This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

<table>
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<tr>
<th>Vocabulary Term</th>
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<td>scalene triangle</td>
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<td>SKAY·leen</td>
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<tr>
<td>vertex angle</td>
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This is a list of key theorems and postulates you will learn in Chapter 4. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

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| Theorem 4.8  
*Leg-Angle Congruence (LA)* |               |                                      |
| Theorem 4.9  
*Isosceles Triangle Theorem*         |               |                                      |
| Theorem 4.10                                   |               |                                      |
| Postulate 4.1  
*Side-Side-Side Congruence (SSS)* |               |                                      |
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*Side-Angle-Side Congruence (SAS)* |               |                                      |
| Postulate 4.3  
*Angle-Side-Angle Congruence (ASA)* |               |                                      |
| Postulate 3.4  
*Hypotenuse-Leg Congruence (HL)*     |               |                                      |
Classify Triangles by Angles One way to classify a triangle is by the measures of its angles.

- If one of the angles of a triangle is an obtuse angle, then the triangle is an **obtuse triangle**.
- If one of the angles of a triangle is a right angle, then the triangle is a **right triangle**.
- If all three of the angles of a triangle are acute angles, then the triangle is an **acute triangle**.
- If all three angles of an acute triangle are congruent, then the triangle is an **equiangular triangle**.

**Example**

Classify each triangle.

a. 

All three angles are congruent, so all three angles have measure 60°. The triangle is an equiangular triangle.

b. 

The triangle has one angle that is obtuse. It is an obtuse triangle.

c. 

The triangle has one right angle. It is a right triangle.

**Exercises**

Classify each triangle as acute, equiangular, obtuse, or right.

1. 

2. 

3. 

4. 

5. 

6.
Classify Triangles by Sides  You can classify a triangle by the measures of its sides. Equal numbers of hash marks indicate congruent sides.

• If all three sides of a triangle are congruent, then the triangle is an **equilateral triangle**.

• If at least two sides of a triangle are congruent, then the triangle is an **isosceles triangle**.

• If no two sides of a triangle are congruent, then the triangle is a **scalene triangle**.

**Example**  Classify each triangle.

a. [Diagram of a triangle with two sides marked]
Two sides are congruent. The triangle is an isosceles triangle.

b. [Diagram of an equilateral triangle]
All three sides are congruent. The triangle is an equilateral triangle.

c. [Diagram of a scalene triangle]
The triangle has no pair of congruent sides. It is a scalene triangle.

**Exercises**

Classify each triangle as **equilateral**, **isosceles**, or **scalene**.

1. [Diagram of a triangle with sides labeled]

2. [Diagram of a triangle with two sides labeled]

3. [Diagram of a triangle with sides labeled]

4. [Diagram of a triangle with sides labeled]

5. [Diagram of a triangle with sides labeled]

6. [Diagram of a triangle with sides labeled]

7. Find the measure of each side of equilateral \(\triangle RST\) with \(RS = 2x + 2\), \(ST = 3x\), and \(TR = 5x - 4\).

8. Find the measure of each side of isosceles \(\triangle ABC\) with \(AB = BC\) if \(AB = 4y\), \(BC = 3y + 2\), and \(AC = 3y\).

9. Find the measure of each side of \(\triangle ABC\) with vertices \(A(-1, 5), B(6, 1),\) and \(C(2, -6)\). Classify the triangle.
Skills Practice
Classifying Triangles

Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1. 

2. 

3. 

4. 

5. 

6. 

Identify the indicated type of triangles.

7. right 

8. isosceles 

9. scalene 

10. obtuse 

ALGEBRA Find \( x \) and the measure of each side of the triangle.

11. \( \triangle ABC \) is equilateral with \( AB = 3x - 2, BC = 2x + 4 \), and \( CA = x + 10 \).

12. \( \triangle DEF \) is isosceles, \( \angle D \) is the vertex angle, \( DE = x + 7, DF = 3x - 1 \), and \( EF = 2x + 5 \).

Find the measures of the sides of \( \triangle RST \) and classify each triangle by its sides.

13. \( R(0, 2), S(2, 5), T(4, 2) \)

14. \( R(1, 3), S(4, 7), T(5, 4) \)
### Practice

#### 4-1

**Classifying Triangles**

Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1.  
   ![Triangle 1](image1)

2.  
   ![Triangle 2](image2)

3.  
   ![Triangle 3](image3)

Identify the indicated type of triangles if $AB \equiv AD \equiv BD \equiv DC$, $BE \equiv ED$, $AB \perp BC$, and $ED \perp DC$.

4. right

5. obtuse

6. scalene

7. isosceles

**ALGEBRA** Find $x$ and the measure of each side of the triangle.

8. $\triangle FGH$ is equilateral with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.

9. $\triangle LMN$ is isosceles, $\angle L$ is the vertex angle, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.

Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

10. $K(-3, 2), P(2, 1), L(-2, -3)$

11. $K(5, -3), P(3, 4), L(-1, 1)$

12. $K(-2, -6), P(-4, 0), L(3, -1)$

13. **DESIGN** Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there?
4-1 Reading to Learn Mathematics

Classifying Triangles

Pre-Activity Why are triangles important in construction?

Read the introduction to Lesson 4-1 at the top of page 178 in your textbook.

• Why are triangles used for braces in construction rather than other shapes?

• Why do you think that isosceles triangles are used more often than scalene triangles in construction?

Reading the Lesson

1. Supply the correct numbers to complete each sentence.
   a. In an obtuse triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).
   b. In an acute triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).
   c. In a right triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).

2. Determine whether each statement is always, sometimes, or never true.
   a. A right triangle is scalene.
   b. An obtuse triangle is isosceles.
   c. An equilateral triangle is a right triangle.
   d. An equilateral triangle is isosceles.
   e. An acute triangle is isosceles.
   f. A scalene triangle is obtuse.

3. Describe each triangle by as many of the following words as apply: acute, obtuse, right, scalene, isosceles, or equilateral.

   a. b. c.

   ![Diagram of triangles with angles labeled]

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to a nonmathematical definition of the same word. How is the use of the word acute, when used to describe acute pain, related to the use of the word acute when used to describe an acute angle or an acute triangle?
4-1 Enrichment

Reading Mathematics

When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example

Consider three points, A, B, and C on a coordinate grid. The y-coordinates of A and B are the same. The x-coordinate of B is greater than the x-coordinate of A. Both coordinates of C are greater than the corresponding coordinates of B. Is triangle ABC acute, right, or obtuse?

To answer this question, first draw a sample triangle that fits the description.

Side AB must be a horizontal segment because the y-coordinates are the same. Point C must be located to the right and up from point B.

From the diagram you can see that triangle ABC must be obtuse.

Answer each question. Draw a simple triangle on the grid above to help you.

1. Consider three points, R, S, and T on a coordinate grid. The x-coordinates of R and S are the same. The y-coordinate of T is between the y-coordinates of R and S. The x-coordinate of T is less than the x-coordinate of R. Is angle R of triangle RST acute, right, or obtuse?

2. Consider three noncollinear points, J, K, and L on a coordinate grid. The y-coordinates of J and K are the same. The x-coordinates of K and L are the same. Is triangle JKL acute, right, or obtuse?

3. Consider three noncollinear points, D, E, and F on a coordinate grid. The x-coordinates of D and E are opposites. The y-coordinates of D and E are the same. The x-coordinate of F is 0. What kind of triangle must \( \triangle DEF \) be: scalene, isosceles, or equilateral?

4. Consider three points, G, H, and I on a coordinate grid. Points G and H are on the positive y-axis, and the y-coordinate of G is twice the y-coordinate of H. Point I is on the positive x-axis, and the x-coordinate of I is greater than the y-coordinate of G. Is triangle GHI scalene, isosceles, or equilateral?
Angles of Triangles

**Angle Sum Theorem** If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

In the figure at the right, \(m\angle A + m\angle B + m\angle C = 180\).

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Find (m\angle T).</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m\angle R + m\angle S + m\angle T = 180)</td>
<td>Angle Sum Theorem</td>
</tr>
<tr>
<td>25 + 35 + (m\angle T = 180)</td>
<td>Substitution</td>
</tr>
<tr>
<td>60 + (m\angle T = 180)</td>
<td>Add.</td>
</tr>
<tr>
<td>(m\angle T = 120)</td>
<td>Subtract 60 from each side.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example 2</th>
<th>Find the missing angle measures.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m\angle 1 + m\angle A + m\angle B = 180)</td>
<td>Angle Sum Theorem</td>
</tr>
<tr>
<td>(m\angle 1 + 58 + 90 = 180)</td>
<td>Substitution</td>
</tr>
<tr>
<td>(m\angle 1 + 148 = 180)</td>
<td>Add.</td>
</tr>
<tr>
<td>(m\angle 1 = 32)</td>
<td>Subtract 148 from each side.</td>
</tr>
<tr>
<td>(m\angle 2 = 32)</td>
<td>Vertical angles are congruent.</td>
</tr>
<tr>
<td>(m\angle 3 + m\angle 2 + m\angle E = 180)</td>
<td>Angle Sum Theorem</td>
</tr>
<tr>
<td>(m\angle 3 + 32 + 108 = 180)</td>
<td>Substitution</td>
</tr>
<tr>
<td>(m\angle 3 + 140 = 180)</td>
<td>Add.</td>
</tr>
<tr>
<td>(m\angle 3 = 40)</td>
<td>Subtract 140 from each side.</td>
</tr>
</tbody>
</table>

**Exercises**

Find the measure of each numbered angle.

1. 
   - \(P = 1\)
   - \(M = 62^\circ\)
   - \(N = 90^\circ\)

2. 
   - \(Q = 1\)
   - \(R = 130^\circ\)

3. 
   - \(U = 1\)
   - \(W = 30^\circ\)
   - \(V = 60^\circ\)

4. 
   - \(P = 1\)
   - \(M = 66^\circ\)
   - \(N = 66^\circ\)

5. 
   - \(W = 30^\circ\)
   - \(T = 60^\circ\)
   - \(R = 2^\circ\)

6. 
   - \(G = 152^\circ\)
   - \(D = 1\)
   - \(A = 20^\circ\)
Exterior Angle Theorem At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an exterior angle of the triangle. For each exterior angle of a triangle, the remote interior angles are the interior angles that are not adjacent to that exterior angle. In the diagram below, $\angle B$ and $\angle A$ are the remote interior angles for exterior $\angle DCB$.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

$$m \angle 1 = m \angle A + m \angle B$$

**Example 1** Find $m \angle 1$.

$$m \angle 1 = m \angle R + m \angle S$$

Exterior Angle Theorem

$$= 60 + 80$$

Substitution

$$= 140$$

Add.

**Example 2** Find $x$.

$$m \angle PQS = m \angle R + m \angle S$$

Exterior Angle Theorem

$$78 = 55 + x$$

Substitution

$$23 = x$$

Subtract 55 from each side.

**Exercises**

Find the measure of each numbered angle.

1.  

2.  

3.  

4.  

5.  

6.  

Find $x$. 
Skills Practice

Angles of Triangles

Find the missing angle measures.

1. \(80^\circ\)

\(73^\circ\)

Find the measure of each angle.

3. \(m\angle 1\)

4. \(m\angle 2\)

5. \(m\angle 3\)

Find the measure of each angle.

6. \(m\angle 1\)

7. \(m\angle 2\)

8. \(m\angle 3\)

Find the measure of each angle.

