reading the Lesson

1. Tell why each figure is *not* a polygon.
   - a. not closed
   - b. curved (not all made up of segments)
   - c. Sides intersect at a point that is not an endpoint.

2. Name each polygon by its number of sides. Then classify it as convex or concave and regular or not regular.
   - a. pentagon, convex, regular
   - b. quadrilateral, concave, not regular
   - c. quadrilateral, convex, not regular

3. What is another name for a regular quadrilateral? a square

4. Match each polygon in the first column with the formula in the second column that can be used to find its perimeter. (s represents the length of each side of a regular polygon.)
   - a. regular dodecagon iv
   - b. square vi
   - c. regular hexagon ii
   - d. rectangle v
   - e. regular octagon iv
   - f. triangle iii
   - i. \( P = 8s \)
   - ii. \( P = 6s \)
   - iii. \( P = a + b + c \)
   - iv. \( P = 12s \)
   - v. \( P = 2l + 2w \)
   - vi. \( P = 4s \)

Helping You Remember

5. One way to remember the meaning of a term is to explain it to another person. How would you explain to a friend what a regular polygon is?
   Sample answer: A regular polygon looks the same no matter what part you look at. The sides are the same length, and the angles are the same size.

Enrichment

Perimeter and Area of Irregular Shapes

Two formulas that are used frequently in mathematics are perimeter and area of a rectangle.

**Perimeter:** \( P = 2l + 2w \)

**Area:** \( A = lw \), where \( l \) is the length and \( w \) is the width

However, many figures are combinations of two or more rectangles creating irregular shapes. To find the area of an irregular shape, it helps to separate the shape into rectangles, calculate the formula for each rectangle, then find the sum of the areas.

**Example** Find the area of the figure at the right.

Separate the figure into two rectangles.

\[
\begin{align*}
A_1 &= 9 \times 2 \\
A_2 &= 3 \times 5 \\
\text{Total Area} &= A_1 + A_2 = 18 + 15 = 33
\end{align*}
\]

The area of the irregular shape is 33 square units.

Find the area and perimeter of each irregular shape.

1. \( A = 12 \text{ in}^2 \) \( P = 20 \text{ in.} \)
2. \( A = 320 \text{ m}^2 \) \( P = 96 \text{ m} \)
3. \( A = 40 \text{ cm}^2 \) \( P = 44 \text{ cm} \)
4. \( A = 90 \text{ ft}^2 \) \( P = 46 \text{ ft} \)

For Exercises 5–8, find the perimeter of the figures in Exercises 1–4.

5. 17 in. 6. 96 m 7. 44 cm 8. 48 ft

9. Describe the steps you used to find the perimeter in Exercise 1.
   See students’ work.