9. \(m\angle 1\)

10. \(m\angle 2\)

11. \(m\angle 3\)

12. \(m\angle 4\)

13. \(m\angle 5\)

Find the measure of each angle.

14. \(m\angle 1\)

15. \(m\angle 2\)
4-2 Practice

Angles of Triangles

Find the missing angle measures.

1.  

2.  

Find the measure of each angle.

3. \( m\angle 1 \)
4. \( m\angle 2 \)
5. \( m\angle 3 \)

Find the measure of each angle.

6. \( m\angle 1 \)
7. \( m\angle 4 \)
8. \( m\angle 3 \)
9. \( m\angle 2 \)
10. \( m\angle 5 \)
11. \( m\angle 6 \)

Find the measure of each angle if \( \angle BAD \) and \( \angle BDC \) are right angles and \( m\angle ABC = 84 \).

12. \( m\angle 1 \)
13. \( m\angle 2 \)

14. CONSTRUCTION The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find \( m\angle 1 \).
Angles of Triangles

Pre-Activity  How are the angles of triangles used to make kites?

Read the introduction to Lesson 4-2 at the top of page 185 in your textbook.

The frame of the simplest kind of kite divides the kite into four triangles.

Describe these four triangles and how they are related to each other.

Reading the Lesson

1. Refer to the figure.
   a. Name the three interior angles of the triangle. (Use three letters to name each angle.)
   b. Name three exterior angles of the triangle. (Use three letters to name each angle.)
   c. Name the remote interior angles of \( \angle EAB \).
   d. Find the measure of each angle without using a protractor.
      i. \( \angle DBC \)
      ii. \( \angle ABC \)
      iii. \( \angle ACF \)
      iv. \( \angle EAB \)

2. Indicate whether each statement is true or false. If the statement is false, replace the underlined word or number with a word or number that will make the statement true.
   a. The acute angles of a right triangle are supplementary.
   b. The sum of the measures of the angles of any triangle is 100.
   c. A triangle can have at most one right angle or acute angle.
   d. If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are congruent.
   e. The measure of an exterior angle of a triangle is equal to the difference of the measures of the two remote interior angles.
   f. If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is 35.
   g. An exterior angle of a triangle forms a linear pair with an interior angle of the triangle.

Helping You Remember

3. Many students remember mathematical ideas and facts more easily if they see them demonstrated visually rather than having them stated in words. Describe a visual way to demonstrate the Angle Sum Theorem.
Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example
In triangle $ABC$, $m \angle A$, is twice $m \angle B$, and $m \angle C$ is 8 more than $m \angle B$. What is the measure of each angle?

Write and solve an equation. Let $x = m \angle B$.

$m \angle A + m \angle B + m \angle C = 180$

$2x + x + (x + 8) = 180$

$4x + 8 = 180$

$4x = 172$

$x = 43$

So, $m \angle A = 2(43)$ or 86, $m \angle B = 43$, and $m \angle C = 43 + 8$ or 51.

Solve each problem.

1. In triangle $DEF$, $m \angle E$ is three times $m \angle D$, and $m \angle F$ is 9 less than $m \angle E$. What is the measure of each angle?

2. In triangle $RST$, $m \angle T$ is 5 more than $m \angle R$, and $m \angle S$ is 10 less than $m \angle T$. What is the measure of each angle?

3. In triangle $JKL$, $m \angle K$ is four times $m \angle J$, and $m \angle L$ is five times $m \angle J$. What is the measure of each angle?

4. In triangle $XYZ$, $m \angle Z$ is 2 more than twice $m \angle X$, and $m \angle Y$ is 7 less than twice $m \angle X$. What is the measure of each angle?

5. In triangle $GHI$, $m \angle H$ is 20 more than $m \angle G$, and $m \angle G$ is 8 more than $m \angle I$. What is the measure of each angle?

6. In triangle $MNO$, $m \angle M$ is equal to $m \angle N$, and $m \angle O$ is 5 more than three times $m \angle N$. What is the measure of each angle?

7. In triangle $STU$, $m \angle U$ is half $m \angle T$, and $m \angle S$ is 30 more than $m \angle T$. What is the measure of each angle?

8. In triangle $PQR$, $m \angle P$ is equal to $m \angle Q$, and $m \angle R$ is 24 less than $m \angle P$. What is the measure of each angle?

9. Write your own problems about measures of triangles.
Corresponding Parts of Congruent Triangles

Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, $\triangle ABC \cong \triangle RST$.

**Example**

If $\triangle XYZ \cong \triangle RST$, name the pairs of congruent angles and congruent sides.

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$

$XY \cong RS$, $XZ \cong RT$, $YZ \cong ST$

**Exercises**

Identify the congruent triangles in each figure.

1.  
2.  
3.  

Name the corresponding congruent angles and sides for the congruent triangles.

4.  
5.  
6.
**4-3 Study Guide and Intervention (continued)**

**Congruent Triangles**

**Identify Congruence Transformations** If two triangles are congruent, you can slide, flip, or turn one of the triangles and they will still be congruent. These are called **congruence transformations** because they do not change the size or shape of the figure. It is common to use prime symbols to distinguish between an original $\triangle ABC$ and a transformed $\triangle A'B'C'$.

**Example** Name the congruence transformation that produces $\triangle A'B'C'$ from $\triangle ABC$.

The congruence transformation is a slide.

$\angle A \cong \angle A'$; $\angle B \cong \angle B'$; $\angle C \cong \angle C'$;

$AB \cong A'B'$; $AC \cong A'C'$; $BC \cong B'C'$

**Exercises**

Describe the congruence transformation between the two triangles as a *slide*, a *flip*, or a *turn*. Then name the congruent triangles.

1. 

2. 

3. 

4. 

5. 

6.
Skills Practice

Congruent Triangles

Identify the congruent triangles in each figure.

1. 

2. 

3. 

4. 

Name the congruent angles and sides for each pair of congruent triangles.

5. \( \triangle ABC \cong \triangle FGH \)

6. \( \triangle PQR \cong \triangle STU \)

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

7. \( \triangle ABC \cong \triangle A'B'C' \)

8. \( \triangle DEF \cong \triangle D'E'F' \)
4-3 Practice

Congruent Triangles

Identify the congruent triangles in each figure.

1.

\[ \triangle ABC \cong \triangle DSC \]

2.

\[ \triangle MNP \cong \triangle LOP \]

Name the congruent angles and sides for each pair of congruent triangles.

3. \( \triangle GKP \cong \triangle LMN \)

4. \( \triangle ANC \cong \triangle RBV \)

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

5. \( \triangle PST \cong \triangle P'S'T' \)

6. \( \triangle LMN \cong \triangle L'M'N' \)

QUILTING For Exercises 7 and 8, refer to the quilt design.

7. Indicate the triangles that appear to be congruent.

8. Name the congruent angles and congruent sides of a pair of congruent triangles.
Reading the Lesson

1. If \( \triangle RST \cong \triangle UWV \), complete each pair of congruent parts.
   \[
   \angle R \cong \ldots \quad \ldots \cong \angle W \quad \angle T \cong \ldots
   \]
   \[
   \overline{RT} \cong \ldots \quad \ldots \cong \overline{UW} \quad \ldots \cong \overline{WV}
   \]

2. Identify the congruent triangles in each diagram.
   a. 
   ![Diagram](image1)
   b. 
   ![Diagram](image2)
   c. 
   ![Diagram](image3)
   d. 
   ![Diagram](image4)

3. Determine whether each statement says that congruence of triangles is reflexive, symmetric, or transitive.
   a. If the first of two triangles is congruent to the second triangle, then the second triangle is congruent to the first.
   b. If there are three triangles for which the first is congruent to the second and the second is congruent to the third, then the first triangle is congruent to the third.
   c. Every triangle is congruent to itself.

Helping You Remember

4. A good way to remember something is to explain it to someone else. Your classmate Ben is having trouble writing congruence statements for triangles because he thinks he has to match up three pairs of sides and three pairs of angles. How can you help him understand how to write correct congruence statements more easily?
Transformations in The Coordinate Plane

The following statement tells one way to map preimage points to image points in the coordinate plane.

\((x, y) \rightarrow (x + 6, y - 3)\)

This can be read, “The point with coordinates \((x, y)\) is mapped to the point with coordinates \((x + 6, y - 3)\).”

With this transformation, for example, \((3, 5)\) is mapped to \((3 + 6, 5 - 3)\) or \((9, 2)\). The figure shows how the triangle \(ABC\) is mapped to triangle \(XYZ\).

1. Does the transformation above appear to be a congruence transformation? Explain your answer.

Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

2. \((x, y) \rightarrow (x - 4, y)\)

3. \((x, y) \rightarrow (x + 8, y + 7)\)

4. \((x, y) \rightarrow (-x, -y)\)

5. \((x, y) \rightarrow \left(-\frac{1}{2}x, y\right)\)
SSS Postulate  You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

SSS Postulate

If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Example

Write a two-column proof.

Given: $\overline{AB} \cong \overline{DB}$ and $C$ is the midpoint of $\overline{AD}$.

Prove: $\triangle ABC \cong \triangle DBC$

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<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. $\overline{AB} \cong \overline{DB}$</td>
<td>1. Given</td>
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<tr>
<td>2. $C$ is the midpoint of $\overline{AD}$</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $\overline{AC} \cong \overline{DC}$</td>
<td>3. Definition of midpoint</td>
</tr>
<tr>
<td>4. $\overline{BC} \cong \overline{BC}$</td>
<td>4. Reflexive Property of $\cong$</td>
</tr>
<tr>
<td>5. $\triangle ABC \cong \triangle DBC$</td>
<td>5. SSS Postulate</td>
</tr>
</tbody>
</table>

Exercises

Write a two-column proof.

1. 

Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$

Prove: $\triangle ABC \cong \triangle XYZ$

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<th>Statements</th>
<th>Reasons</th>
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2. 

Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

Prove: $\triangle RST \cong \triangle UTS$

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<th>Statements</th>
<th>Reasons</th>
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</table>
SAS Postulate  Another way to show that two triangles are congruent is to use the Side-Angle-Side (SAS) Postulate.

| SAS Postulate | If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. |

**Example**  For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.

**a.**

In $\triangle ABC$, the angle is not “included” by the sides $AB$ and $AC$. So the triangles cannot be proved congruent by the SAS Postulate.

**b.**

The right angles are congruent and they are the included angles for the congruent sides.  
$\triangle DEF \cong \triangle JGH$ by the SAS Postulate.

**c.**

The included angles, $\angle 1$ and $\angle 2$, are congruent because they are alternate interior angles for two parallel lines.  
$\triangle PSR \cong \triangle RQP$ by the SAS Postulate.

**Exercises**  For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.

1.  2.  3.  

4.  5.  6.  

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4-4 Skills Practice

Proving Congruence—SSS, SAS

Determine whether \( \triangle ABC \cong \triangle KLM \) given the coordinates of the vertices. Explain.

1. \( A(-3, 3), B(-1, 3), C(-3, 1), K(1, 4), L(3, 4), M(1, 6) \)

2. \( A(-4, -2), B(-4, 1), C(-1, -1), K(0, -2), L(0, 1), M(4, 1) \)

3. Write a flow proof.
   
   \[ \begin{align*}
   \text{Given:} & \quad \overline{PR} \equiv \overline{DE}, \overline{PT} \equiv \overline{DF} \\
   & \quad \angle R \equiv \angle E, \angle T \equiv \angle F \\
   \text{Prove:} & \quad \triangle PRT \cong \triangle DEF
   \end{align*} \]

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write \textit{not possible}.

4. 

5. 

6. 

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Determine whether \( \triangle DEF \cong \triangle PQR \) given the coordinates of the vertices. Explain.

1. \( D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0) \)

2. \( D(-7, -3), E(-4, -1), F(-2, -5), P(2, -2), Q(5, -4), R(0, -5) \)

3. Write a flow proof.
   Given: \( RS \cong TS \)
   \( V \) is the midpoint of \( RT \).
   Prove: \( \triangle RSV \cong \triangle TSV \)

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write not possible.

4. 

5. 

6. 

7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths \( A'B' \) and \( AB \) are equal?
Pre-Activity  How do land surveyors use congruent triangles?

Read the introduction to Lesson 4-4 at the top of page 200 in your textbook.

Why do you think that land surveyors would use congruent right triangles rather than other congruent triangles to establish property boundaries?

Reading the Lesson

1. Refer to the figure.
   a. Name the sides of \( \triangle LMN \) for which \( \angle L \) is the included angle.
   b. Name the sides of \( \triangle LMN \) for which \( \angle N \) is the included angle.
   c. Name the sides of \( \triangle LMN \) for which \( \angle M \) is the included angle.

2. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate that you would use. If not, write not possible.
   a. \( \triangle ABC \)
   b. \( \triangle DEF \)
   c. \( \overline{EH} \) and \( \overline{DG} \) bisect each other.
   d. \( \triangle RST \)

Helping You Remember

3. Find three words that explain what it means to say that two triangles are congruent and that can help you recall the meaning of the SSS Postulate.
**Congruent Parts of Regular Polygonal Regions**

Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.

1. Divide each square into four congruent parts. Use three different ways.

2. Divide each pentagon into five congruent parts. Use three different ways.

3. Divide each hexagon into six congruent parts. Use three different ways.

4. What hints might you give another student who is trying to divide figures like those into congruent parts?
ASA Postulate  The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate  If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example  Find the missing congruent parts so that the triangles can be proved congruent by the ASA Postulate. Then write the triangle congruence.

a.  \( \triangle ABC \) and \( \triangle DEF \)

Two pairs of corresponding angles are congruent, \( \triangle A \cong \triangle D \) and \( \triangle C \cong \triangle F \). If the included sides \( AC \) and \( DF \) are congruent, then \( \triangle ABC \cong \triangle DEF \) by the ASA Postulate.

b.  \( \angle R \cong \angle Y \) and \( \overline{SR} \cong \overline{XY} \). If \( \angle S \cong \angle X \), then \( \triangle RST \cong \triangle YXW \) by the ASA Postulate.

Exercises

What corresponding parts must be congruent in order to prove that the triangles are congruent by the ASA Postulate? Write the triangle congruence statement.

1.  
2.  
3.  
4.  
5.  
6.  

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AAS Theorem Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

| AAS Theorem | If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent. |

You now have five ways to show that two triangles are congruent.
- definition of triangle congruence
- ASA Postulate
- SSS Postulate
- AAS Theorem
- SAS Postulate

**Example** In the diagram, \(\angle BCA \cong \angle DCA\). Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Postulate?

\(\overline{AC} \cong \overline{AC}\) by the Reflexive Property of congruence. The congruent angles cannot be \(\angle 1\) and \(\angle 2\), because \(\overline{AC}\) would be the included side.

If \(\angle B \cong \angle D\), then \(\triangle ABC \cong \triangle ADC\) by the AAS Theorem.

**Exercises**

In Exercises 1 and 2, draw and label \(\triangle ABC\) and \(\triangle DEF\). Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem.

1. \(\angle A \cong \angle D; \angle B \cong \angle E\)
2. \(BC \cong EF; \angle A \cong \angle D\)

3. Write a flow proof.
   **Given:** \(\angle S \cong \angle U; \overline{TR}\) bisects \(\angle STU\).
   **Prove:** \(\angle SRT \cong \angle URT\)
4-5 Skills Practice
Proving Congruence—ASA, AAS

Write a flow proof.

1. Given: \( \angle N \cong \angle L \)
   \( \overline{JK} \cong \overline{MK} \)
Prove: \( \triangle JKN \cong \triangle MKL \)

2. Given: \( \overline{AB} \cong \overline{CB} \)
   \( \angle A \cong \angle C \)
   \( \overline{DB} \) bisects \( \angle ABC \).
Prove: \( \overline{AD} \cong \overline{CD} \)

3. Write a paragraph proof.

   Given: \( \overline{DE} \parallel \overline{FG} \)
   \( \angle E \cong \angle G \)
Prove: \( \triangle DFG \cong \triangle FDE \)
1. Write a flow proof.
   **Given:** \( S \) is the midpoint of \( QT \).
   \( QR \parallel TU \)
   **Prove:** \( \triangle QSR \cong \triangle TSU \)

2. Write a paragraph proof.
   **Given:** \( \angle D \equiv \angle F \)
   \( GE \) bisects \( \angle DEF \).
   **Prove:** \( DG \equiv FG \)

ARCHITECTURE  For Exercises 3 and 4, use the following information.

An architect used the window design in the diagram when remodeling an art studio. \( AB \) and \( CB \) each measure 3 feet.

3. Suppose \( D \) is the midpoint of \( AC \). Determine whether \( \triangle ABD \cong \triangle CBD \).
   Justify your answer.

4. Suppose \( \angle A \equiv \angle C \). Determine whether \( \triangle ABD \cong \triangle CBD \). Justify your answer.
Reading the Lesson

1. Explain in your own words the difference between how the ASA Postulate and the AAS Theorem are used to prove that two triangles are congruent.

2. Which of the following conditions are sufficient to prove that two triangles are congruent?
   A. Two sides of one triangle are congruent to two sides of the other triangle.
   B. The three sides of one triangles are congruent to the three sides of the other triangle.
   C. The three angles of one triangle are congruent to the three angles of the other triangle.
   D. All six corresponding parts of two triangles are congruent.
   E. Two angles and the included side of one triangle are congruent to two sides and the included angle of the other triangle.
   F. Two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of the other triangle.
   G. Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
   H. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
   I. Two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of the other triangle.

3. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate or theorem that you would use. If not, write not possible.
   a. [Diagram]
   b. T is the midpoint of RU.

Helping You Remember

4. A good way to remember mathematical ideas is to summarize them in a general statement. If you want to prove triangles congruent by using three pairs of corresponding parts, what is a good way to remember which combinations of parts will work?
Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider \( \triangle ABD \) and \( \triangle CDB \) whose vertices have coordinates \( A(0, 0) \), \( B(2, 5) \), \( C(9, 5) \), and \( D(7, 0) \). Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that \( \triangle ABD \cong \triangle CDB \). You may wish to make a sketch to help get you started.

2. Consider \( \triangle PQR \) and \( \triangle KLM \) whose vertices are the following points.
   
   \[
   P(1, 2) \quad Q(3, 6) \quad R(6, 5) \\
   K(-2, 1) \quad L(-6, 3) \quad M(-5, 6)
   \]
   
   Briefly describe how you can show that \( \triangle PQR \cong \triangle KLM \).

3. If you know the coordinates of all the vertices of two triangles, is it always possible to tell whether the triangles are congruent? Explain.
Properties of Isosceles Triangles

An isosceles triangle has two congruent sides. The angle formed by these sides is called the vertex angle. The other two angles are called base angles. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (Isosceles Triangle Theorem)
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example 1

Find $x$.

$$\triangle ABC$$

$BC = BA$, so $m\angle A = m\angle C$.

- Isos. Triangle Theorem
- $5x - 10 = 4x + 5$  
  - Substitution
- $x - 10 = 5$  
  - Subtract 4x from each side.
- $x = 15$  
  - Add 10 to each side.

Example 2

Find $x$.

$$\triangle STW$$

$m\angle S = m\angle T$, so $SR = TR$.

- Converse of Isos. Thm.
- $3x - 13 = 2x$  
  - Substitution
- $3x = 2x + 13$  
  - Add 13 to each side.
- $x = 13$  
  - Subtract 2x from each side.

Exercises

Find $x$.

1. $\triangle RQP$
2. $\triangle STV$
3. $\triangle WYZ$
4. $\triangle KDP$
5. $\triangle BDL$
6. $\triangle TPS$

7. Write a two-column proof.

Given: $\angle 1 \equiv \angle 2$

Prove: $AB \equiv CB$

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Properties of Equilateral Triangles

An equilateral triangle has three congruent sides. The Isosceles Triangle Theorem can be used to prove two properties of equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60°.

Example

Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.

Proof:

<table>
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<tr>
<th>Statements</th>
<th>Reasons</th>
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</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC ) is equilateral; ( PQ \parallel BC ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle A = m\angle B = m\angle C = 60 )</td>
<td>2. Each ( \angle ) of an equilateral ( \triangle ) measures 60°.</td>
</tr>
<tr>
<td>3. ( \angle 1 \equiv \angle B, \angle 2 \equiv \angle C )</td>
<td>3. If ( \parallel ) lines, then corres. ( \angle )s are ( \equiv ).</td>
</tr>
<tr>
<td>4. ( m\angle 1 = 60, m\angle 2 = 60 )</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. ( \triangle APQ ) is equilateral.</td>
<td>5. If a ( \triangle ) is equiangular, then it is equilateral.</td>
</tr>
</tbody>
</table>

Exercises

Find \( x \).

1. \( \triangle DFE \)

2. \( \triangle GJH \)

3. \( \triangle KLM \) is equilateral.

4. \( \triangle PQV \)

5. \( \triangle XYZ \)

6. \( \triangle RMO \)

7. Write a two-column proof.

Given: \( \triangle ABC \) is equilateral; \( \angle 1 \equiv \angle 2 \).

Prove: \( \angle ADB \equiv \angle CDB \)

Proof:

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<th>Statements</th>
<th>Reasons</th>
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Skills Practice

**Isosceles Triangles**

Refer to the figure.

1. If $\overline{AC} \cong \overline{AD}$, name two congruent angles.

2. If $\overline{BE} \cong \overline{BC}$, name two congruent angles.

3. If $\angle EBA \cong \angle EAB$, name two congruent segments.

4. If $\angle CED \cong \angle CDE$, name two congruent segments.

$\triangle ABF$ is isosceles, $\triangle CDF$ is equilateral, and $m \angle AFD = 150$. Find each measure.

5. $m \angle CFD$  
6. $m \angle AFB$

7. $m \angle ABF$  
8. $m \angle A$

In the figure, $\overline{PL} \cong \overline{RL}$ and $\overline{LR} \cong \overline{BR}$.

9. If $m \angle RLP = 100$, find $m \angle BRL$.

10. If $m \angle LPR = 34$, find $m \angle B$.

11. Write a two-column proof.
    
    **Given:** $\overline{CD} \cong \overline{CG}$  
    $\overline{DE} \cong \overline{GF}$
    
    **Prove:** $\overline{CE} \cong \overline{CF}$
4-6 Practice  
Isosceles Triangles

Refer to the figure.

1. If $\overline{RV} \cong \overline{RT}$, name two congruent angles.

2. If $\overline{RS} \cong \overline{SV}$, name two congruent angles.

3. If $\angle SRT \cong \angle STR$, name two congruent segments.

4. If $\angle STV \cong \angle SVT$, name two congruent segments.

Triangles $GHM$ and $HJM$ are isosceles, with $\overline{GH} \cong \overline{MH}$ and $\overline{HJ} \cong \overline{MJ}$. Triangle $KLM$ is equilateral, and $\angle HMK = 50$. Find each measure.

5. $m \angle KML$  
6. $m \angle HMG$  
7. $m \angle GHM$

8. If $m \angle HJM = 145$, find $m \angle MHJ$.

9. If $m \angle G = 67$, find $m \angle GHM$.

10. Write a two-column proof.

   **Given:**  
   $DE \parallel BC$  
   $\angle 1 \cong \angle 2$  

   **Prove:** $AB \cong AC$

11. **SPORTS** A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle.
Pre-Activity  How are triangles used in art?

Read the introduction to Lesson 4-6 at the top of page 216 in your textbook.

• Why do you think that isosceles and equilateral triangles are used more often than scalene triangles in art?

• Why might isosceles right triangles be used in art?

Reading the Lesson

1. Refer to the figure.

   a. What kind of triangle is \( \triangle QRS \)?

   b. Name the legs of \( \triangle QRS \).

   c. Name the base of \( \triangle QRS \).

   d. Name the vertex angle of \( \triangle QRS \).

   e. Name the base angles of \( \triangle QRS \).

2. Determine whether each statement is always, sometimes, or never true.

   a. If a triangle has three congruent sides, then it has three congruent angles.

   b. If a triangle is isosceles, then it is equilateral.

   c. If a right triangle is isosceles, then it is equilateral.

   d. The largest angle of an isosceles triangle is obtuse.

   e. If a right triangle has a 45° angle, then it is isosceles.

   f. If an isosceles triangle has three acute angles, then it is equilateral.

   g. The vertex angle of an isosceles triangle is the largest angle of the triangle.

3. Give the measures of the three angles of each triangle.

   a. an equilateral triangle

   b. an isosceles right triangle

   c. an isosceles triangle in which the measure of the vertex angle is 70°

   d. an isosceles triangle in which the measure of a base angle is 70°

   e. an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles

Helping You Remember

4. If a theorem and its converse are both true, you can often remember them most easily by combining them into an “if-and-only-if” statement. Write such a statement for the Isosceles Triangle Theorem and its converse.
Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

1. Given: $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m \angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.

Find $m \angle ABC$.

2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m \angle DAC = 44$ and $CE$ bisects $\angle ACD$.

Find $m \angle DEC$.

3. Given: $m \angle UZY = 90$, $m \angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).

Find $m \angle WZY$.

4. Given: $m \angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.

Find $m \angle JKM$. 
Position and Label Triangles  A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make the computations as simple as possible.

Example  Position an equilateral triangle on the coordinate plane so that its sides are \( a \) units long and one side is on the positive \( x \)-axis.
Start with \( R(0, 0) \). If \( RT \) is \( a \), then another vertex is \( T(a, 0) \).
For vertex \( S \), the \( x \)-coordinate is \( \frac{a}{2} \). Use \( b \) for the \( y \)-coordinate, so the vertex is \( S\left(\frac{a}{2}, b\right) \).

Exercises  Find the missing coordinates of each triangle.

1. \( \triangle ABC \) with \( A(0, 0) \) and \( B(2p, 0) \)
2. \( \triangle TQR \) with \( T(a, ?) \)
3. \( \triangle FEG \) with \( F(?, b) \)

Position and label each triangle on the coordinate plane.

4. isosceles triangle \( \triangle RST \) with base \( RS \) 4\( a \) units long
5. isosceles right \( \triangle DEF \) with legs \( e \) units long
6. equilateral triangle \( \triangle EQI \) with vertex \( Q(0, a) \) and sides 2\( b \) units long
**Write Coordinate Proofs** Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

**Example** Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.

First, position and label an isosceles triangle on the coordinate plane. One way is to use \( T(a, 0), R(-a, 0), \) and \( S(0, c) \). Then \( U(0, 0) \) is the midpoint of \( RT \).

**Given:** Isosceles \( \triangle RST; U \) is the midpoint of base \( RT \).

**Prove:** \( SU \perp RT \)

**Proof:**
\( U \) is the midpoint of \( RT \) so the coordinates of \( U \) are \( \left( \frac{-a + a}{2}, \frac{0 + 0}{2} \right) = (0, 0) \). Thus \( SU \) lies on the \( y \)-axis, and \( \triangle RST \) was placed so \( RT \) lies on the \( x \)-axis. The axes are perpendicular, so \( SU \perp RT \).

**Exercises**

Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.
Skills Practice

Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

1. right \( \triangle FGH \) with legs \( a \) units and \( b \) units
2. isosceles \( \triangle KLP \) with base \( KP \) 6\( b \) units long
3. isosceles \( \triangle AND \) with base \( AD \) 5\( a \) long

Find the missing coordinates of each triangle.

4. 
5. 
6. 

7. 
8. 
9. 

10. Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

Given: isosceles right \( \triangle ABC \) with \( \angle ABC \) the right angle and \( M \) the midpoint of \( \overline{AC} \)
Prove: \( BM \perp AC \)
Practice

4-7

Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

1. equilateral $\triangle SWY$ with sides $\frac{1}{4}a$ long
2. isosceles $\triangle BLP$ with base $BL$ $3b$ units long
3. isosceles right $\triangle DGJ$ with hypotenuse $DJ$ and legs $2a$ units long

Find the missing coordinates of each triangle.

4. $\triangle S(?, ?)$
5. $\triangle E(0, ?)$
6. $\triangle M(0, ?)$

NEIGHBORHOODS For Exercises 7 and 8, use the following information.

Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Write a coordinate proof to prove that Karina’s high school, her home, and the mall are at the vertices of a right triangle.

Given: $\triangle SKM$
Prove: $\triangle SKM$ is a right triangle.

8. Find the distance between the mall and Karina’s home.
Pre-Activity  How can the coordinate plane be useful in proofs?

Read the introduction to Lesson 4-7 at the top of page 222 in your textbook.

From the coordinates of A, B, and C in the drawing in your textbook, what do you know about \( \triangle ABC \)?

Reading the Lesson

1. Find the missing coordinates of each triangle.
   a. \( R(?, b) \) \( T(a, ?) \) \( S(?, ?) \)
   b. \( E(?, a) \) \( F(?, ?) \) \( D(?, ?) \)

2. Refer to the figure.
   a. Find the slope of \( SR \) and the slope of \( ST \).
   b. Find the product of the slopes of \( SR \) and \( ST \). What does this tell you about \( SR \) and \( ST \)?
   c. What does your answer from part b tell you about \( \triangle RST \)?
   d. Find \( SR \) and \( ST \). What does this tell you about \( SR \) and \( ST \)?
   e. What does your answer from part d tell you about \( \triangle RST \)?
   f. Combine your answers from parts c and e to describe \( \triangle RST \) as completely as possible.
   g. Find \( m\angle SRT \) and \( m\angle STR \).
   h. Find \( m\angle OSR \) and \( m\angle OST \).

Helping You Remember

3. Many students find it easier to remember mathematical formulas if they can put them into words in a compact way. How can you use this approach to remember the slope and midpoint formulas easily?
How Many Triangles?

Each puzzle below contains many triangles. Count them carefully. Some triangles overlap other triangles.

How many triangles are there in each figure?

1.  
2.  
3.  
4.  
5.  
6.  

How many triangles can you form by joining points on each circle? List the vertices of each triangle.

7.  
8.  
9.  

NAME ______________________________ DATE ____________ PERIOD _____
4-1 Study Guide and Intervention
Classifying Triangles

Classify Triangles by Angles

One way to classify a triangle is by the measures of its angles.

- If one of the angles of a triangle is an obtuse angle, then the triangle is an obtuse triangle.
- If one of the angles of a triangle is a right angle, then the triangle is a right triangle.
- If all three of the angles of a triangle are acute angles, then the triangle is an acute triangle.
- If all three angles of an acute triangle are congruent, then the triangle is an equiangular triangle.

Classify Triangles by Sides

You can classify a triangle by the measures of its sides. Equal numbers of hash marks indicate congruent sides.

- If all three sides of a triangle are congruent, then the triangle is an equilateral triangle.
- If at least two sides of a triangle are congruent, then the triangle is an isosceles triangle.
- If no two sides of a triangle are congruent, then the triangle is a scalene triangle.

Example

Classify each triangle.

a. The triangle has one angle that is obtuse. It is an obtuse triangle.

b. The triangle has one right angle. It is a right triangle.

c. The triangle has one angle that is obtuse. It is an obtuse triangle.

Exercises

Classify each triangle as equilateral, isosceles, or scalene.

1. equilateral
2. isosceles
3. scalene
4. acute
5. obtuse
6. equiangular
7. Find the measure of each side of equilateral \( \triangle RST \) where \( RS = 2x + 2, ST = 3x \), and \( TR = 5x - 4 \).
8. Find the measure of each side of isosceles \( \triangle ABC \) with \( AB = BC \) if \( AB = 4y, BC = 3y + 2, \) and \( AC = 3y \).
9. Find the measure of each side of \( \triangle ABC \) with vertices \( A(-1, 5), B(6, 1), \) and \( C(2, -6) \).
   Classify the triangle.
   \( AB = BC = \sqrt{65}, AC = \sqrt{130}; \triangle ABC \) is isosceles.
**Skills Practice**

**Classifying Triangles**

Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1. equiangular
2. obtuse
3. right
4. acute
5. obtuse
6. acute

Identify the indicated type of triangles.

7. right
   - $\triangle ABE, \triangle BCE$
8. isosceles
   - $\triangle BCD, \triangle BDE$
9. scalene
10. obtuse
    - $\triangle ABE, \triangle BCE$
     - $\triangle BDE$

**ALGEBRA** Find $x$ and the measure of each side of the triangle.

11. $\triangle ABC$ is equilateral with $AB = 3x - 2$, $BC = 2x + 4$, and $CA = x + 10$.
    - $x = 6$, $AB = 16$, $BC = 16$, $CA = 16$

12. $\triangle DEF$ is isosceles, $\angle D$ is the vertex angle, $DE = x + 7$, $DF = 3x - 1$, and $EF = 2x + 5$.
    - $x = 4$, $DE = 11$, $DF = 11$, $EF = 13$

Find the measures of the sides of $\triangle RST$ and classify each triangle by its sides.

13. $R(0, 2)$, $S(2, 5)$, $T(4, 2)$
    - $RS = \sqrt{13}$, $ST = \sqrt{13}$, $RT = 4$; isosceles

14. $R(1, 3)$, $S(4, 7)$, $T(5, 4)$
    - $RS = 5$, $ST = \sqrt{17}$, $RT = \sqrt{17}$; scalene

**Practice (Average)**

**Classifying Triangles**

Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1. obtuse
2. acute
3. right
4. acute
5. obtuse
6. scalene
7. isosceles
8. $\triangle ABC, \triangle CDE$
9. $\triangle BDE, \triangle BDC$
10. $\triangle ABE, \triangle BCE$
11. $\triangle BCD, \triangle BDE$

**ALGEBRA** Find $x$ and the measure of each side of the triangle.

8. $\triangle FGH$ is equilateral with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.
    - $x = 7$, $FG = 12$, $GH = 12$, $FH = 12$

9. $\triangle LMN$ is isosceles, $\angle L$ is the vertex angle, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.
    - $x = 3$, $LM = 7$, $LN = 7$, $MN = 13$

Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

10. $K(-3, 2)$, $P(2, 1)$, $L(-2, -3)$
    - $KP = \sqrt{26}$, $PL = 4\sqrt{2}$, $LK = \sqrt{26}$; isosceles

11. $K(5, -3)$, $P(3, 4)$, $L(-1, 1)$
    - $KP = \sqrt{53}$, $PL = 5$, $LK = 2\sqrt{13}$; scalene

12. $K(-2, -6)$, $P(-4, 0)$, $L(3, -1)$
    - $KP = 2\sqrt{10}$, $PL = 5\sqrt{2}$, $LK = 5\sqrt{2}$; isosceles

13. **DESIGN** Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there? 5
4-1 Reading to Learn Mathematics
Classifying Triangles

Pre-Activity Why are triangles important in construction?

- Why are triangles used for braces in construction rather than other shapes? Sample answer: Triangles lie in a plane and are rigid shapes.
- Why do you think that isosceles triangles are used more often than scalene triangles in construction? Sample answer: Isosceles triangles are symmetrical.

Reading the Lesson
1. Supply the correct numbers to complete each sentence.
   a. In an obtuse triangle, there are 2 acute angle(s), 0 right angle(s), and 1 obtuse angle(s).
   b. In an acute triangle, there are 3 acute angle(s), 0 right angle(s), and 0 obtuse angle(s).
   c. In a right triangle, there are 2 acute angle(s), 1 right angle(s), and 0 obtuse angle(s).

2. Determine whether each statement is always, sometimes, or never true.
   a. A right triangle is scalene. sometimes
   b. An obtuse triangle is isosceles. sometimes
   c. An equilateral triangle is a right triangle. never
   d. An equilateral triangle is isosceles. always
   e. An acute triangle is isosceles. sometimes
   f. A scalene triangle is obtuse. sometimes

3. Describe each triangle by as many of the following words as apply: acute, obtuse, right, scalene, isosceles, or equilateral.
   a. 
   acute, scalene
   b. 
   obtuse, isosceles
   c. 
   right, scalene

Helping You Remember
4. A good way to remember a new mathematical term is to relate it to a nonmathematical definition of the same word. How is the use of the word acute, when used to describe acute pain, related to the use of the word acute when used to describe an acute angle or an acute triangle? Sample answer: Both are related to the meaning of acute as sharp. An acute pain is a sharp pain, and an acute angle can be thought of as an angle with a sharp point. In an acute triangle all of the angles are acute.

4-1 Enrichment

Reading Mathematics
When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example
Consider three points, A, B, and C on a coordinate grid. The y-coordinates of A and B are the same. The x-coordinate of A is greater than the x-coordinate of B. Both coordinates of C are greater than the corresponding coordinates of B. Is triangle ABC acute, right, or obtuse?

To answer this question, first draw a sample triangle that fits the description.

Side AB must be a horizontal segment because the y-coordinates are the same. Point C must be located from the right and up from point B.

From the diagram you can see that triangle ABC must be obtuse.

Answer each question. Draw a simple triangle on the grid above to help you.

1. Consider three points, R, S, and T on a coordinate grid. The x-coordinates of R and S are the same. The y-coordinate of T is between the y-coordinates of R and S. The x-coordinate of T is less than the x-coordinate of R. Is angle R of triangle RST acute, right, or obtuse? acute

2. Consider three noncollinear points, J, K, and L on a coordinate grid. The y-coordinates of J and K are the same. The x-coordinates of K and L are the same. Is triangle JKL acute, right, or obtuse? right

3. Consider three noncollinear points, D, E, and F on a coordinate grid. The x-coordinates of D and E are opposites. The y-coordinates of D and E are the same. The y-coordinate of F is 0. What kind of triangle must \( \triangle DEF \) be: scalene, isosceles, or equilateral? isosceles

4. Consider three points, G, H, and I on a coordinate grid. Points G and H are on the positive y-axis, and the y-coordinate of G is twice the y-coordinate of H. Point I is on the positive x-axis, and the x-coordinate of I is greater than the y-coordinate of G. Is triangle GHJ scalene, isosceles, or equilateral? scalene
4-2

Study Guide and Intervention

Angles of Triangles

Angle Sum Theorem  If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

**Example 1**

Find \( m\angle T \).

\[
\angle R + \angle S + \angle T = 180
\]

\[
25 + 35 + \angle T = 180
\]

\[
\angle T = 120
\]

**Example 2**

Find the missing angle measures.

\[
m\angle 1 + m\angle A + m\angle B = 180
\]

\[
m\angle 1 + 58 + 90 = 180
\]

\[
m\angle 1 = 32
\]

**Exterior Angle Theorem**

At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an exterior angle of the triangle. For each exterior angle of a triangle, the remote interior angles are the interior angles that are not adjacent to that exterior angle. In the diagram below, \( \angle B \) and \( \angle A \) are the remote interior angles for exterior \( \angle DCB \).

**Exterior Angle Theorem**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. \( m\angle 1 = m\angle A + m\angle B \)

**Example 1**

Find \( m\angle 1 \).

\[
m\angle 1 = m\angle R + m\angle S
\]

\[
= 60 + 80
\]

\[
m\angle 1 = 140
\]

**Example 2**

Find \( x \).

\[
m\angle PQS = m\angle R + m\angle S
\]

\[
= 135 + x
\]

\[
x = 45
\]

**Exercises**

Find the measure of each numbered angle.

1. \( m\angle 1 = 28 \)

2. \( m\angle 1 = 120 \)

3. \( m\angle 1 = 30, m\angle 2 = 60 \)

4. \( m\angle 1 = 56, m\angle 2 = 56, m\angle 3 = 74 \)

5. \( m\angle 1 = 30, m\angle 2 = 60 \)

6. \( m\angle 1 = 8 \)
Find the missing angle measures.

1. TIGERS
   - Angle 1: 27°

2. Angle 2: 18°, Angle 3: 17°

Find the measure of each angle.

3. \( \angle 1 \): 55°
4. \( \angle 2 \): 55°
5. \( \angle 3 \): 70°

Find the measure of each angle.

6. \( \angle 1 \): 125°
7. \( \angle 2 \): 55°
8. \( \angle 3 \): 95°

Find the measure of each angle.

9. \( \angle 1 \): 140°
10. \( \angle 2 \): 40°
11. \( \angle 3 \): 65°
12. \( \angle 4 \): 75°
13. \( \angle 5 \): 115°

Find the measure of each angle.

14. \( \angle 1 \): 27°
15. \( \angle 2 \): 27°

Find the measure of each angle if \( \angle BAD \) and \( \angle BDC \) are right angles and \( \angle ABC = 84° \).

12. \( \angle 1 \): 26°
13. \( \angle 2 \): 32°

14. CONSTRUCTION The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find \( \angle 1 \).

   - \( \angle 1 \): 55°
Pre-Activity  How are the angles of triangles used to make kites?

Read the introduction to Lesson 4-2 at the top of page 185 in your textbook. The frame of the simplest kind of kite divides the kite into four triangles. Describe these four triangles and how they are related to each other. Sample answer: There are two pairs of right triangles that have the same size and shape.

Reading the Lesson

1. Refer to the figure.
   a. Name the three interior angles of the triangle. (Use three letters to name each angle.) ∠BAC, ∠ABC, ∠BCA
   b. Name three exterior angles of the triangle. (Use three letters to name each angle.) ∠EAB, ∠DBC, ∠FCA
   c. Name the remote interior angles of ∠EAB, ∠ABC, ∠BCA
   d. Find the measure of each angle without using a protractor.
      i. ∠DBC 62
      ii. ∠ABC 118
      iii. ∠ACF 157
      iv. ∠EAB 141

2. Indicate whether each statement is true or false. If the statement is false, replace the underlined word or number with a word or number that will make the statement true.
   a. The acute angles of a right triangle are supplementary. false; complementary
   b. The sum of the measures of the angles of any triangle is 180. false; 180
   c. A triangle can have at most one right angle or acute angle. false; obtuse
   d. If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the two triangles are congruent. true
   e. The measure of an exterior angle of a triangle is equal to the difference of the measures of the two remote interior angles. false; sum
   f. If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is 35. false; 25
   g. An exterior angle of a triangle forms a linear pair with an interior angle of the triangle. true

Helping You Remember

3. Many students remember mathematical ideas and facts more easily if they see them demonstrated visually rather than having them stated in words. Describe a visual way to demonstrate the Angle Sum Theorem.
   Sample answer: Cut off the angles of a triangle and place them side-by-side on one side of a line so that their vertices meet at a common point. The result will show three angles whose measures add up to 180.

Example

Describe these four triangles and how they are related to each other.

ABC is twice \( \angle 1 \) and \( \angle 2 \) is 8 more than \( m\angle B \) and \( \angle C \) is 8 more than \( m\angle B \). What is the measure of each angle?

Write and solve an equation. Let \( x = m\angle B \).

\[
m\angle A + m\angle B + m\angle C = 180
\]

\[
2x + x + (x + 8) = 180
\]

\[
x + 8 = 180
\]

\[
x = 172
\]

So, \( m\angle A = 2(43) \) or 86, \( m\angle B = 43 \), and \( m\angle C = 43 + 8 \) or 51.

Solve each problem.

1. In triangle DEF, \( m\angle E \) is three times \( m\angle D \), and \( m\angle F \) is 9 less than \( m\angle E \). What is the measure of each angle?

   \( m\angle D = 27 \), \( m\angle E = 81 \), \( m\angle F = 72 \)

2. In triangle RST, \( m\angle T \) is 5 more than \( m\angle R \), and \( m\angle S \) is 10 less than \( m\angle T \). What is the measure of each angle?

   \( m\angle R = 60 \), \( m\angle S = 55 \), \( m\angle T = 65 \)

3. In triangle JKL, \( m\angle K \) is four times \( m\angle J \), and \( m\angle L \) is 5 times \( m\angle J \). What is the measure of each angle?

   \( m\angle J = 18 \), \( m\angle K = 72 \), \( m\angle L = 90 \)

4. In triangle XYZ, \( m\angle Z \) is 2 more than twice \( m\angle X \), and \( m\angle Y \) is 7 less than twice \( m\angle X \). What is the measure of each angle?

   \( m\angle X = 37 \), \( m\angle Y = 67 \), \( m\angle Z = 76 \)

5. In triangle GHI, \( m\angle H \) is 20 more than \( m\angle G \), and \( m\angle H \) is 8 more than \( m\angle I \). What is the measure of each angle?

   \( m\angle G = 56 \), \( m\angle H = 76 \), \( m\angle I = 48 \)

6. In triangle MNO, \( m\angle M \) is equal to \( m\angle N \), and \( m\angle O \) is 5 more than three times \( m\angle N \). What is the measure of each angle?

   \( m\angle M = m\angle N = 35 \), \( m\angle O = 110 \)

7. In triangle STU, \( m\angle U \) is half \( m\angle T \), and \( m\angle S \) is 30 more than \( m\angle T \). What is the measure of each angle?

   \( m\angle S = 90 \), \( m\angle T = 60 \), \( m\angle U = 30 \)

8. In triangle PQR, \( m\angle P \) is equal to \( m\angle Q \), and \( m\angle R \) is 24 less than \( m\angle P \). What is the measure of each angle?

   \( m\angle P = m\angle Q = 68 \), \( m\angle R = 44 \)

9. Write your own problems about measures of triangles.

   See students' work.
Identify Congruence Transformations

If two triangles are congruent, you can slide, flip, or turn one of the triangles and they will still be congruent. These are called congruence transformations because they do not change the size or shape of the figure. It is common to use prime symbols to distinguish between an original \( \triangle ABC \) and a transformed \( \triangle A'B'C' \).

Example

Name the congruence transformation that produces \( \triangle A'B'C' \) from \( \triangle ABC \).

The congruence transformation is a slide.

\[ \angle A \rightarrow \angle A'; \; \angle B \rightarrow \angle B'; \; \angle C \rightarrow \angle C'; \]
\[ AB \rightarrow A'B'; \; AC \rightarrow A'C'; \; BC \rightarrow B'C' \]

Exercises

Describe the congruence transformation between the two triangles as a slide, a flip, or a turn. Then name the congruent triangles.

1. slide; \( \triangle RST \rightarrow \triangle R'S'T' \)
2. flip; \( \triangle MNP \rightarrow \triangle M'N'P' \)
3. turn; \( \triangle OPQ \rightarrow \triangle O'P'Q' \)
4. flip; \( \triangle ABC \rightarrow \triangle A'B'C \)
5. slide; \( \triangle ABC \rightarrow \triangle A'B'C' \)
6. turn; \( \triangle MNP \rightarrow \triangle MN'P' \)
4-3 Skills Practice

Identify the congruent triangles in each figure.

1. \( \triangle JPL = \triangle TVS \)
2. \( \triangle ABC = \triangle WXY \)
3. \( \triangle PQR = \triangle PSR \)
4. \( \triangle DEF = \triangle DGF \)

Name the congruent angles and sides for each pair of congruent triangles.

5. \( \triangle ABC = \triangle FGH \)
   \( \angle A = \angle F, \angle B = \angle G, \angle C = \angle H; \overline{AB} = \overline{FG}, \overline{BC} = \overline{GH}, \overline{AC} = \overline{FH} \)
6. \( \triangle PQR = \triangle STU \)
   \( \angle P = \angle S, \angle Q = \angle T, \angle R = \angle U; \overline{PQ} = \overline{ST}, \overline{QR} = \overline{TU}, \overline{PR} = \overline{SU} \)

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

7. \( \triangle ABC = \triangle A'B'C' \)
8. \( \triangle DEF = \triangle D'E'F' \)
   \( AB = 2\sqrt{2}, A'B' = 2\sqrt{2}, \)
   \( BC = 2\sqrt{2}, B'C' = 2\sqrt{2}, \)
   \( AC = 4, A'C' = 4, \angle A = \angle A', \)
   \( \angle B = \angle B', \angle C = \angle C'; \text{slide} \)
   \( DE = 4, D'E' = 4, EF = 5, \)
   \( E'F' = 5, DF = 3, D'F' = 3, \)
   \( \angle D = \angle D', \angle E = \angle E', \)
   \( \angle F = \angle F'; \text{flip} \)

7. Indicate the triangles that appear to be congruent.
   \( \triangle ABI = \triangle EBF, \triangle CBD = \triangle HBG \)

8. Name the congruent angles and congruent sides of a pair of congruent triangles.
   Sample answer: \( \angle A = \angle E, \angle ABI = \angle EBF, \angle I = \angle F, \)
   \( AB = EB, BI = BF, AI = EF \)
   \( \triangle ABC = \triangle DRS \)
   \( \triangle LMN = \triangle QPN \)
   \( \angle G = \angle L, \angle K = \angle M, \angle P = \angle N; \overline{LM} = \overline{KP}, \overline{MN} = \overline{GP} = \overline{LN} \)
   \( \angle A = \angle R, \angle N = \angle B, \angle C = \angle V; \overline{AN} = \overline{RR}, \overline{NC} = \overline{BV}, \overline{AC} = \overline{RV} \)

Verifying that each of the following transformations preserves congruence, and name the congruence transformation.

7. \( \triangle ABC = \triangle A'B'C' \)
8. \( \triangle DEF = \triangle D'E'F' \)
   \( PS = \sqrt{13}, P'S' = \sqrt{13}, \)
   \( ST = \sqrt{5}, S'T' = \sqrt{5}, PT = \sqrt{10}, \)
   \( P'T' = \sqrt{10}, \angle P = \angle P', \)
   \( \angle S = \angle S', \angle T = \angle T'; \text{flip} \)
   \( \angle M = \angle M', \angle N = \angle N'; \text{flip} \)
   \( \angle L = \angle L', \angle G = \angle G', \angle K = \angle K'; \text{slide} \)

7. Indicate the triangles that appear to be congruent.
   \( \triangle ABI = \triangle EBF, \triangle CBD = \triangle HBG \)

8. Name the congruent angles and congruent sides of a pair of congruent triangles.
   Sample answer: \( \angle A = \angle E, \angle ABI = \angle EBF, \angle I = \angle F, \)
   \( AB = EB, BI = BF, AI = EF \)
4-3 Reading to Learn Mathematics

**Congruent Triangles**

**Pre-Activity** Why are triangles used in bridges?

Read the introduction to Lesson 4-3 at the top of page 192 in your textbook.

In the bridge shown in the photograph in your textbook, diagonal braces were used to divide squares into two isosceles right triangles. Why do you think these braces are used on the bridge? Sample answer: The diagonal braces make the structure stronger and prevent it from being deformed when it has to withstand a heavy load.

Reading the Lesson

1. If \( \triangle RST \cong \triangle UVW \), complete each pair of congruent parts.
   \[
   \angle R = \angle U \\
   \overline{RT} = \overline{UV} \\
   \angle T = \angle V \\
   \overline{ST} = \overline{WV}
   \]

2. Identify the congruent triangles in each diagram.
   a. \( \triangle ABC \cong \triangle ADC \)
   b. \( \triangle PQS \cong \triangle RQS \)
   c. \( \triangle MNO \cong \triangle QPO \)
   d. \( \triangle RTV \cong \triangle USV \)

3. Determine whether each statement says that congruence of triangles is **reflexive**, **symmetric**, or **transitive**.
   a. If the first of two triangles is congruent to the second triangle, then the second triangle is congruent to the first. **symmetric**
   b. If there are three triangles for which the first is congruent to the second and the second is congruent to the third, then the first triangle is congruent to the third. **transitive**
   c. Every triangle is congruent to itself. **reflexive**

**Helping You Remember**

4. A good way to remember something is to explain it to someone else. Your classmate Ben is having trouble writing congruence statements for triangles because he thinks he has to match up three pairs of sides and three pairs of angles. How can you help him understand how to write correct congruence statements more easily? Sample answer: Write the three vertices of one triangle in any order. Then write the corresponding vertices of the second triangle in the same order. If the angles are written in the correct correspondence, the sides will automatically be in the correct correspondence also.

1. Does the transformation above appear to be a congruence transformation? Explain your answer. Yes; the transformation slides the figure to the lower right without changing its size or shape.

Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

2. \( (x, y) \rightarrow (x - 4, y) \) **yes**
3. \( (x, y) \rightarrow (x + 8, y + 7) \) **yes**
4. \( (x, y) \rightarrow (-x, -y) \) **yes**
5. \( (x, y) \rightarrow \left( -\frac{1}{2}x, y \right) \) **no**
**4-4 Study Guide and Intervention**

**Proving Congruence—SSS, SAS**

**SSS Postulate** You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

- **SSS Postulate**
  - If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

- **SAS Postulate**
  - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

---

**Example**

**Write a two-column proof.**

Given: \( AB = DB \) and \( C \) is the midpoint of \( AD \).

**Prove:** \( \triangle ABC \cong \triangle DBC \)

---

**Exercises**

**Write a two-column proof.**

1. \( \triangle ABC \cong \triangle XYZ \)
   - **Given:** \( \triangle ABC \cong \triangle XYZ \)
   - **Prove:** \( \triangle ABC \cong \triangle XYZ \)
   - **Statements**
     - 1. \( AB = XY \)
     - 2. \( AC = XZ \)
     - 3. \( BC = YZ \)
   - **Reasons**
     - 1. Given
     - 2. Given
     - 3. Given

2. \( \triangle RST \cong \triangle UTS \)
   - **Given:** \( RS = UT \), \( RT = US \)
   - **Prove:** \( \triangle RST \cong \triangle UTS \)
   - **Statements**
     - 1. \( RS = UT \)
     - 2. \( RT = US \)
     - 3. \( ST = TS \)
   - **Reasons**
     - 1. Given
     - 2. Given
     - 3. Reflexive Property of Congruence

---

**Example**

For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.

**Exercises**

For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.

1. \( \triangle TRU \equiv \triangle PMN \) by the SAS Postulate.
2. \( \angle XQY \) and \( \angle WQZ \) are not the included angles for the congruent segments. The triangles are not congruent by the SAS Postulate.
3. \( \angle MPL \equiv \angle NPL \) because both are right angles. \( \triangle MPL \equiv \triangle NPL \) by the SAS Postulate.
4. The triangles cannot be proved congruent by the SAS Postulate.
5. \( \angle D \equiv \angle B \) because both are right angles. The two triangles are congruent by the SAS Postulate.
6. The congruent angles are the included angles for the congruent sides. \( \triangle FJH \equiv \triangle GHJ \) by the SAS Postulate.
4-4 Skills Practice

Proving Congruence—SSS, SAS

Determine whether \( \triangle ABC \equiv \triangle KLM \) given the coordinates of the vertices. Explain.

1. \( A(-3, 3), B(-1, 3), C(-3, 1), K(1, 4), L(3, 4), M(1, 6) \)
   \[ AB = 2, KL = 2, BC = 2\sqrt{2}, LM = 2\sqrt{2}, AC = 2, KM = 2. \]
   The corresponding sides have the same measure and are congruent, so \( \triangle ABC \equiv \triangle KLM \) by SSS.

2. \( A(-4, -2), B(-4, 1), C(1, 1), K(0, -2), L(0, 1), M(4, 1) \)
   \[ AB = 3, KL = 3, BC = \sqrt{13}, LM = 4, AC = \sqrt{10}, KM = 5. \]
   The corresponding sides are not congruent, so \( \triangle ABC \) is not congruent to \( \triangle KLM \).

3. Write a flow proof.
   Given: \( PR = DE, PT = DF \)
   \( \angle R = \angle E, \angle T = \angle F \)
   Prove: \( \triangle PRT \equiv \triangle DEF \)
   
   **Proof:**
   
   \[ \begin{align*}
   \overline{PR} & \equiv \overline{DE} \quad \text{Given} \\
   \angle R & \equiv \angle E \quad \text{Given} \\
   \angle T & \equiv \angle F \quad \text{Given} \\
   \angle P & \equiv \angle D \quad \text{Third Angle Theorem} \\
   \end{align*} \]

   \( \triangle PRT \equiv \triangle DEF \quad \text{SAS} \)

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write not possible.

4. SSS
5. SAS or SSS
6. not possible

4-4 Practice (Average)

Proving Congruence—SSS, SAS

Determine whether \( \triangle DEF \equiv \triangle PQR \) given the coordinates of the vertices. Explain.

1. \( D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0) \)
   \[ DE = 5\sqrt{2}, PQ = 5\sqrt{3}, EF = 2\sqrt{10}, QR = 2\sqrt{10}, DF = 5\sqrt{2}, PR = 5\sqrt{2}. \]
   \( \triangle DEF \equiv \triangle POR \) by SSS since corresponding sides have the same measure and are congruent.

2. \( D(-7, -3), E(-4, -1), F(-2, -5), P(2, -2), Q(5, -4), R(0, -5) \)
   \[ DE = \sqrt{13}, PQ = \sqrt{13}, EF = 2\sqrt{5}, QR = \sqrt{26}, DF = 2\sqrt{5}, PR = \sqrt{13}. \]
   Corresponding sides are not congruent, so \( \triangle DEF \) is not congruent to \( \triangle POR \).

3. Write a flow proof.
   Given: \( RS = TS \)
   \( V \) is the midpoint of \( RT \).
   Prove: \( \triangle RSV \equiv \triangle TSV \)
   
   **Proof:**
   
   \[ \begin{align*}
   RS & \equiv TS \quad \text{Given} \\
   V & \text{is the midpoint of } RT \quad \text{Definition of midpoint} \\
   \end{align*} \]

   \( \triangle RSV \equiv \triangle TSV \quad \text{SSS} \)

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write not possible.

4. not possible
5. SAS or SSS
6. SSS

7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths \( AB' \) and \( AB \) are equal?
   
   Since \( \angle ACB \) and \( \angle A'C'B' \) are vertical angles, they are congruent. In the figure, \( \angle AC \equiv \angle A'C \) and \( BC \equiv B'C \). So \( \triangle ABC \equiv \triangle A'B'C \) by SAS. By CPCTC, the lengths \( A'B' \) and \( AB \) are equal.
4-4 Proving Congruence—SSS, SAS

Pre-Activity How do land surveyors use congruent triangles?

Read the introduction to Lesson 4-4 at the top of page 200 in your textbook. Why do you think that land surveyors would use congruent right triangles rather than other congruent triangles to establish property boundaries?

Sample answer: Land is usually divided into rectangular lots, so their boundaries meet at right angles.

Reading the Lesson

1. Refer to the figure.
   a. Name the sides of \( \triangle LMN \) for which \( \angle L \) is the included angle.
      \( LM, LN \)
   b. Name the sides of \( \triangle LMN \) for which \( \angle N \) is the included angle.
      \( NL, NM \)
   c. Name the sides of \( \triangle LMN \) for which \( \angle M \) is the included angle.
      \( ML, MN \)

2. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate that you would use. If not, write not possible.
   a. \( \triangle ABD \cong \triangle CBD \); SAS
   b. not possible
   c. \( EF \) and \( DG \) bisect each other.
   d. \( \triangle DEF \cong \triangle GHF \); SAS
      \( \triangle RSU \cong \triangle TSU \); SSS

Helping You Remember

3. Find three words that explain what it means to say that two triangles are congruent and that can help you recall the meaning of the SSS Postulate.

Sample answer: Congruent triangles are triangles that are the same size and shape, and the SSS Postulate ensures that two triangles with three corresponding sides congruent will be the same size and shape.

4-4 Enrichment

Congruent Parts of Regular Polygonal Regions

Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.

1. Divide each square into four congruent parts. Use three different ways. Sample answers are shown.

2. Divide each pentagon into five congruent parts. Use three different ways. Sample answers are shown.

3. Divide each hexagon into six congruent parts. Use three different ways. Sample answers are shown.

4. What hints might you give another student who is trying to divide figures like those into congruent parts? See students’ work.
Study Guide and Intervention

Proving Congruence—ASA, AAS

ASA Postulate  The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example

Find the missing congruent parts so that the triangles can be proved congruent by the ASA Postulate. Then write the triangle congruence.

a. Two pairs of corresponding angles are congruent, \( \angle A = \angle D \) and \( \angle C = \angle F \). If the included sides \( AC \) and \( DF \) are congruent, then \( \triangle ABC \cong \triangle DEF \) by the ASA Postulate.

b. \( \angle R = \angle Y \) and \( SR = XY \). If \( \angle S = \angle X \), then \( \triangle RST \cong \triangle YXW \) by the ASA Postulate.

Exercises

What corresponding parts must be congruent in order to prove that the triangles are congruent by the ASA Postulate? Write the triangle congruence statement.

1. \( \overline{DC} \cong \overline{BC} \); \( \triangle CDE \cong \triangle CBA \)

2. \( \overline{WY} \cong \overline{WY} \); \( \angle XYW = \angle ZYW \); \( \overline{WXY} \cong \overline{WZY} \)

3. \( \overline{ABE} \cong \overline{CBD} \); \( \overline{ABE} \cong \overline{CBD} \)

4. \( \overline{BD} \cong \overline{DB} \); \( \angle ADB \cong \angle CDB \); \( \overline{ABD} \cong \overline{CDB} \)

5. \( \overline{ST} \cong \overline{VT} \); \( \angle RST \cong \angle UVT \)

6. \( \angle ACB \cong \angle E \); \( \triangle ABC \cong \triangle CDE \)

4-5 Study Guide and Intervention

Proving Congruence—ASA, AAS

AAS Theorem  Another way to show that two triangles are congruent is the Angle-Side-Angle (AAS) Theorem.

AAS Theorem

If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

Example

In the diagram, \( \angle BCA = \angle DCA \). Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Postulate?

\( AC \cong AC \) by the Reflexive Property of congruence. The congruent angles cannot be \( \angle 1 \) and \( \angle 2 \), because \( AC \) would be the included side. If \( \angle B = \angle D \), then \( \triangle ABC \cong \triangle ADC \) by the AAS Theorem.

Exercises

In Exercises 1 and 2, draw and label \( \triangle ABC \) and \( \triangle DEF \). Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem.

1. \( \angle A = \angle D; \angle A = \angle D \)

2. \( \angle B = \angle E; \angle B = \angle E \)

If \( \overline{BC} \cong \overline{EF} \) (or if \( \overline{AC} \cong \overline{DF} \), then \( \triangle ABC \cong \triangle DEF \) by the AAS Theorem.

3. Write a flow proof.

Given: \( \angle S \cong \angle U; TR \) bisects \( \overline{STU} \).

Prove: \( \angle SRT \cong \angle URT \)

\( \overline{TR} \) bisects \( \overline{STU} \)

Given

\( \angle S = \angle U \)

Detail: \( \angle S \cong \angle U \)

\( \angle SRT \cong \angle URT \)

AAS

\( \overline{TS} \cong \overline{UT} \)

CPCTC

\( \overline{RT} \cong \overline{RT} \)

Reflex Prop. of \( \cong \)
Write a flow proof.

1. Given: \( \angle N = \angle L \)
   \( JK = MK \)
   Prove: \( \triangle KN = \triangle MKL \)

   \begin{align*}
   \angle N &= \angle L \\
   JK &= MK \\
   \triangle KN &= \triangle MKL \\
   \text{Given} \\
   \text{Vertical & congruent} \\
   \end{align*}

2. Given: \( \overline{AB} = \overline{CB} \)
   \( \angle A = \angle C \)
   \( \overline{DB} \) bisects \( \angle ABC \).
   Prove: \( \overline{AD} = \overline{CD} \)

   \begin{align*}
   \overline{AB} &= \overline{CB} \\
   \angle A &= \angle C \\
   \triangle ABD &= \triangle CBD \\
   \text{Given} \\
   \text{Def. of bisector} \\
   \end{align*}

3. Write a paragraph proof.
   Given: \( \overline{DE} \parallel \overline{FG} \)
   \( \angle E = \angle G \)
   Prove: \( \triangle DFG = \triangle FDE \)

   Proof: Since it is given that \( \overline{DE} \parallel \overline{FG} \), it follows that \( \angle EDF = \angle FDE \). Because alt. int. \( \Delta \) are \( \cong \), it is given that \( \angle E = \angle G \). By the Reflexive Property, \( \overline{DF} = \overline{FD} \). So \( \triangle DFG = \triangle FDE \) by AAS.
**Answers (Lesson 4-5)**

**Reading to Learn Mathematics**

**How are congruent triangles used in construction?**

Congruent Triangles in the Coordinate Plane

Read the introduction to Lesson 4-5 at the top of page 207 in your textbook.

Which of the triangles in the photograph in your textbook appear to be congruent? There may be more than one way to do this.

1. Consider \(\triangle ABD\) and \(\triangle CDB\).

2. Consider \(\triangle PQR\) and \(\triangle KLM\) whose vertices are the following points.

3. If you know the coordinates of all the vertices of two triangles, is it always possible to tell whether the triangles are congruent? Explain.

4. Yes, you can use the Distance Formula and SSS.

**Helping You Remember**

A good way to remember mathematical ideas is to summarize them in a general statement. If you want to prove triangles congruent by using three pairs of corresponding parts, what is a good way to remember which combinations of parts will work?

Sample answer: At least one pair of corresponding parts must be sides. If you use two pairs of corresponding sides and the angles between the pairs, then the sides must both be included by the angles or must both be nonincluded sides.

Sample answer: Show that the slopes of \(AB\) and \(CD\) are equal and that the slopes of \(AD\) and \(BG\) are equal. Conclude that \(\triangle ABG\) and \(\triangle CDG\) are congruent by SSA. Yes, you can use the Distance Formula and SSS.

Sample answer: In ASA, you use two pairs of congruent angles and the included side. In AAS, you use two pairs of congruent angles and a nonincluded side.
Study Guide and Intervention

**Isosceles Triangles**

Properties of Isosceles Triangles
An isosceles triangle has two congruent sides. The angle formed by these sides is called the vertex angle. The other two angles are called base angles. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. *(Isosceles Triangle Theorem)*
  - If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.
  - If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

**Example 1**
Find $x$.

**Example 2**
Find $x$.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x-13=2x$</td>
<td>$3x=2x+13$</td>
</tr>
<tr>
<td>$x=13$</td>
<td>Add 13 to each side.</td>
</tr>
</tbody>
</table>

**Exercises**

Find $x$.

1. $x = 35$
2. $x = 12$
3. $x = 15$
4. $x = 12$
5. $x = 20$
6. $x = 36$

7. Write a two-column proof.
   **Given:** $\angle 1 \cong \angle 2$
   **Prove:** $\overline{AB} \cong \overline{CB}$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle 1 \cong \angle 2$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle 2 \cong \angle 3$</td>
<td>Vertical angles are congruent.</td>
</tr>
<tr>
<td>$\angle 1 \cong \angle 3$</td>
<td>Transitive Property of $\cong$</td>
</tr>
<tr>
<td>$\overline{AB} \cong \overline{CB}$</td>
<td>If two angles of a triangle are $\cong$, then the sides opposite the angles are $\cong$.</td>
</tr>
</tbody>
</table>

**Isosceles Triangle Theorem**

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures $60^\circ$.

**Example**
Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.

**Proof:**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$</td>
<td>$\triangle ABC$ is equilateral; $\overline{PQ} \parallel \overline{BC}$</td>
</tr>
<tr>
<td>$m\angle A = m\angle B = m\angle C = 60^\circ$</td>
<td>$m\angle A = m\angle B = m\angle C = 60^\circ$</td>
</tr>
<tr>
<td>$\angle 1 \cong \angle 2 \cong \angle 3$</td>
<td>If $\parallel$ lines, then corres. $\angle$s are $\cong$.</td>
</tr>
<tr>
<td>$m\angle 1 = 60^\circ, m\angle 2 = 60^\circ$</td>
<td>$4. $ $\triangle ABC$ is equilateral.</td>
</tr>
<tr>
<td>$\overline{PQ}$ is equilateral.</td>
<td>$5. $ If a $\triangle$ is equiangular, then it is equilateral.</td>
</tr>
</tbody>
</table>

**Exercises**

Find $x$.

1. $x = 10$
2. $x = 5$
3. $x = 10$
4. $x = 10$
5. $x = 12$
6. $x = 15$
7. Write a two-column proof.
   **Given:** $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$
   **Prove:** $\angle ADB \cong \angle CDB$

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\triangle ABC$ is equilateral.</td>
<td>$\triangle ABC$ is equilateral.</td>
</tr>
<tr>
<td>$\overline{AB} \cong \overline{CB} \cong \angle A \cong \angle C$</td>
<td>$\overline{AB} \cong \overline{CB} \cong \angle A \cong \angle C$</td>
</tr>
<tr>
<td>$\angle 1 \cong \angle 2$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle ADB \cong \angle CBD$</td>
<td>$4. $ $\triangle ABD \cong \triangle CBD$</td>
</tr>
<tr>
<td>$\angle ADB \cong \angle CDB$</td>
<td>$5. $ CPCTC</td>
</tr>
</tbody>
</table>

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Glencoe Geometry
Isosceles Triangles

Refer to the figure.

1. If \( \overline{AC} \cong \overline{AD} \), name two congruent angles.
   \( \angle ACD \cong \angle CDA \)

2. If \( \overline{BE} = \overline{BC} \), name two congruent angles.
   \( \angle BEC \cong \angle BCE \)

3. If \( \angle ERA = \angle EAB \), name two congruent segments.
   \( EB = EA \)

4. If \( \angle ECD = \angle CDE \), name two congruent segments.
   \( CE = CD \)

\( \triangle ABF \) is isosceles, \( \triangle CDF \) is equilateral, and \( m\angle AFD = 150^\circ \). Find each measure.

5. \( m\angle CFD = 60^\circ \)
6. \( m\angle AFB = 55^\circ \)
7. \( m\angle ABF = 70^\circ \)
8. \( m\angle A = 55^\circ \)
9. \( m\angle JLM = 81 \)
10. \( m\angle HMG = 70 \)
11. \( m\angle GHM = 40 \)

In the figure, \( PL = RL \) and \( LR = RR \).
9. If \( m\angle LRP = 100 \), find \( m\angle BRL = 20 \)
10. If \( m\angle LPR = 34 \), find \( m\angle B = 68 \)
11. Write a two-column proof.

   Given: \( \overline{CD} = \overline{CG} \)
   \( \overline{DE} = \overline{GF} \)

   Prove: \( \overline{CE} = \overline{CF} \)

   Proof:

<table>
<thead>
<tr>
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<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{CD} = \overline{CG} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle D = \angle G )</td>
<td>2. If 2 sides of a ( \triangle ) are ( \cong ), then the ( \triangle ) opposite those sides are ( \cong ).</td>
</tr>
<tr>
<td>3. ( \overline{DE} = \overline{GF} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overline{CDE} = \overline{CGF} )</td>
<td>4. SAS</td>
</tr>
<tr>
<td>5. ( \overline{CE} = \overline{CF} )</td>
<td>5. CPCTC</td>
</tr>
</tbody>
</table>

11. **SPORTS** A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle. 81, 81
4-6 Reading to Learn Mathematics

Isosceles Triangles

Pre-Activity How are triangles used in art?
Read the introduction to Lesson 4-6 at the top of page 216 in your textbook.

- Why do you think that isosceles and equilateral triangles are used more often than scalene triangles in art? Sample answer: Their symmetry is pleasing to the eye.
- Why might isosceles right triangles be used in art? Sample answer: Two congruent isosceles right triangles can be placed together to form a square.

Reading the Lesson
1. Refer to the figure.
   a. What kind of triangle is \( \triangle PQS \)? isosceles
   b. Name the legs of \( \triangle PQS \). \( PQ, QS \)
   c. Name the base of \( \triangle PQS \). \( Q \)
   d. Name the vertex angle of \( \triangle PQS \). \( \angle S \)
   e. Name the base angles of \( \triangle PQS \). \( \angle Q, \angle R \)

2. Determine whether each statement is always, sometimes, or never true.
   a. If a triangle has three congruent sides, then it has three congruent angles. always
   b. If a triangle is isosceles, then it is equilateral. sometimes
   c. If a right triangle is isosceles, then it is equilateral. never
   d. The largest angle of an isosceles triangle is obtuse. sometimes
   e. If a right triangle has a 45° angle, then it is isosceles. always
   f. If an isosceles triangle has three acute angles, then it is equilateral. sometimes
   g. The vertex angle of an isosceles triangle is the largest angle of the triangle. sometimes

3. Give the measures of the three angles of each triangle.
   a. an equilateral triangle 60, 60, 60
   b. an isosceles right triangle 45, 45, 90
   c. an isosceles triangle in which the measure of the vertex angle is 70 70, 55, 55
   d. an isosceles triangle in which the measure of a base angle is 70 70, 70, 40
   e. an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles 90, 45, 45

Helping You Remember
4. If a theorem and its converse are both true, you can often remember them most easily by combining them into an “if-and-only-if” statement. Write such a statement for the Isosceles Triangle Theorem and its converse. Sample answer: Two sides of a triangle are congruent if and only if the angles opposite those sides are congruent.
Study Guide and Intervention

**Triangles and Coordinate Proof**

Position and Label Triangles  A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines:

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make the computations as simple as possible.

Example  Position an equilateral triangle on the coordinate plane so that its sides are $a$ units long and one side is on the positive $x$-axis. Start with $R(0, 0)$. If $RT$ is $a$, then another vertex is $T(a, 0)$. For vertex $S$, the $x$-coordinate is $\frac{a}{2}$. Use $b$ for the $y$-coordinate, so the vertex is $S(\frac{a}{2}, b)$.

Exercises

1. Find the missing coordinates of each triangle.
   - $C(p, q)$
   - $T(2a, 2a)$
   - $E(-2g, 0)$, $F(0, b)$

2. Position and label each triangle on the coordinate plane.
   - 4. isosceles triangle $\triangle RST$ with base $RS$
     - $4a$ units long
   - 5. isosceles right $\triangle DEF$ with legs $a$ units long
   - 6. equilateral triangle $\triangle EFG$ with vertex $G(0, a)$ and sides $2b$ units long

Sample answers are given.

Write Coordinate Proofs  Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

Example  Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.

First, position and label an isosceles triangle on the coordinate plane. One way is to use $T(a, 0)$, $R(-a, 0)$, and $S(0, c)$. Then $U(0, 0)$ is the midpoint of $RT$.

Given: Isosceles $\triangle RST$; $U$ is the midpoint of base $RT$.

Proof: $SU \perp RT$

Proof:

$U$ is the midpoint of $RT$ so the coordinates of $U$ are $\left(-\frac{a + a}{2}, \frac{0 + 0}{2}\right) = (0, 0)$. Thus $SU$ lies on the $y$-axis, and $\triangle RST$ was placed so $RT$ lies on the $x$-axis. The axes are perpendicular, so $SU \perp RT$.

Exercises

1. Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.

Sample answer: Position and label right $\triangle ABC$ with the coordinates $A(0, 0)$, $B(0, 2b)$, and $C(2a, 0)$.

The midpoint $P$ of $BC$ is $\left(\frac{0 + 2a}{2}, \frac{2b + 0}{2}\right) = (a, b)$.

The midpoint $Q$ of $AC$ is $\left(\frac{0 + 2a}{2}, \frac{0 + 0}{2}\right) = (a, 0)$.

The midpoint $R$ of $AB$ is $\left(\frac{0 + 0}{2}, \frac{0 + 2b}{2}\right) = (0, b)$.

The slope of $\overline{RP}$ is $\frac{b - b}{a - 0} = 0$, so the segment is horizontal.

The slope of $\overline{PQ}$ is $\frac{b - 0}{a - a} = 0$ which is undefined, so the segment is vertical.

$\triangle RQP$ is a right angle because any horizontal line is perpendicular to any vertical line. $\triangle PRQ$ has a right angle, so $\triangle PRQ$ is a right triangle.
Skills Practice

4-7 Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

1. Right \( \triangle FGH \) with legs \( a \) units and \( b \) units
2. Isosceles \( \triangle KLP \) with base \( KP \) 66 units long
3. Isosceles \( \triangle AND \) with base \( AD \) 5a long

Find the missing coordinates of each triangle.

4. \( A(0, 2a) \)
5. \( Z(b, c) \)
6. \( M(0, c) \)
7. \( Q(4a, 0) \)
8. \( R(\frac{7}{2}, b, c) \)
9. \( T(0, b) \)

10. Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

   Given: Isosceles right \( \triangle ABC \) with \( \angle ABC \) the right angle and \( M \) the midpoint of \( \overline{AC} \)

   Prove: \( \overline{BM} \perp \overline{AC} \)

   Proof:

   The Midpoint Formula shows that the coordinates of \( M \) are \( \left( \frac{0 + 2a}{2}, \frac{2a + 0}{2} \right) \) or \((a, a)\). The slope of \( \overline{AC} \) is \( \frac{2a - 0}{0 - 2a} = -1 \). The slope of \( \overline{BM} \) is \( \frac{a - 0}{a - 0} = 1 \). The product of the slopes is \(-1\), so \( \overline{BM} \perp \overline{AC} \).

Practice (Average)

4-7 Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

1. Equilateral \( \triangle SWY \) with sides 4 \( \frac{1}{2} \) a long
2. Isosceles \( \triangle BLP \) with base \( BL \) 3b units long
3. Isosceles right \( \triangle DGF \) with hypotenuse \( \overline{DJ} \) and legs 2a units long

Find the missing coordinates of each triangle.

4. \( S(1\frac{1}{2}, b, c) \)
5. \( C(3a, 0), E(0, c) \)
6. \( M(0, c), N(-2b, 0) \)

NEIGHBORHOODS For Exercises 7 and 8, use the following information.

Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Write a coordinate proof to prove that Karina's high school, her home, and the mall are at the vertices of a right triangle.

   Given: \( \triangle SKM \)

   Prove: \( \triangle SKM \) is a right triangle.

   Proof:

   Slope of \( \overline{SK} = \frac{4 - 0}{6 - 0} = \frac{2}{3} \)

   Slope of \( \overline{SM} = \frac{-3 - 0}{-2 - 0} = \frac{3}{2} \)

   Since the slope of \( \overline{SM} \) is the negative reciprocal of the slope of \( \overline{SK} \), \( \overline{SM} \perp \overline{SK} \). Therefore, \( \triangle SKM \) is right triangle.

8. Find the distance between the mall and Karina's home.

   \[ KM = \sqrt{(2 - 6)^2 + (3 - 4)^2} = \sqrt{64 + 1} = \sqrt{65} \approx 8.1 \text{ miles} \]
How Many Triangles?

How many triangles are there in each figure?

1. 5
2. 8
3. 5
4. 13
5. 6

Some triangles overlap other triangles.

1. How many triangles do you know about in the drawing in your textbook?

2. How many triangles can you form by joining points on each circle?

List the vertices of each triangle.

1. A, B, C
2. D, E, F
3. G, H, I
4. J, K, L
5. M, N, O

How Many Triangles?

The following are the vertices of each triangle.

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